CSCI 2824-Spring 2015: In-class exercise #3
Date: Wednesday April 1, 2015

Last name: First name: Student ID:

(1) How many bit strings of length between 1 and 4 are there?

♠ Here is the reasoning:

- Let a_k be the number of bit strings of length k.
- Then $a_k = 2^k$.
- Hence the answer is $a_1 + a_2 + a_3 + a_4 = 2^1 + 2^2 + 2^3 + 2^4 = 2(2^4 - 1) = 30$.

(2) How many bit strings of length 5 begin with one 1 and end with one 0?

♠ Only 3 out of the 5 bits are up to us to choose. So the answer is $2^3 = 8$.

(3) How many bit strings of length 5 either begin with two 1's or end with two 1's?

♠ Using the inclusion-exclusion principle, the answer is:

(# len-5 bit str starting with two 1's)+ (# len-5 bit str ending with two 1's)− (# len-5 bit str starting and ending with two 1's)

= (# len-3 bit str)+ (# len-3 bit str)- (# len-1 bit str)

$= 2^3 + 2^3 - 2 = 14$.

(4) How many strings of four decimal digits do not contain the same digit four times?

♠ Using the complement rule, the answer is:

(# len-4 decimal str) − (# len-4 decimal str containing the same digits four times)

$= 10^4 - 10 = 9990$.

(5) Among the integers between 1 and 100,000, how many integers are multiples of 3 or of 4?

♠ Let A be the set of all multiples of 3 between 1 and 100000, and B be the set of all multiples of 3 between 1 and 100000. Then $A \cap B$ is the set of all multiples of 12 between 1 and 100000, and

$$\begin{aligned}
A &= \{3n \mid 1 \le 3n \le 100000 \ \wedge \ n \in \mathbb{Z}\} \\
&= \left\{3n \mid \frac{1}{3} \le n \le \frac{100000}{3} \ \wedge \ n \in \mathbb{Z}\right\} \\
&= \left\{3n \mid \left\lceil \frac{1}{3} \right\rceil \le n \le \left\lfloor \frac{100000}{3} \right\rfloor \ \wedge \ n \in \mathbb{Z}\right\}, \\
B &= \{4n \mid 1 \le 4n \le 100000 \ \wedge \ n \in \mathbb{Z}\} \\
&= \left\{4n \mid \frac{1}{4} \le n \le \frac{100000}{4} \ \wedge \ n \in \mathbb{Z}\right\} \\
&= \left\{4n \mid \left\lceil \frac{1}{4} \right\rceil \le n \le 25000 \ \wedge \ n \in \mathbb{Z}\right\}, \\
A \cap B &= \{12n \mid 1 \le 12n \le 100000 \ \wedge \ n \in \mathbb{Z}\} \\
&= \left\{12n \mid \frac{1}{12} \le n \le \frac{100000}{12} \ \wedge \ n \in \mathbb{Z}\right\} \\
&= \left\{12n \mid \left\lceil \frac{1}{12} \right\rceil \le n \le \left\lfloor \frac{100000}{12} \right\rfloor \ \wedge \ n \in \mathbb{Z}\right\}.
\end{aligned}$$

This means that $|A| = \left\lfloor \frac{100000}{3} \right\rfloor = 33333$, $|B| = 25000$ and $|A \cap B| = \left\lfloor \frac{100000}{12} \right\rfloor = 8333$. Hence

$$|A \cup B| = |A| + |B| - |A \cap B| = 50000.$$

(6) How many strings of four decimal digits contain 0 at least once?

♠ Using the complement rule, the answer is:

$$\begin{aligned}
&(\#\text{ len-4 decimal str}) - (\#\text{ len-4 decimal str not containing 0}) \\
&= 10^4 - 9^4 = 3439.
\end{aligned}$$

(7) How many distinct arrangements of the letters in AABBA are there?

♠ We can think of counting the number of distinct 2-combinations from $\{1, 2, 3, 4, 5\}$, wherein each 2-combination gives the position of the two B's. Hence the answer is $C(5, 2) = 10$.

(8) How many bit strings of length 8 have three 1's, no two of which are adjacent?

♠ We can think of counting the number of distinct 3-combinations from $\{1, 2, 3, 4, 5, 6\}$. Each such 3-combination $\{a_1, a_2, a_3\}$ can give us a length-8 bit string satisfying the requirement: assuming that $a_1 < a_2 < a_3$, the corresponding bit string has the three 1's in the positions a_1, $a_2 + 1$ and $a_3 + 1$. Note that this method gives us a one-to-one correspondence (i.e., a bijective function) between the set of all 3-combinations and the set of all bit strings of length 8 with three 1's and no two of which are adjacent.

Therefore the answer is $C(6, 3) = 20$.

(9) In how many ways can a photographer at a wedding arrange 4 people in a row, including the bride and groom, so that the bride must be next to the groom?

♠ If we think of the bride and the groom as one entity, then there would be $2 \times 3! = 12$ ways. (The factor 2 is to account for the two possibilities that the bride can stand on the left or right of the groom.)

(10) We have three tee shirts. Two are red (indistinguishable) and one is violet. We have three people interested in these tee-shirts: Alice, Bob and Carol. How many possibilities ways are there of choosing a color for every person?

♠ Note that exactly one of three people gets the violet tee shirt. Once we fix the person who gets the violet tee shirt, the other two persons are "automatically" assigned the red tee shirts. Hence there are $C(3, 1) = 3$ ways.