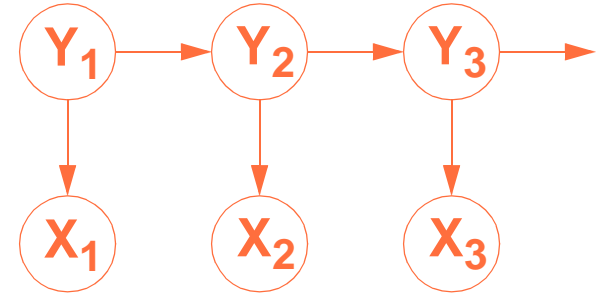


# Conditional Random Fields

# Motivation

## HMM

- generative model
- specifies  $P(X, Y)$
- can be used to compute  $P(X)$ ,  $P(Y|X)$ ,  $P(X|Y)$ , etc.



**What do we typically do with HMM (e.g., speech recognition)?**

**What independence assumptions does it make?**

- Given  $Y_3$ , what can we say about  $X_3$  and  $X_2$ ?
- Is this assumption sensible?

# Markov Random Fields

E.g., Image segmentation

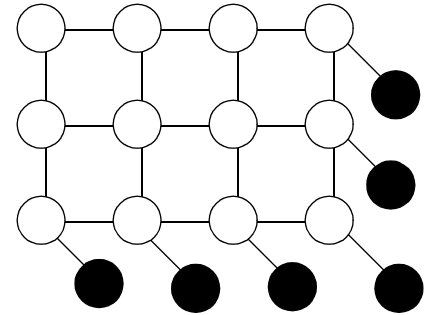
Undirected graphical model

Set of random variables,  $\{Y_i\}$

Contextual constraints (spatial, temporal) connect neighbors

Neighborhood relations define cliques

subsets of RVs in which every pair of distinct RVs are neighbors



# Directed Vs. Undirected Graphical Model

**Joint probability in *directed* graphical model:**

$$P(Y) = \prod_i P(Y_i \mid PA_i)$$

i: index over nodes

**Joint probability in undirected graphical model:**

$$P(Y) \sim \prod_c V_c(Y_c) = \exp( \sum_c \ln V_c(Y_c) )$$

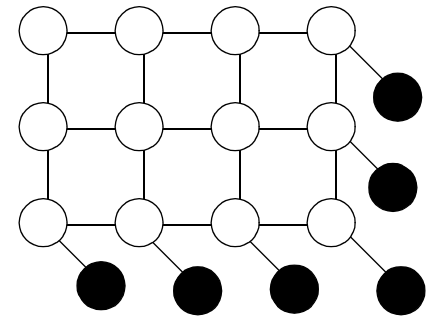
c: index over cliques

$Y_c$ : elements of  $Y$  in clique  $c$  (article uses  $Y|_c$ )

$V_c$ : potential function that depends on configuration of clique

For discrete RVs,  $V_c(Y_c) = \sum_{y_c} \lambda_{c,y_c} f_{y_c}(Y_c)$

'goodness' of configuration  
binary 'feature'  
(is  $Y_c = y_c$ ?)



# Conditional Random Field

**MRF specifies joint distribution on Y**

**For any probability distribution, you can condition it on some other variables X**

**CRF = MRF conditioned on X**

MRF:  $P(Y_i | Y_j \text{ for all } j \neq i) = P(Y_i | N_i)$  where  $N_i$  are the neighbors of  $i$

CRF:  $P(Y_i | X, Y_j \text{ for all } j \neq i) = P(Y_i | X, N_i)$  where  $N_i$  are the neighbors of  $i$

$$P(Y|X) \sim \exp\left(\sum_{c, y_c} \lambda_{c, y_c} f_{y_c}(\{X, Y\}_c)\right)$$

# CRF For Sequential Data

## Framework

sequence of observations  $X = \{X_i \text{ for } i = 1 \dots n\}$

sequence of labels  $Y = \{Y_i \text{ for } i = 1 \dots n\}$

goal: infer  $Y$  given  $X$

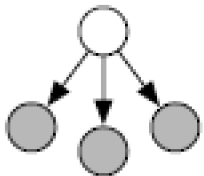
## Applications

speech recognition

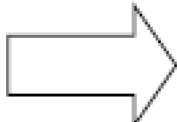
part of speech tagging

pretty much anything we use an HMM for, because it is typical to be given observation sequence  $X$

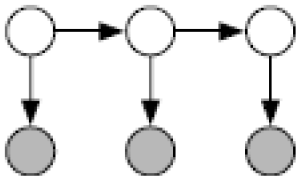
# Relationships Among Models



Naive Bayes



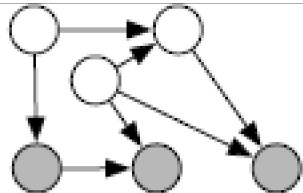
**SEQUENCE**



HMMs



**GENERAL GRAPHS**



Generative directed models



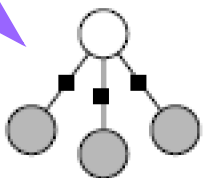
**CONDITIONAL**



**CONDITIONAL**



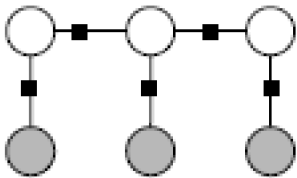
**CONDITIONAL**



Logistic Regression



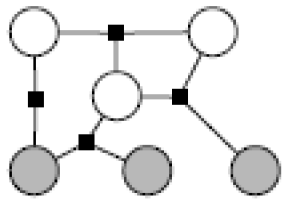
**SEQUENCE**



Linear-chain CRFs



**GENERAL GRAPHS**



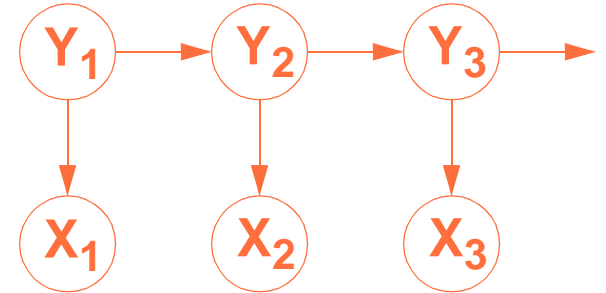
General CRFs

factor graph

# Relation To Other Sequential Models

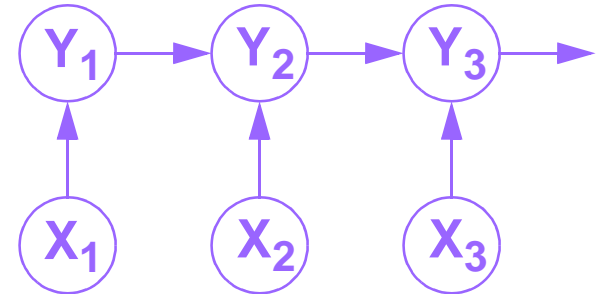
## HMM

- generative model
- assumes cond. independence of X's
- does generality matter for recognition problems?



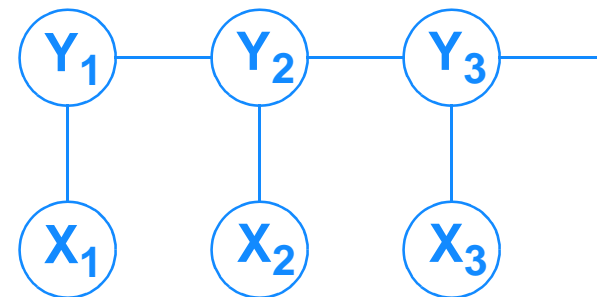
## MEMM (Maximum Entropy Markov Model)

- specifies  $P(Y|X)$
- does *not* require independence of X's
- many free parameters
- label bias problem



## CRF

- specifies  $P(Y|X)$
- does *not* require independence of X's
- fewer free parameters
- not subject to label bias problem

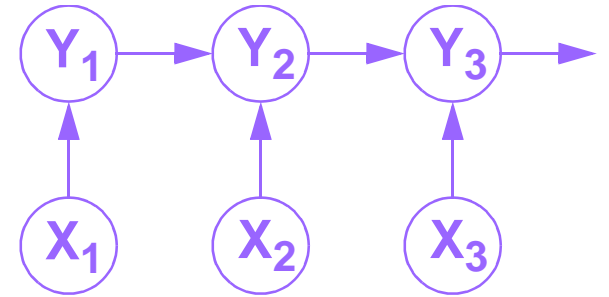




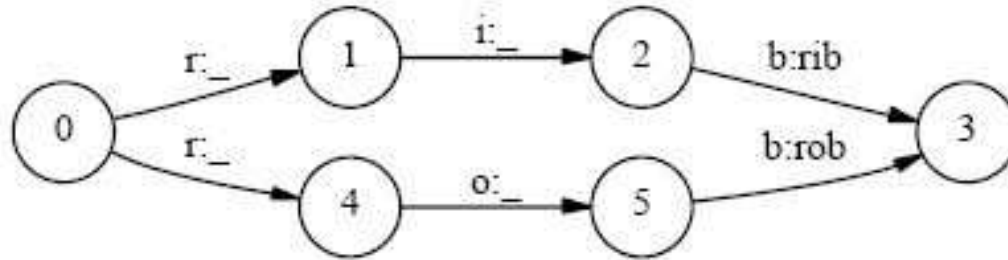
# Label Bias Problem

Consider  $P(Y_{i+1} | X_i, Y_i)$  in MEMM:

$$\sum_j P(Y_{i+1}=j | X_i, Y_i) = 1.$$



If only one value of  $Y_{i+1}$  is possible given  $Y_i$ , evidence is ignored.



## Claim

Less robust to inaccurate modeling assumptions than CRF

# Training Objective

## HMM

Given observation sequence  $\{X_1, X_2, \dots, X_N\}$

Search for model parameters that maximize the likelihood of the observations.

$$L(\theta|X) = \prod_{i=1}^N P(X_i|\theta)$$

## CRF

Given an observation sequence  $\{(X_1, Y_1), (X_2, Y_2), \dots, (X_N, Y_N)\}$

Search for model parameters that maximize the likelihood the conditional sequence

$$L(\theta|X, Y) = \prod_{i=1}^N P(Y_i|\theta, X_i)$$

# Training Procedure

Algorithm exactly analogous to training procedure for HMM

1. Run forward-backward algorithm to obtain  $P(Y_i|X,\theta)$
2. Adjust  $\theta$  such that the inferred  $Y_i$  better match the training states

# Simulation Studies

## Part-of-speech tagging

label each word in English sentence with one of 45 syntactic tags

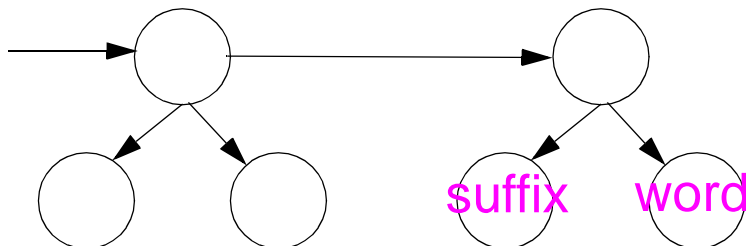
results for first-order HMM, MEMM, CRF in first 3 rows

<i>model</i>	<i>error</i>	<i>oov error</i>
HMM	5.69%	45.99%
MEMM	6.37%	54.61%
CRF	5.55%	48.05%
MEMM <sup>+</sup>	4.81%	26.99%
CRF <sup>+</sup>	4.27%	23.76%

<sup>+</sup>Using spelling features

addition of orthographic features: begins with a number or upper case letter, contains a hyphen, whether it ends in specific suffix (-ing, -ogy, -ed, -s, -ly, etc.)

additional features rely on conditional *nonindependence*, vs. HMM



**STOP HERE**

# Simulation Studies

## Label bias problem

CRF error 4.6%, MEMM error is 42%

## Synthetic data

from mixture of first- and second-order models

We generate data from a mixed-order HMM with state transition probabilities given by  $p_\alpha(\mathbf{y}_i | \mathbf{y}_{i-1}, \mathbf{y}_{i-2}) = \alpha p_2(\mathbf{y}_i | \mathbf{y}_{i-1}, \mathbf{y}_{i-2}) + (1 - \alpha) p_1(\mathbf{y}_i | \mathbf{y}_{i-1})$  and, similarly, emission probabilities given by  $p_\alpha(\mathbf{x}_i | \mathbf{y}_i, \mathbf{x}_{i-1}) = \alpha p_2(\mathbf{x}_i | \mathbf{y}_i, \mathbf{x}_{i-1}) + (1 - \alpha) p_1(\mathbf{x}_i | \mathbf{y}_i)$ .

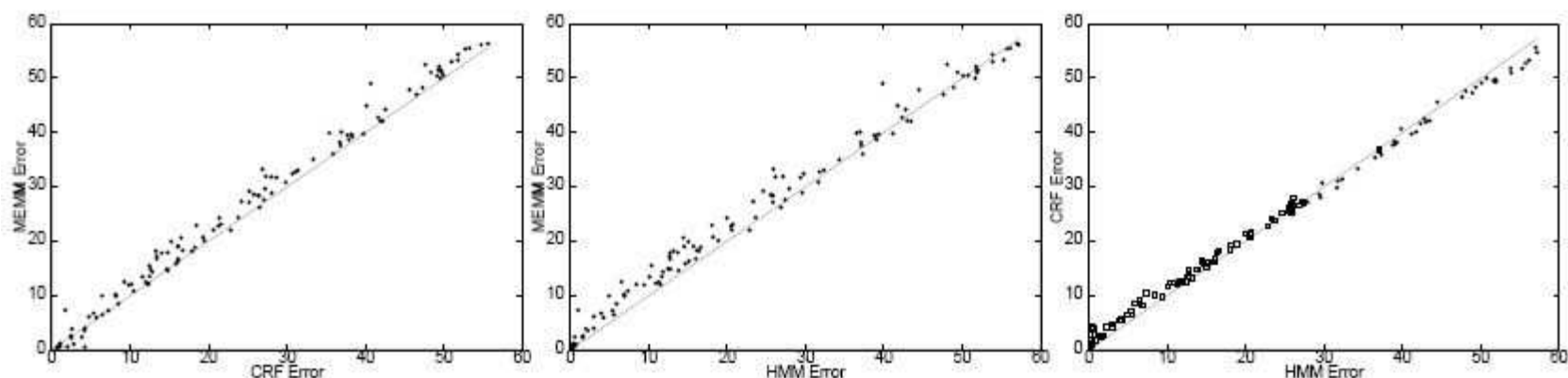


Figure 3. Plots of  $2 \times 2$  error rates for HMMs, CRFs, and MEMMs on randomly generated synthetic data sets, as described in Section 5.2. As the data becomes “more second order,” the error rates of the test models increase. As shown in the left plot, the CRF typically significantly outperforms the MEMM. The center plot shows that the HMM outperforms the MEMM. In the right plot, each open square represents a data set with  $\alpha < \frac{1}{2}$ , and a solid circle indicates a data set with  $\alpha \geq \frac{1}{2}$ . The plot shows that when the data is mostly second order ( $\alpha \geq \frac{1}{2}$ ), the discriminatively trained CRF typically outperforms the HMM. These experiments are not designed to demonstrate the advantages of the additional representational power of CRFs and MEMMs relative to HMMs.

# Lafferty, McCallum, & Pereira: Classic Paper Organization

## 1. existing approaches: HMM, MEMM

## 2. existing approaches have deficiencies

assumption of independence of observations

label bias problem

## 3. technique that overcomes the deficiencies: CRF

general case

special case (sequential structure)

## 4. algorithms for training CRF

## 5. simulations to show superiority of CRF over HMM, MEMM

label bias

mixed-order Markov model

part-of-speech tagging

# Decontaminating Human Judgments by Removing Sequential Dependencies

**Michael C. Mozer**  
University of Colorado

**Hal Pashler**  
UCSD

**Matt Wilder, Rob Lindsey, Matt Jones**  
University of Colorado

**Mike Jones**  
Indiana University



# Sequential Effects In Judgment

• **Primacy Effect** – First item in a list has the most influence on judgment

• **Recency Effect** – Last item in a list has the most influence on judgment

• **Order Effect** – The order of items in a list can influence judgment

• **Serial Position Effect** – The combination of primacy and recency effects

• **Recency Bias** – The tendency to remember the most recent information

• **Primacy Bias** – The tendency to remember the first information

• **Order Bias** – The tendency to judge items based on their order

• **Serial Position Bias** – The combination of primacy and recency biases

• **Recency Effect** – The last item in a list has the most influence on judgment

• **Primacy Effect** – The first item in a list has the most influence on judgment

• **Order Effect** – The order of items in a list can influence judgment

• **Serial Position Effect** – The combination of primacy and recency effects

• **Recency Bias** – The tendency to remember the most recent information

# Sequential Effects In Judgment

**On a 1-10 scale, make a moral judgment about the following actions, with 1 indicating 'not particularly bad or wrong', and 10 indicating 'extremely evil':**

- (1) Stealing a towel from a hotel
- (2) Keeping a dime you found on the ground
- (3) Poisoning a barking dog

# Sequential Effects In Judgment

**On a 1-10 scale, make a moral judgment about the following actions, with 1 indicating 'not particularly bad or wrong', and 10 indicating 'extremely evil':**

- (1) Stealing a towel from a hotel
- (2) Keeping a dime you found on the ground
- (3) Poisoning a barking dog

**Suppose instead the sequence had been:**

- (1') Testifying falsely for pay
- (2') Using guns on striking workers
- (3') Poisoning a barking dog

**Rating of action (3) is reliably higher than rating of action (3')  
(Parducci, 1968)**

# Sequential Effects In Judgment

Rate these movies on a 1-5 scale



## Netflix competition

anchoring effects (early vs. late in rating session)

slow drift

If ratings are contaminated by trial-to-trial sequence, can we *decontaminate* ratings to get scores that are more meaningfully related to an individual's internal sensation/impression/evaluation?

# Strategy

- 1. Collect data on a simple judgment task for which we have ground truth knowledge of the subjects' internal sensations**
- 2. Use half of the subjects (*training subjects*) to build statistical/probabilistic models of decontamination**
- 3. Evaluate ability of models to decontaminate on the other half of subjects (*test subjects*)**

# Sequential Effects: Cognitive Models Vs. Decontamination Models

## Cognitive model

Given past sequence of stimuli and responses, and current stimulus, predict current response (or response latency)

$$S(1), S(2), \dots S(t), R(1), R(2), \dots R(t-1) \Rightarrow R(t)$$

## Decontamination model

Given complete sequence of responses, predict complete sequence of sensations

$$R(1), R(2), \dots, R(T) \Rightarrow S(1), S(2), \dots, S(T)$$

# Gap Detection Task

On a 1-10 scale, judge how big the gap is between these two dots:



# Gap Detection Task

On a 1-10 scale, judge how big the gap is between these two dots:





# Gap Detection Task

On a 1-10 scale, judge how big the gap is between these two dots:



# Gap Detection Task

On a 1-10 scale, judge how big the gap is between these two dots:



**Absolute identification task (10 stimuli, 10 responses)**

**10 initial trials where subject is shown all 10 stimuli and is told the correct response**

**No further feedback**

**Like 1000's of similar studies in the literature, except without any feedback to make it more like Netflix rating task.**

# Experiments

## Experiment 1

180 trials, 2 blocks of 90 trials

Within each block, all pairs of  $\{\text{gap}(t-1), \text{gap}(t)\}$  presented once, excluding repetitions

Within each 10 trials of block, all gaps presented once

$\text{gap} = .08 K$  ( $K = 1, \dots, 10$ )

## Experiment 2

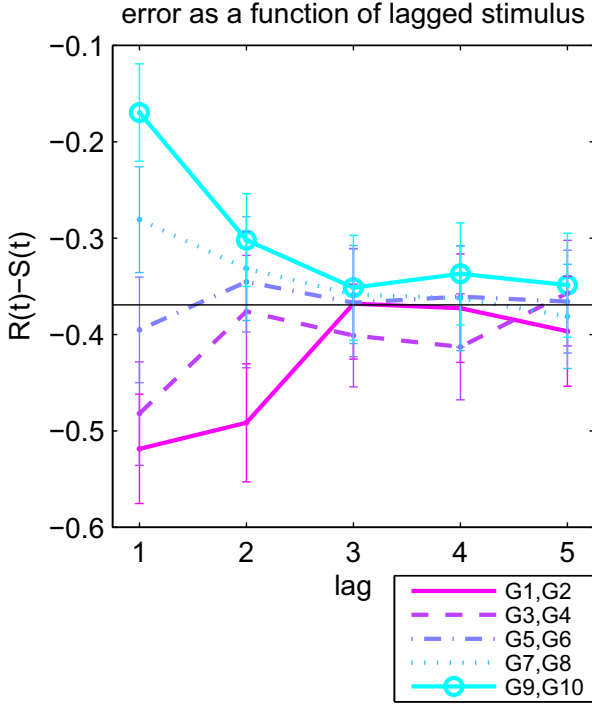
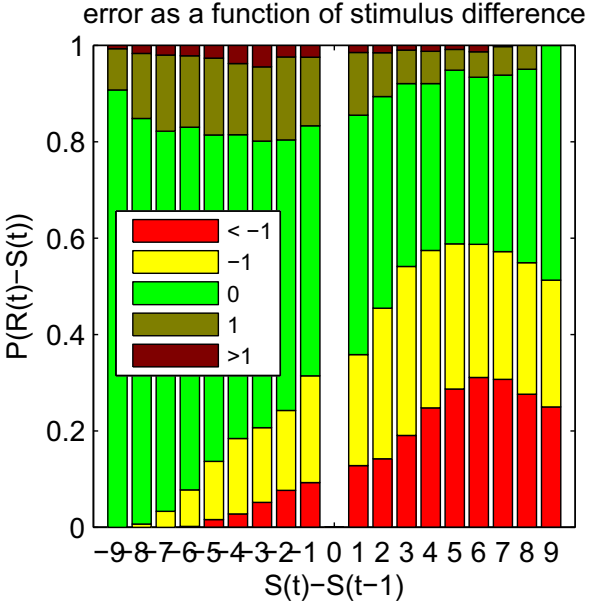
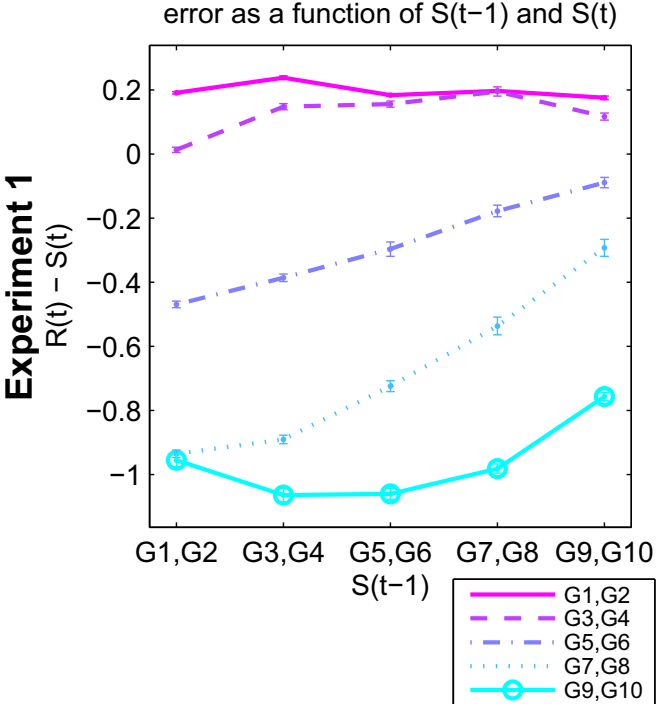
200 trials, 2 blocks of 100 trials

Within each block, all pairs of  $\{\text{gap}(t-1), \text{gap}(t)\}$  presented once, including repetitions

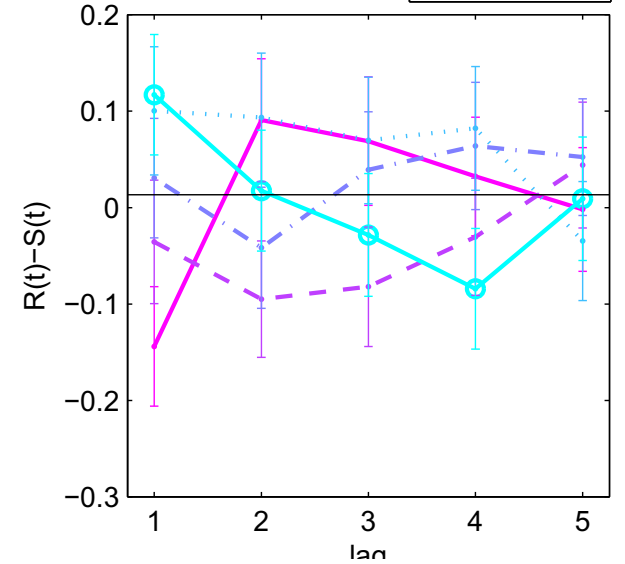
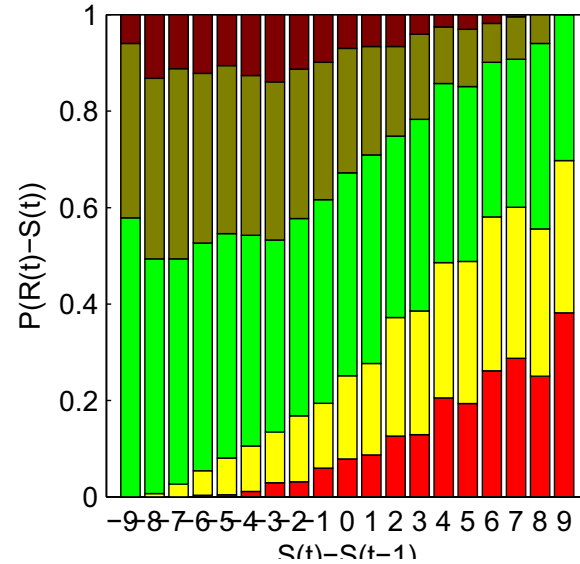
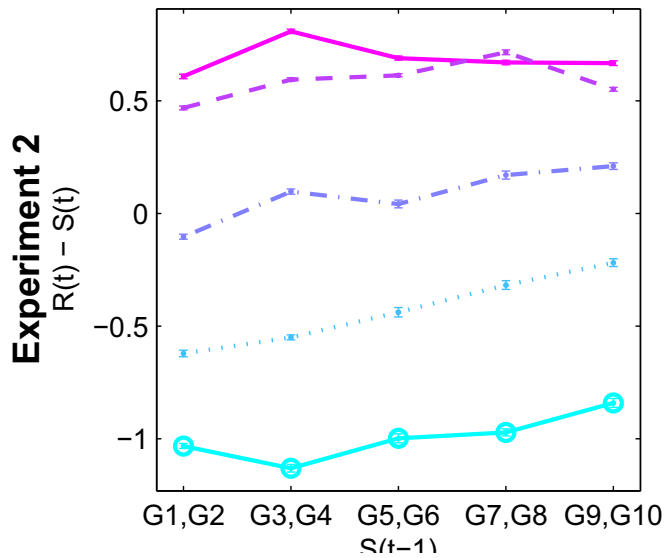
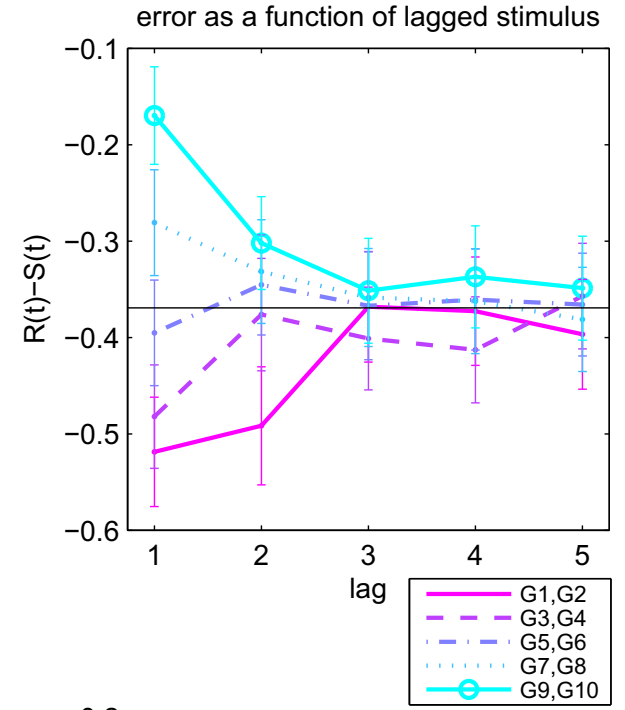
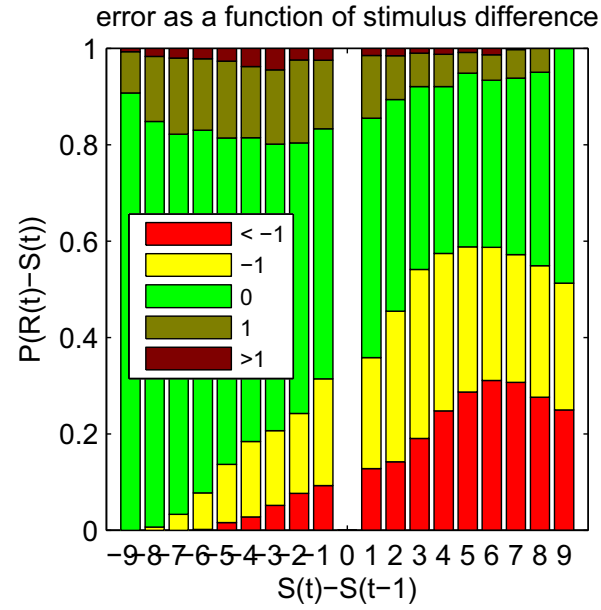
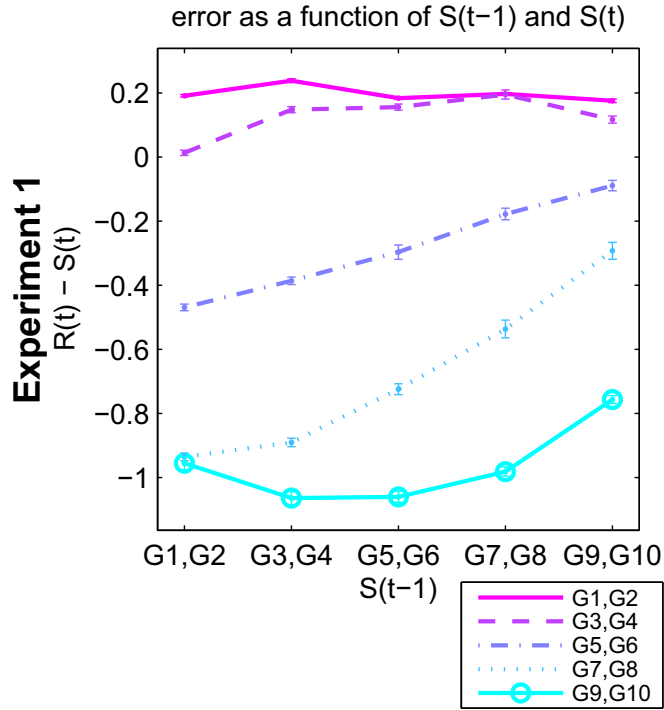
No subblock structure

$\text{gap} = .06 + .08 K$  ( $K = 1, \dots, 10$ )

# Results



# Results



# Decontamination Models

## 1. Regression

$$\hat{S}(t) = \beta_0 + \beta_1 R(t) + \beta_2 R(t-1)$$

## 2. Look up table

$$\hat{S}(t) = \text{Table}(R(t), R(t-1))$$

## 3. Regression + look up table

$$\hat{S}(t) = \beta_0 + \beta_1 R(t) + \beta_2 R(t-1) + \text{Table}(R(t), R(t-1))$$

## 4. Conditional random field regression

$$\hat{S}(t) = \beta_0 + \beta_1 R(t) + \beta_2 R(t-1) + \beta_3 \hat{S}(t-1)$$

Inference via forward-backward algorithm

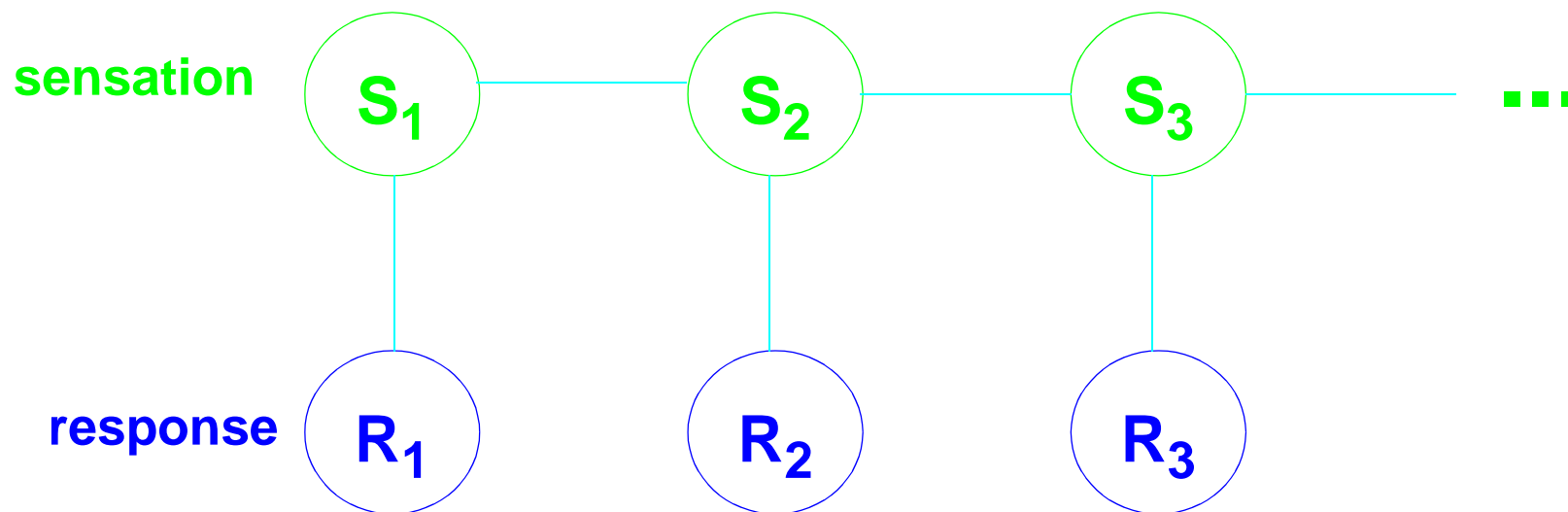
## 5. Conditional random field look up table

$$\hat{S}(t) = \text{Table}(R(t), R(t-1), \hat{S}(t-1))$$

## 6. Conditional random field regression + look up table

$$\hat{S}(t) = \beta_0 + \beta_1 R(t) + \beta_2 R(t-1) + \beta_3 \hat{S}(t-1) + \text{Table}(R(t), R(t-1), \hat{S}(t-1))$$

# Conditional Random Fields



$$P(S_{1,T}|R_{1,T}) = \frac{1}{Z(R_{1,T})} \exp \left\{ \sum_{t=1}^T \sum_{k=1}^K \lambda_k f_k(t, S_{t-1,t}, R_{1,T}) \right\}$$

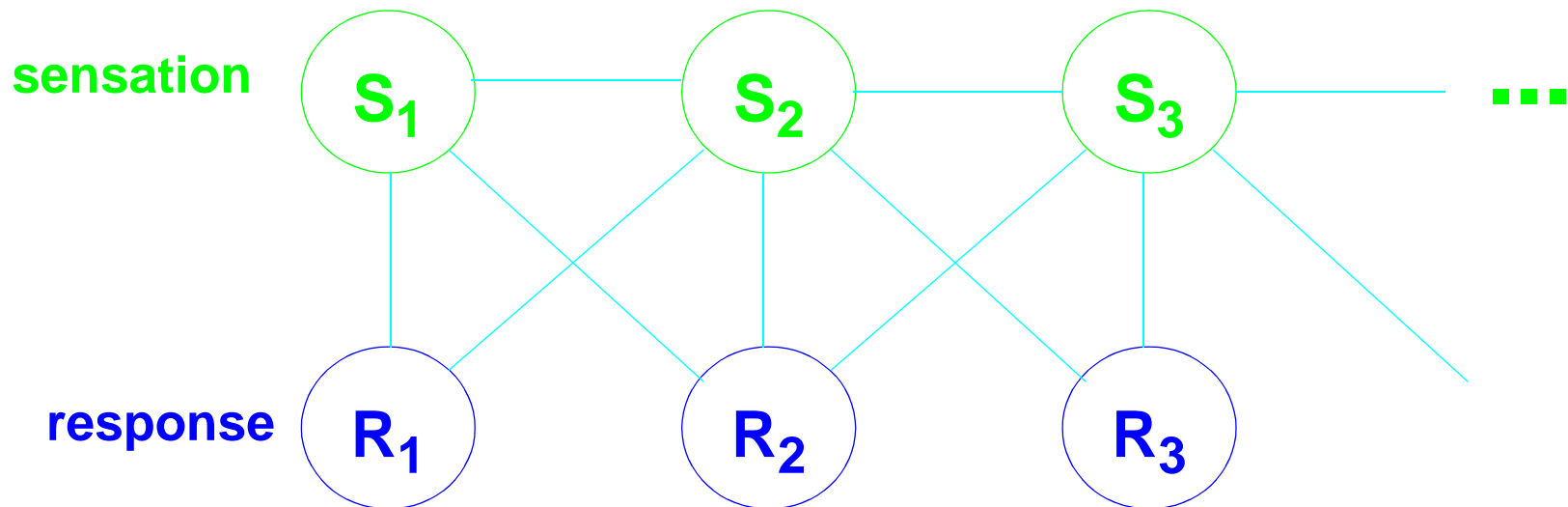
# Conditional Random Fields

$$P(S_{1,T}|R_{1,T}) = \frac{1}{Z(R_{1,T})} \exp \left\{ \sum_{t=1}^T \sum_{k=1}^K \lambda_k f_k(t, S_{t-1,t}, R_{1,T}) \right\}$$

for regression:  $\Phi_t = -(\text{REG}_t(m, n) - S_t)^2$

$$\beta_0 + \beta_1 R(t) + \beta_2 R(t-1) + \beta_3 \hat{S}(t-1)$$

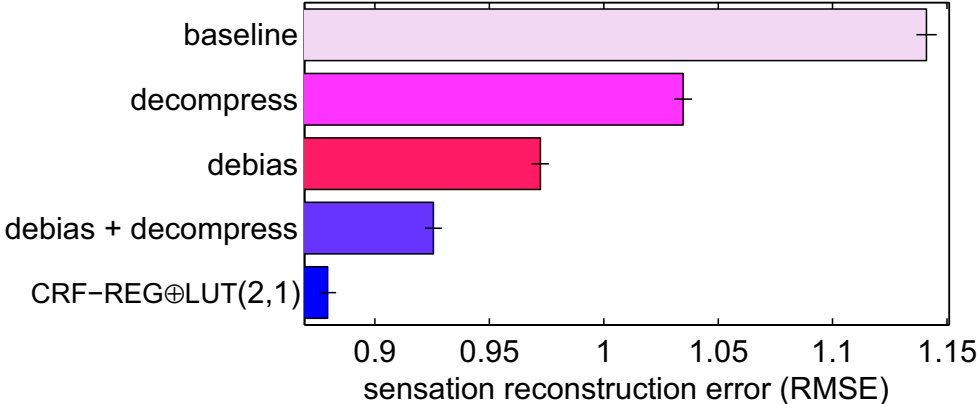
terms:  $S_t, R_t S_t, S_t^2, R_t S_{t-1}, R_{t-1} S_t, S_t S_{t-1}$



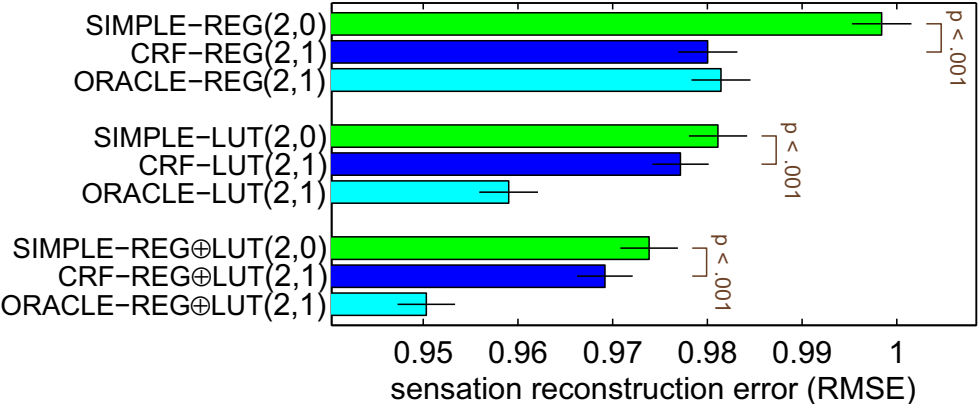
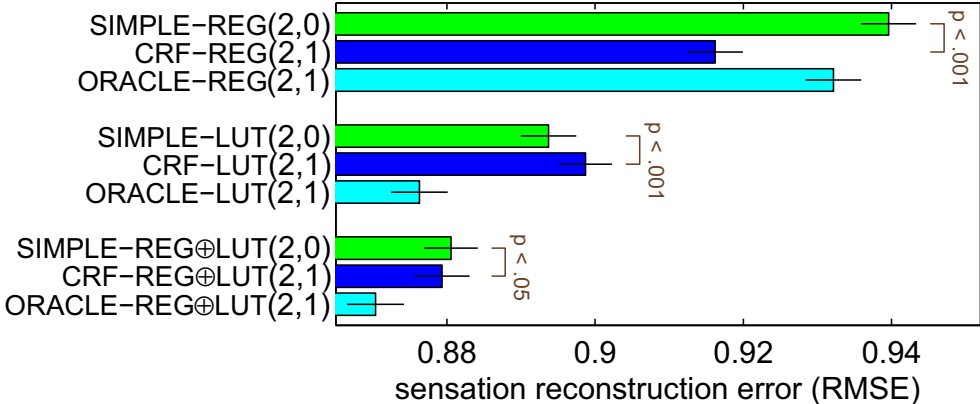
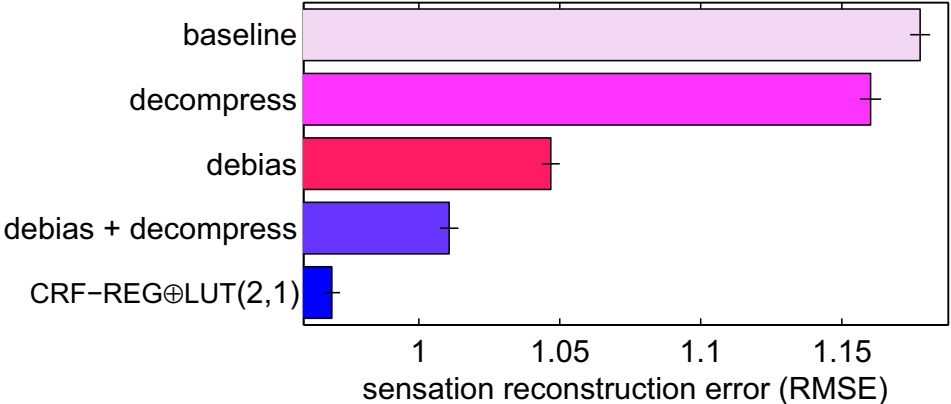


# Decontamination Results

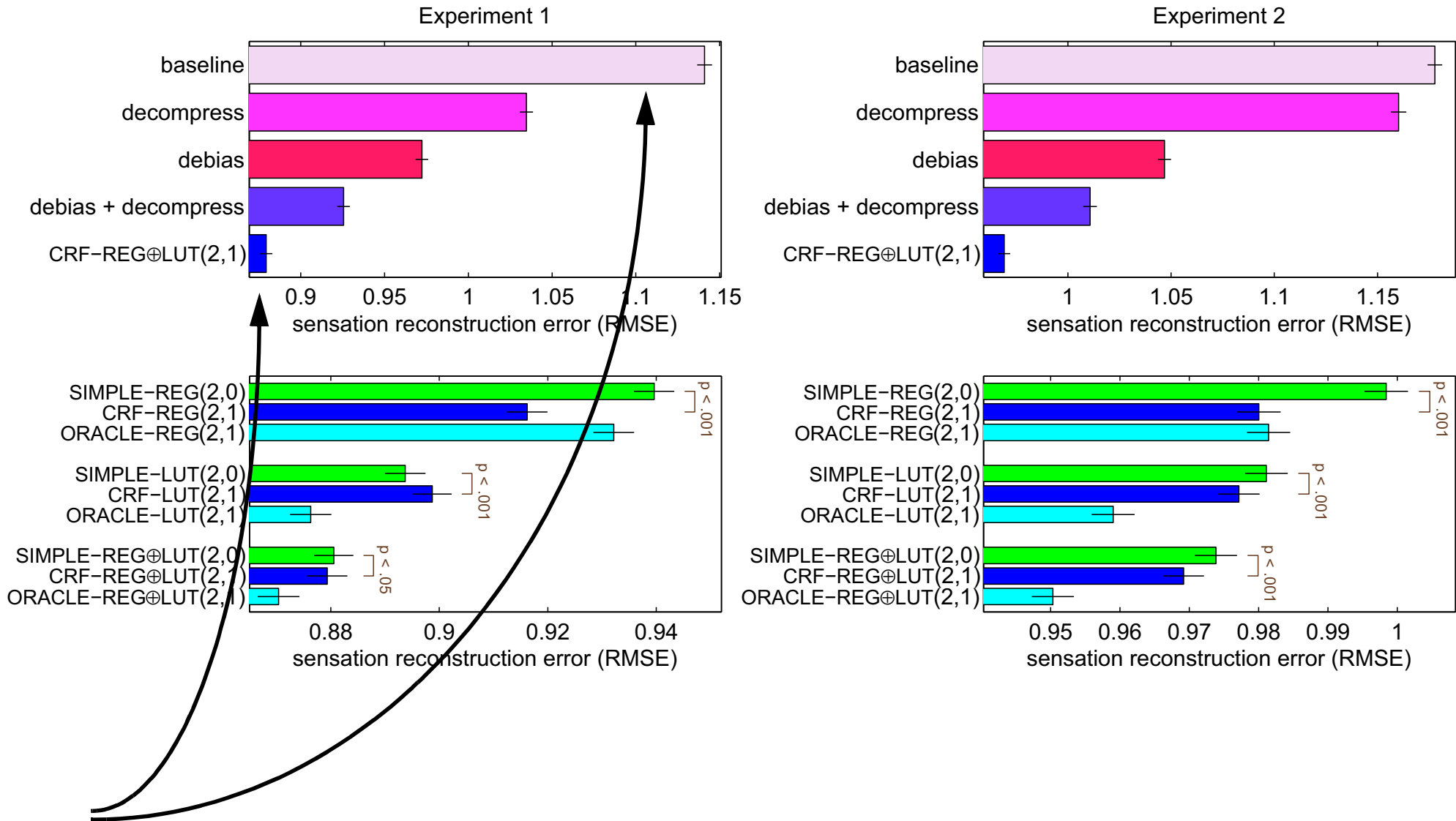
Experiment 1



Experiment 2



# Decontamination Results



**Bottom line: 20% reduction in error over using subject's response vs. decontaminated estimate**



