

CSCI 2824-Spring 2014: Work sheet on induction.

P1 Let T_n be a recurrence defined as follows:

$$T_0 = 1, T_{n+1} = (n + 1) + \frac{1}{2}T_n, n \in \mathbb{N}.$$

We wish to prove that $T_n \geq 2n + 1$ for all $n \in \mathbb{N}$.

Proof: Proof is by weak induction on n .

Base Case:

(Write down and verify base case)

Answer

Ind. Hyp.

(Write down the statement of the ind. hyp.)

Answer

Proof of Ind. Hyp.

(prove the ind. hyp.)

Answer

P2 Let $F_n, n \geq 0$ be the Fibonacci series. **Theorem:** For all $n \in \mathbb{N}$, $\sum_{j=0}^n F_j = F_{n+2} - 1$.

Proof: Proof is by **weak induction** on n .

Base Case:

(Write down and verify base case)

Answer

Ind. Hyp.

(Write down the statement of the ind. hyp.)

Answer

Proof of Ind. Hyp.

(prove the ind. hyp.)

Answer

P3 Let F_n , $n \geq 0$ be the Fibonacci series.

Theorem: For all $n \in \mathbb{N}$, If $n \geq 2$ then $F_n \geq 1.2^n$.

Proof: Proof is by **strong induction** on n .

Base Case:

(Write down and verify base case)

Answer

Ind. Hyp.

(Write down the statement of the ind. hyp.)

Answer

Proof of Ind. Hyp.

(prove the ind. hyp.)

Answer