

# Experience Guided Search: A Bayesian Perspective on Cognitive Control

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**Matt Jones (U. Colorado, Psychology)**

**Count the number of pennies.**

**Which coin doesn't touch the others?**

**Find Jefferson.**

**Are there more heads or tails?**

**Are all the coins U.S.?**

**Are any coins the wrong size?**



# Attentional Control

**The ability to deploy attention based on task demands**

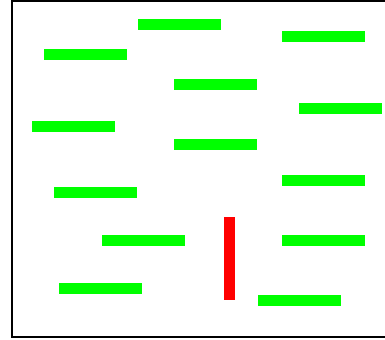
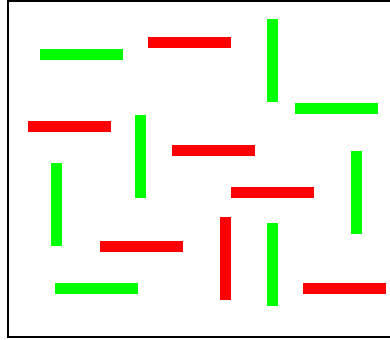
# Visual Search

Search for a *target* object among *distractors*.



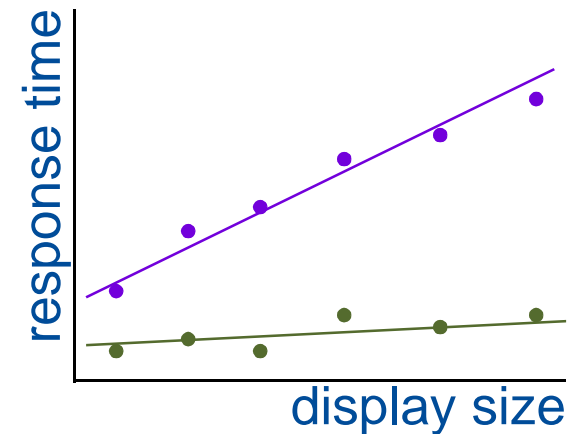
# Visual Search in the Lab

e.g., find the red vertical bar



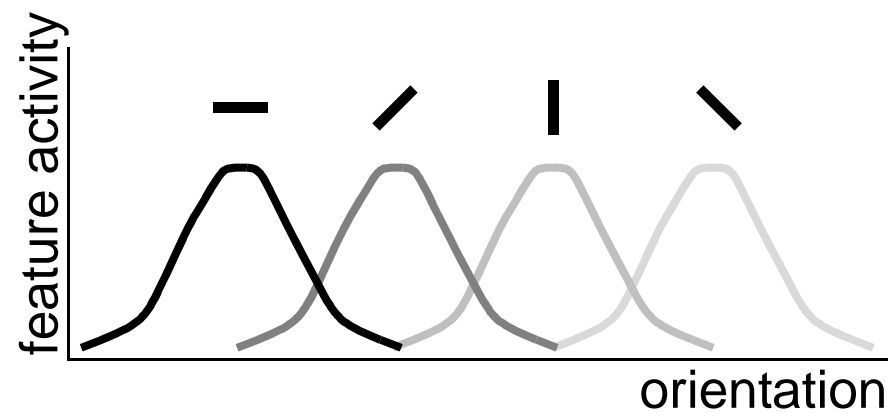
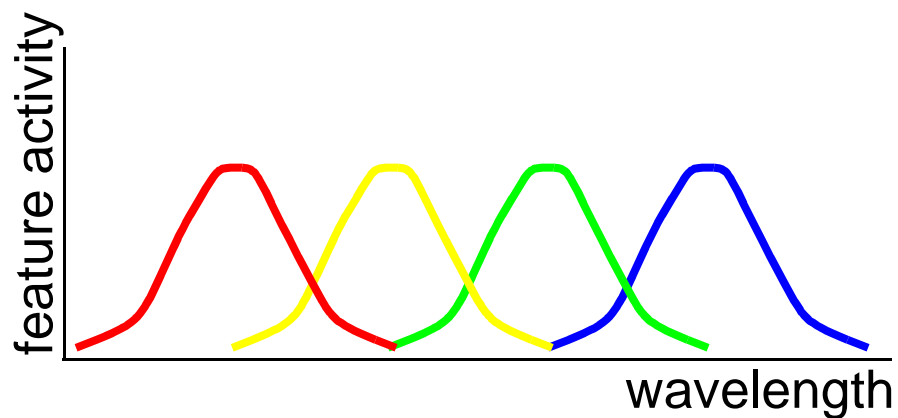
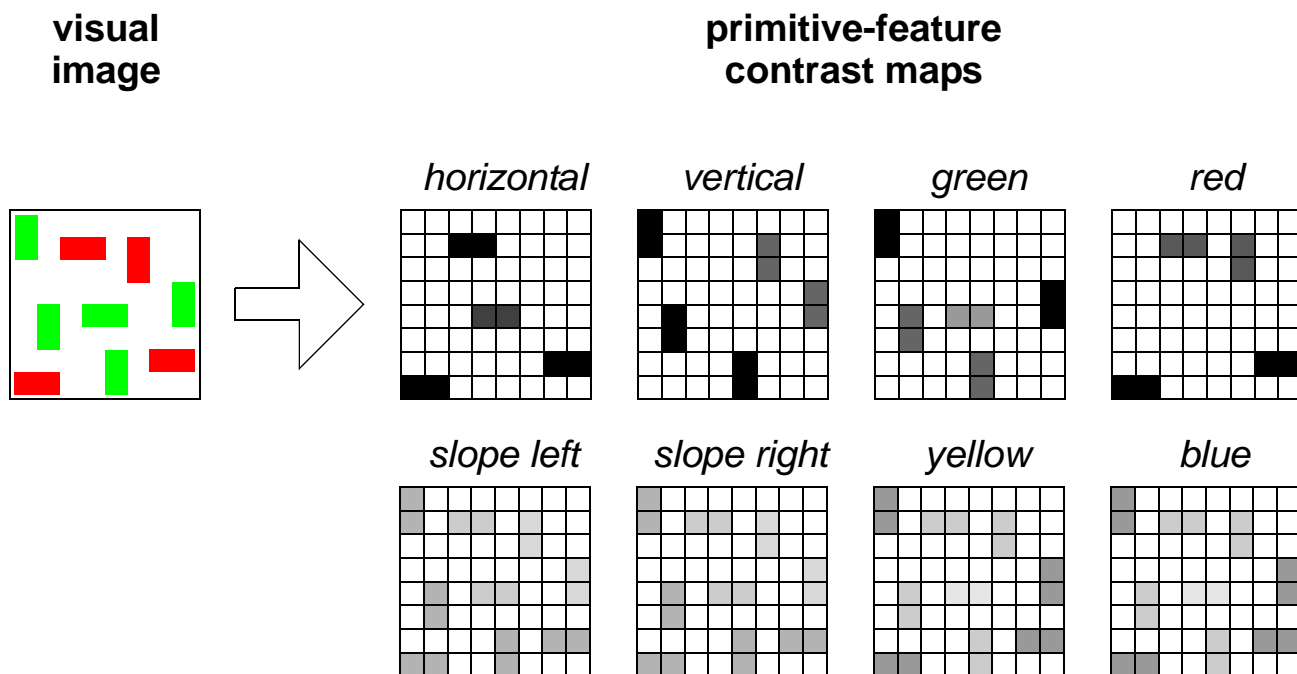
Examine time to detect target presence/absence as a function of display size

Task difficulty ~ search slope

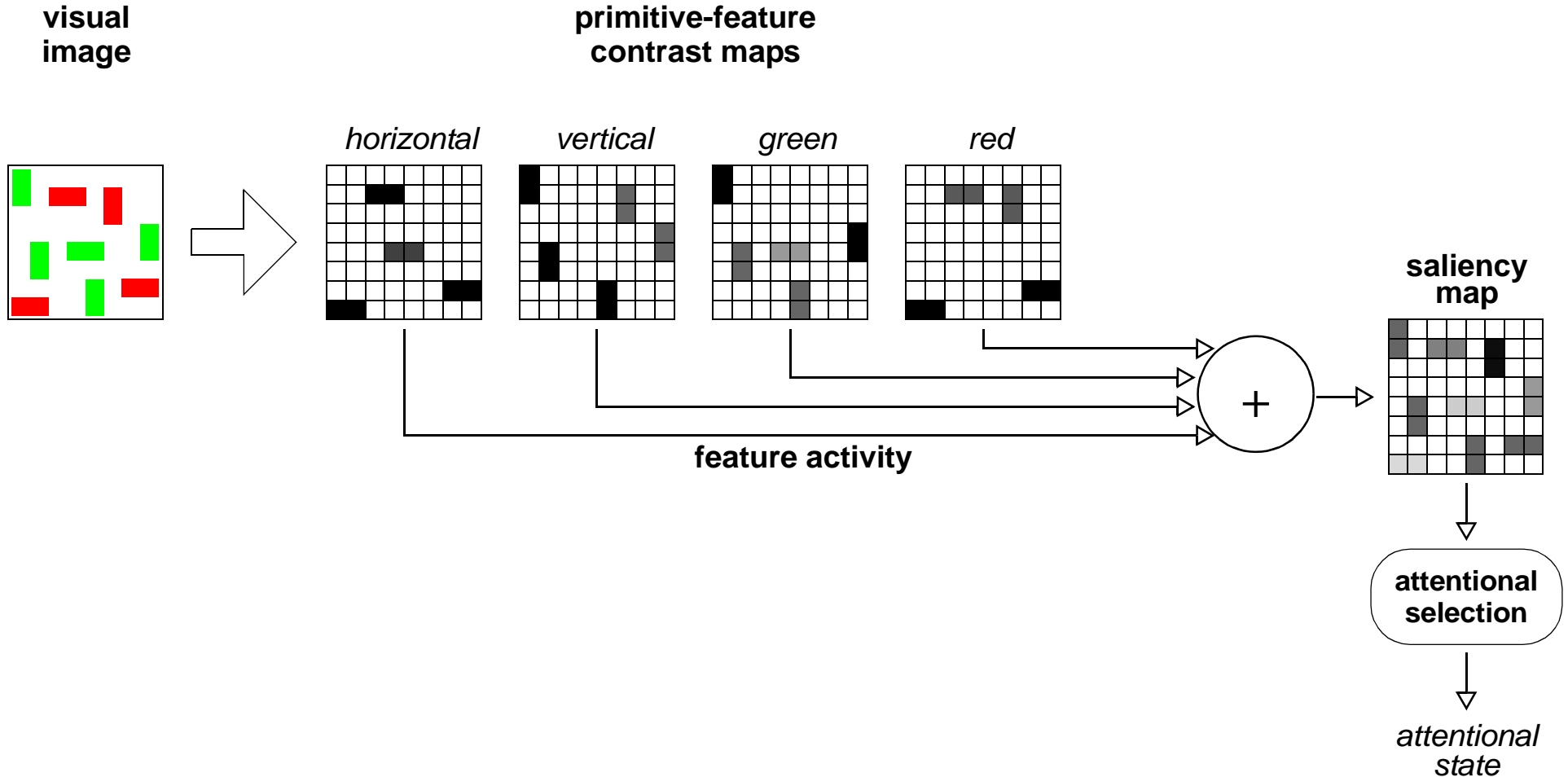


# Guided Search Model (Wolfe, 1994, 1997, 2007)

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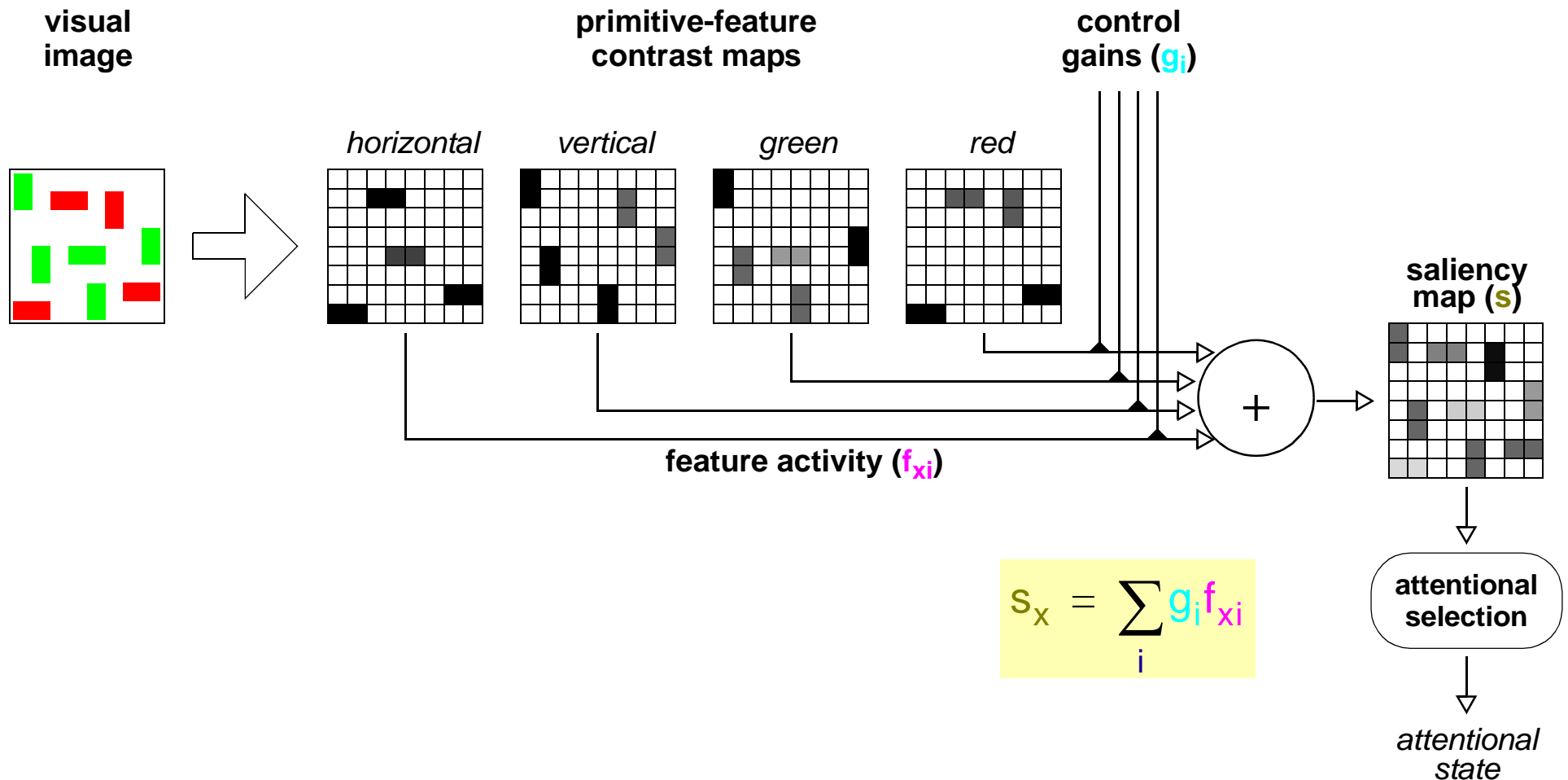
**Saliency map prioritizes locations for search.**

**GS2.0 response rule**

$$\text{Response\_time} = \mu_0 + \mu_1 \text{ target\_ranking}$$



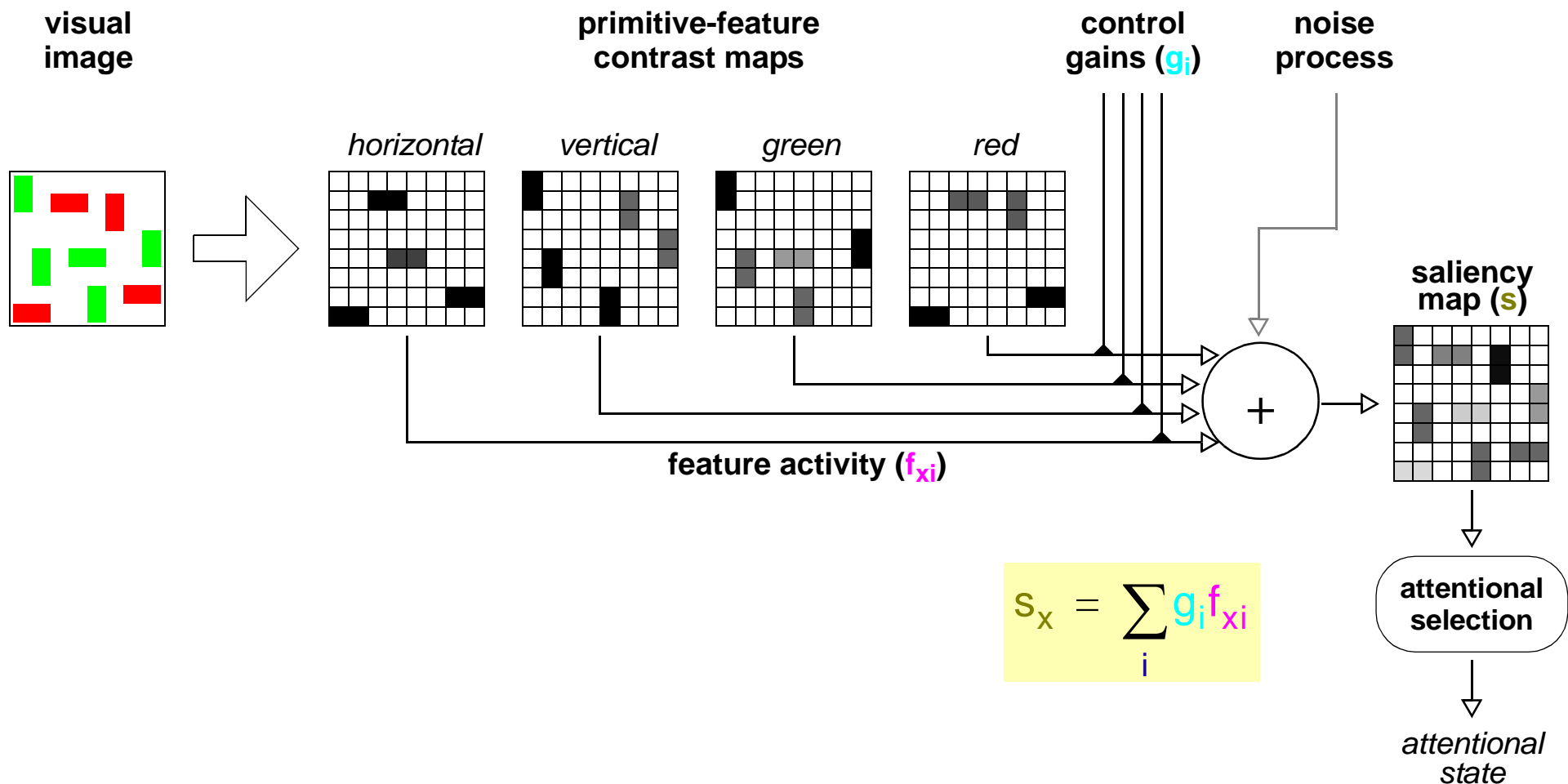
# Guided Search Model (Wolfe, 1994, 1997, 2007)



**Gains guide attention to task-relevant locations.**

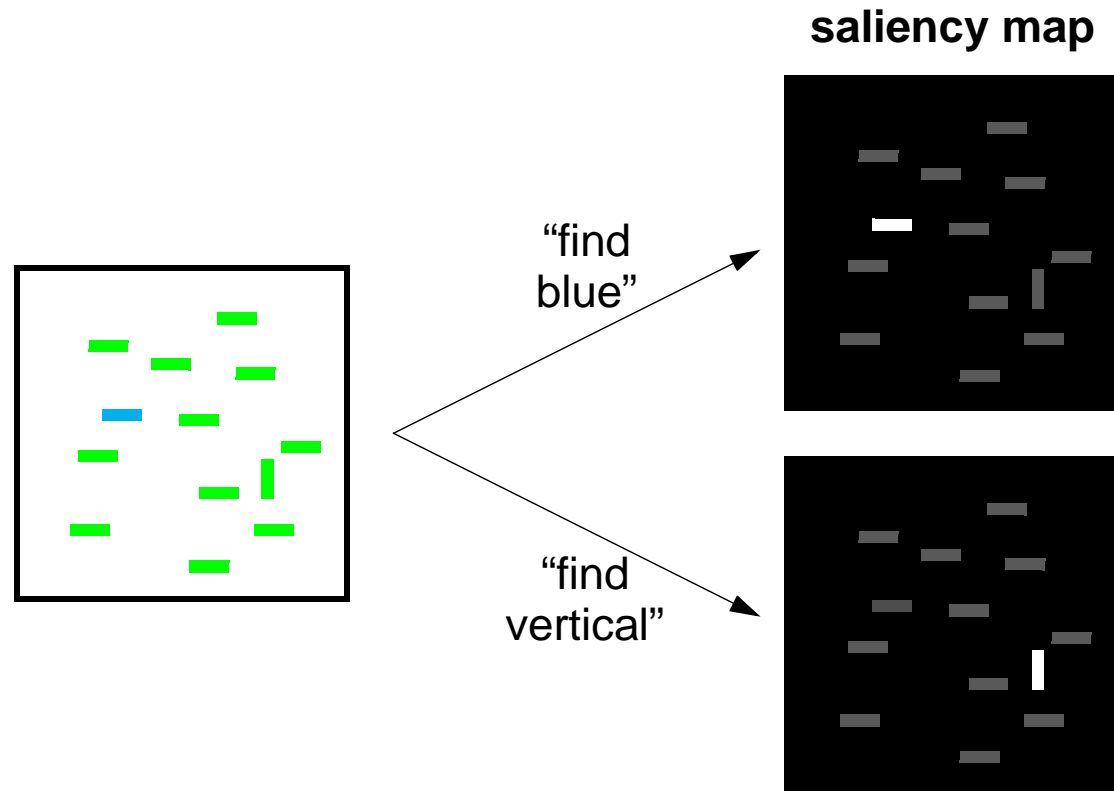
a.k.a. attentional weights, attentional set

# Guided Search Model (Wolfe, 1994, 1997, 2007)



**Noise corruption of saliency map.**

# How Guided Search Is Supposed To Work



# How Are Gains Determined?

**Guided Search doesn't specify**

## **Common intuition**

Gain ~ how well feature discriminates targets from nontargets

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## Common intuition

Gain ~ how well feature discriminates targets from nontargets

$\rho_{i1}$

**average activity  
of feature  $i$   
at locations  
containing target**

$\rho_{i0}$

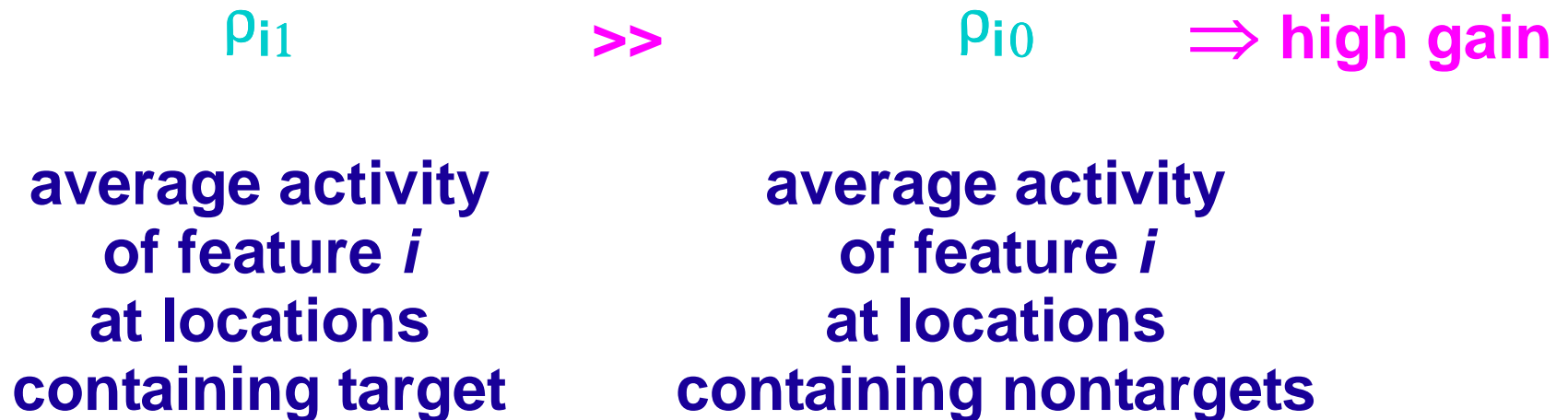
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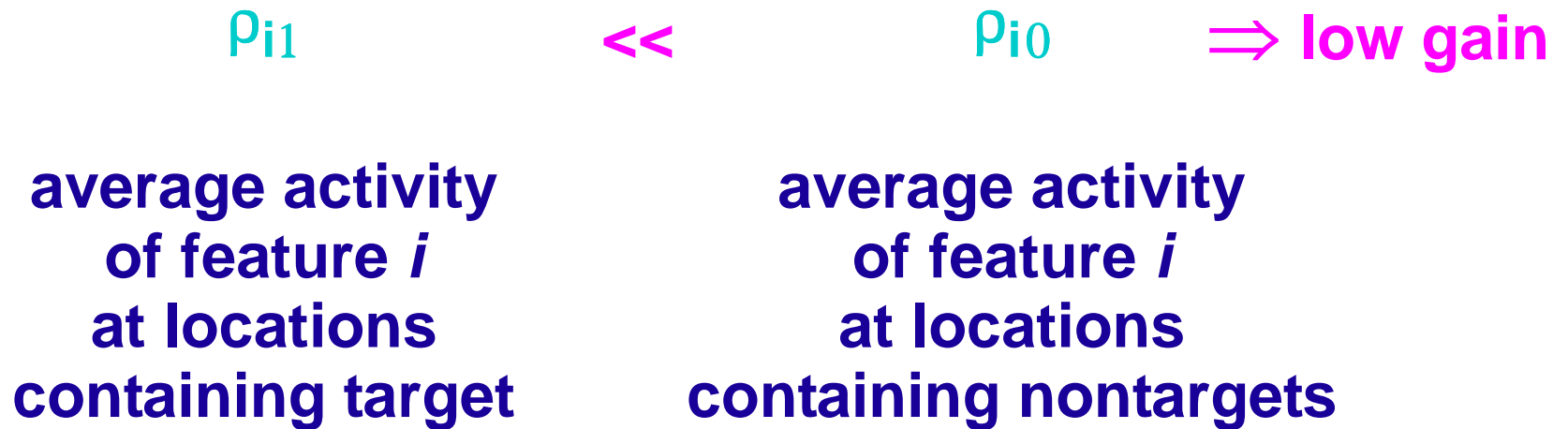


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Guided Search doesn't specify

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# Experience-Guided Search



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## Adopt same basic architecture as Guided Search

- feature extraction
- contrast enhancement

**But frame the model's objective in probabilistic terms...**

**Saliency**  $\equiv P(T_x | F_x, \rho)$

- task statistics – learned thru experience
- feature activity (vector) at location x
- target at location x? (1=true, 0=false)

$$\text{Saliency} \equiv P(T_x | F_x, \rho)$$

task statistics – learned thru experience  
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## Bayes' Rule

$$P(T_x | F_x, \rho) = \frac{P(T_x)P(F_x | T_x, \rho)}{\sum_{t=0}^1 P(T_x = t)P(F_x | T_x = t, \rho)}$$

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$P(F_x | T_x, \rho)$  is a task-specific model of the environment.

Indicates visual system response ( $F_x$ ) for targets ( $T_x=1$ ) vs. nontargets ( $T_x=0$ )

## Modeling game

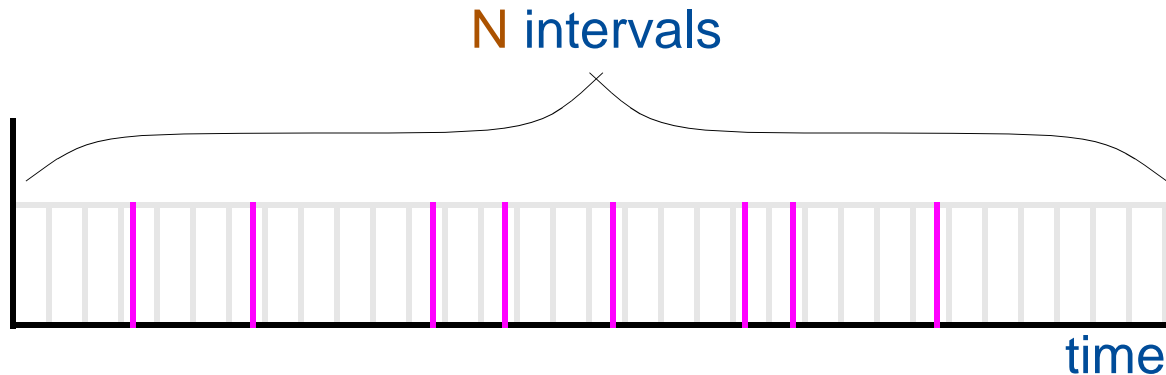
Specify a set of qualitative assumptions that define the environment model, and explore the consequences.

# Key Assumptions

## 1. Feature responses are conditionally independent

$$P(\mathbf{F}_x | T_x, \rho) = \prod_i P(F_{xi} | T_x, \rho)$$

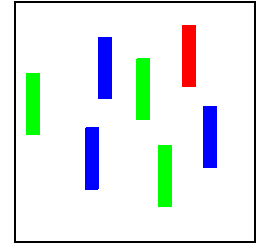
## 2. Feature detection is carried out by rate-coded spiking neuron



$F_{xi}$ : count of the number of spikes observed for feature  $i$  at location  $x$

$\rho_{it}$ : spike rate for feature  $i$  if the target is of type  $t$

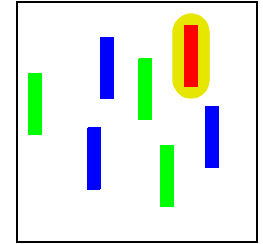
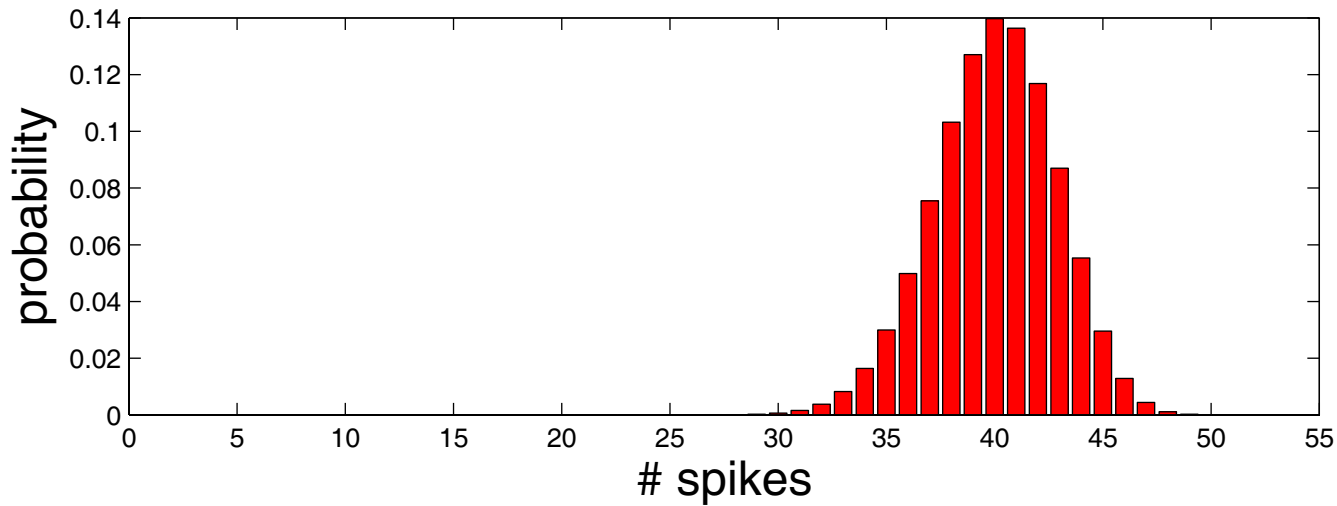
**Consider search for a red object among non-red.  
What will the response of the red feature detector be?**





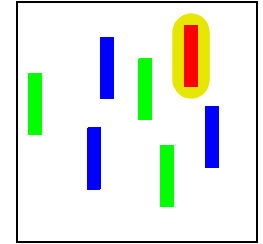
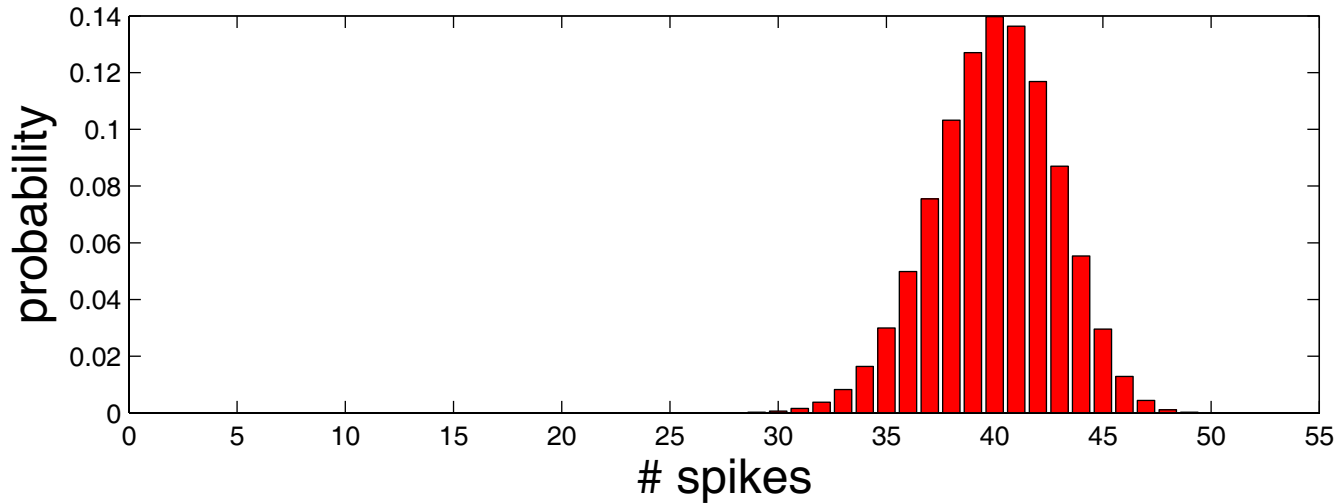
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$P(F_{\text{red}} | T = 1, p_{\text{red}, 1} = 0.8)$  for  $N=50$

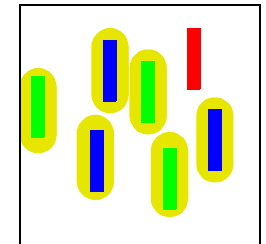
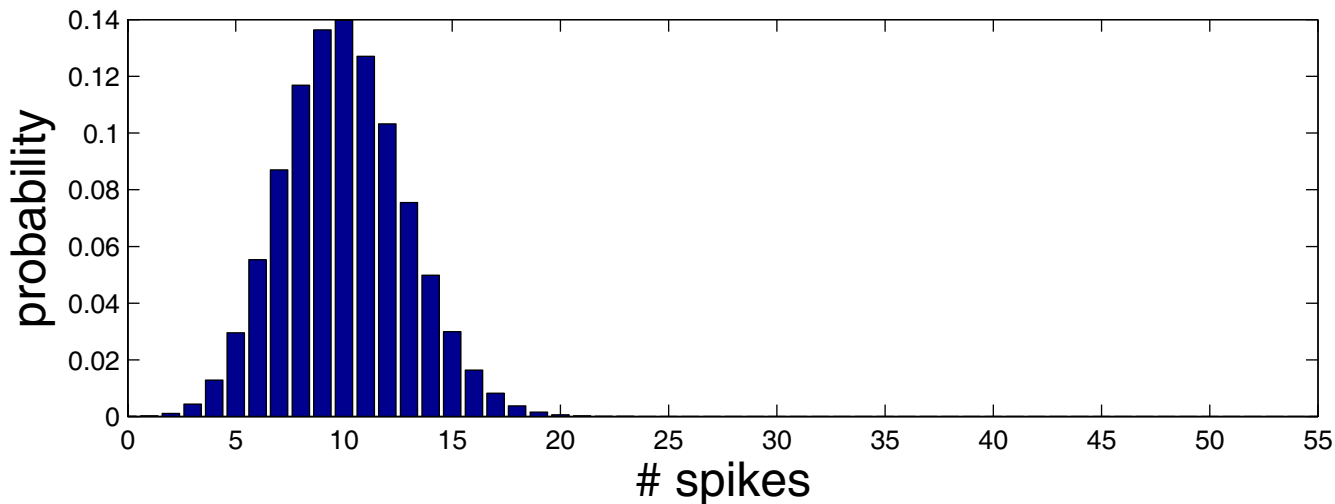


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What will the response of the red feature detector be?

$P(F_{\text{red}} | T = 1, p_{\text{red}, 1} = 0.8)$  for  $N=50$



$P(F_{\text{red}} | T = 0, p_{\text{red}, 0} = 0.2)$  for  $N=50$



$$P(F_{xi} | T_x, \rho) \sim \text{Binomial}(\rho_{it}, N)$$

number of time intervals

spiking rate of feature i  
for target (t=1) or distractor (t=0)

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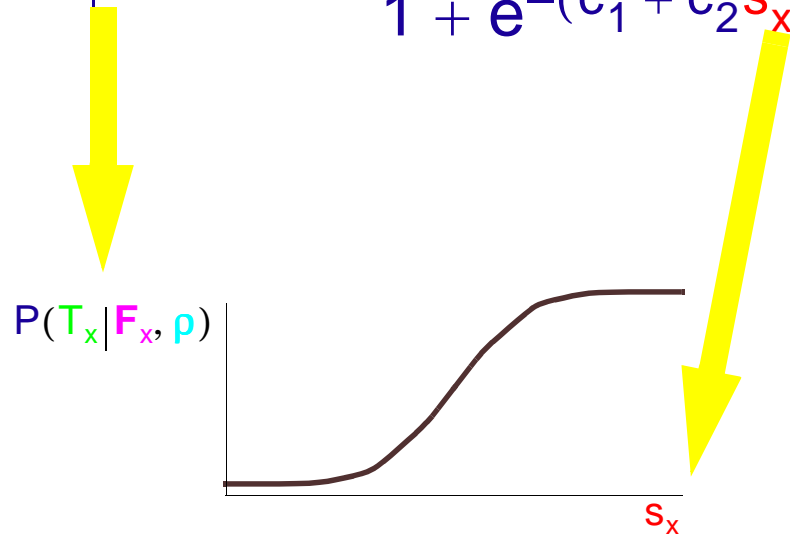
spiking rate of feature i  
for target (t=1) or distractor (t=0)

$$\sim \text{Gaussian}(N\rho_{it}, N\rho_{it}(1 - \rho_{it}))$$

**These assumptions lead to...**

$$P(T_x | F_x, \rho) = \frac{1}{1 + e^{-(c_1 + c_2 S_x)}}$$

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Because attentional priority depends on relative saliency, we can substitute  $s_x$  for  $P(T_x | F_x, \rho)$ .

$$P(T_x | F_x, \rho) = \frac{1}{1 + e^{-(c_1 + c_2 s_x)}}$$

response of feature i  
in location x

$$s_x = \sum_i \sum_{t=0}^1 \frac{1 - 2t}{\rho_{it}(1 - \rho_{it})} (f_{xi} - \rho_{it})^2$$



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$$S_x = \sum_i c_i f_{xi} + \tilde{c}_i f_{xi}^2$$

$$s_x = \sum_i \sum_{t=0}^1 \frac{1-2t}{p_{it}(1-p_{it})} (f_{xi} - p_{it})^2$$

$$\frac{2(p_{i1} - p_{i0})}{(1-p_{i0})(1-p_{i1})}$$

$$\frac{1}{p_{i0}(1-p_{i0})} - \frac{1}{p_{i1}(1-p_{i1})}$$

$$s_x = \sum_i c_i f_{xi} + \tilde{c}_i f_{xi}^2$$

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**Experience-  
Guided Search**

$$s_x = \sum_i c_i f_{xi} + \tilde{c}_i f_{xi}^2$$

**Guided Search**

$$s_x = \sum_i c_i f_{xi}$$

# Differences Between EGS and GS

1. EGS includes terms quadratic in  $f_{xi}$
2. GS determines gains via heuristics or optimization;  
in EGS, gains are function of environment/task statistics

**Experience-Guided Search**

$$S_x = \sum_i c_i f_{xi} + \tilde{c}_i f_{xi}^2$$

**Guided Search**

$$S_x = \sum_i c_i f_{xi}$$

# How Are Environment Statistics ( $\rho$ ) Learned?

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As locations are inspected during a trial, a supervisory process labels each element as target or nontarget.



Given these observations, update (learn) the  $\rho_{it}$  via Bayesian inference.

# Assumptions Underlying Learning

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## 1. Prior (bias) that all features are considered relevant in the absence of experience

Achieved by treating  $\rho$  as a Beta random variable with imaginary-count prior

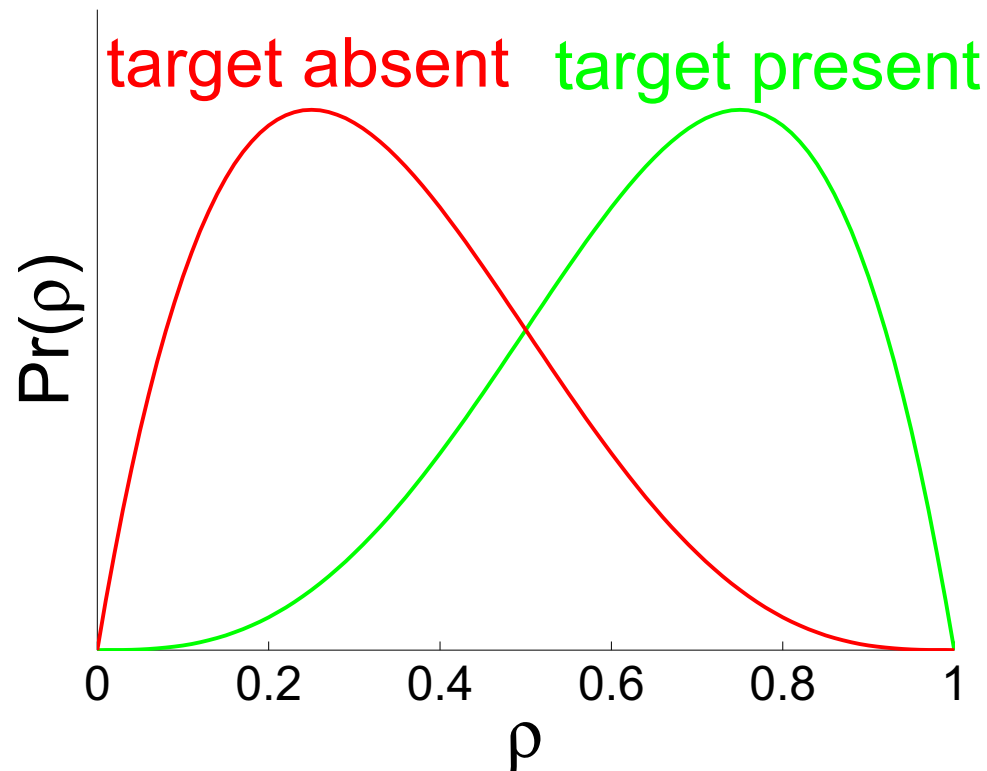


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With probability  $\lambda$ , environment and/or task can change.

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With probability  $\lambda$ , environment and/or task can change.

**From these two claims, we have *three* free parameters total.**

Qualitative performance does not depend on parameters as long as  $\lambda > 0$  and  $E[\rho_{i0}] < E[\rho_{i1}]$

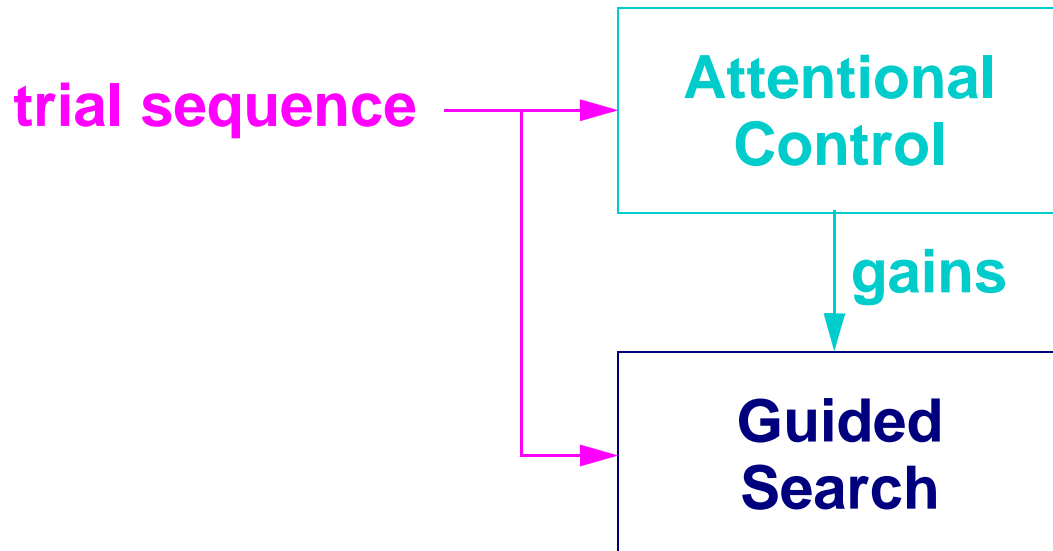
# Summary So Far

**Under a probabilistic generative model of the task environment, we obtain**

- an expression for saliency given feature activations and task statistics
- an inference rule for updating task statistics following each trial

## Three free parameters in model

- bias that all features are task relevant (2 parameters)
- environmental change probability (1 parameter)
- + a few leftover parameters of GS (e.g., RT scaling)



# What It Boils Down To



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- **Generate stimulus sequence corresponding to experiment.**

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- **Initialize task statistics**

$$\alpha_{i1} = \beta_{i0} = \varphi$$

$$\alpha_{i0} = \beta_{i1} = \theta$$

where  $\bar{p}_{it} = \frac{\alpha_{it}}{(\alpha_{it} + \beta_{it})}$

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- Generate stimulus sequence corresponding to experiment.
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- Compute saliency at each location  $x$

$$s_x = \sum_i \frac{2(\rho_{i1} - \rho_{i0})}{(1 - \rho_{i0})(1 - \rho_{i1})} \tilde{f}_{xi} + \left[ \frac{1}{\rho_{i0}(1 - \rho_{i0})} - \frac{1}{\rho_{i1}(1 - \rho_{i1})} \right] \tilde{f}_{xi}^2$$

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- **Generate stimulus sequence corresponding to experiment.**
- **Initialize task statistics**
- **On each trial, perform feature extraction on display.**
- **Compute saliency at each location  $x$**
- **Determine response time based on ranking**

$$\text{ResponseTime} = \mu_0 + \mu_1 \text{ SaliencyRankingOfTarget}$$

# What It Boils Down To

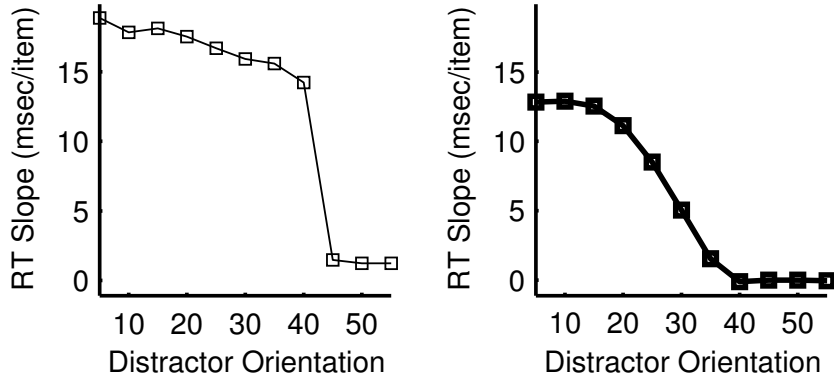
- **Generate stimulus sequence corresponding to experiment.**
- **Initialize task statistics**
- **On each trial, perform feature extraction on display.**
- **Compute saliency at each location  $x$**
- **Determine response time based on ranking**
- **Update task statistics based on current trial feature activity**

$$\alpha_{it} \leftarrow \lambda \alpha_{it}^0 + (1 - \lambda) \left( \alpha_{it} + \sum_{x \in \chi_t} \tilde{f}_{xi} \right)$$
$$\beta_{it} \leftarrow \lambda \beta_{it}^0 + (1 - \lambda) \left( \beta_{it} + \sum_{x \in \chi_t} 1 - \tilde{f}_{xi} \right)$$

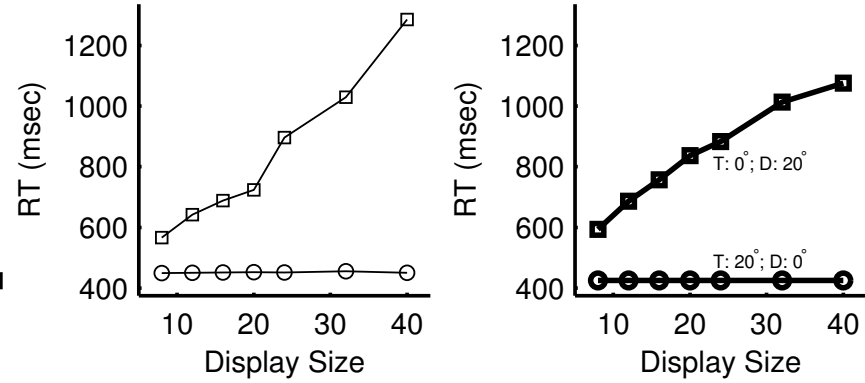
Approximate inference, but excellent approximation

# EGS Replicates GS

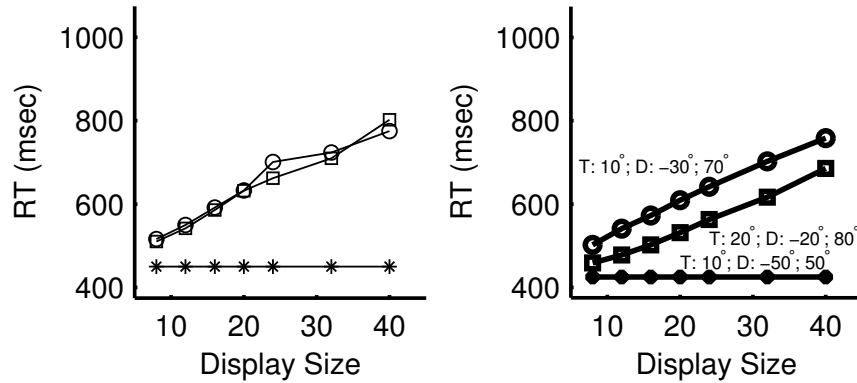
**GS** (A) Vertical Bar Among Homogeneous Distractors **EGS**



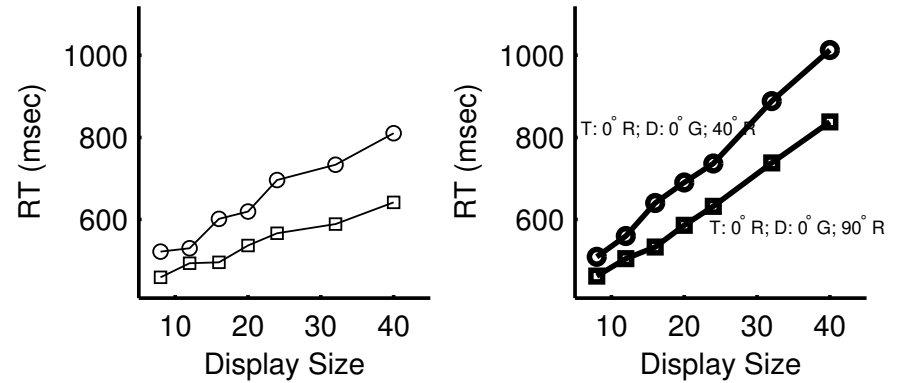
**GS** (D) Feature Search Asymmetry **EGS**



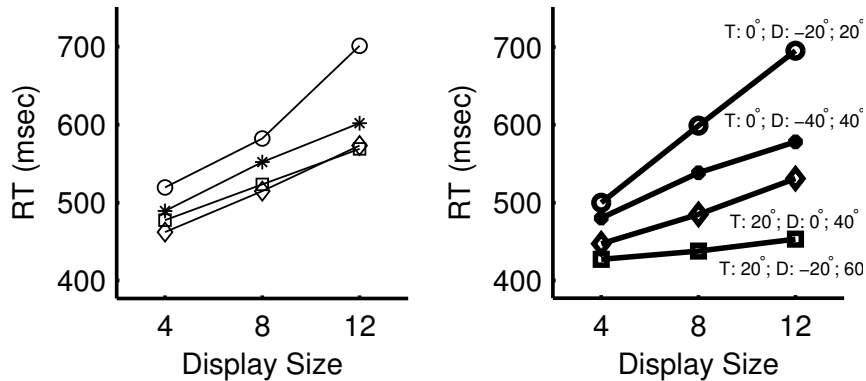
(B) Categorical Search



(E) Conjunction Search Varying Distractor Confusability

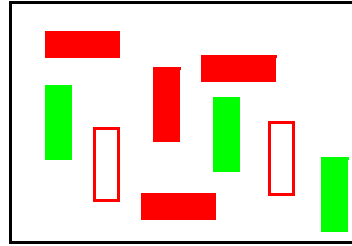


(C) Target-Distractor Similarity

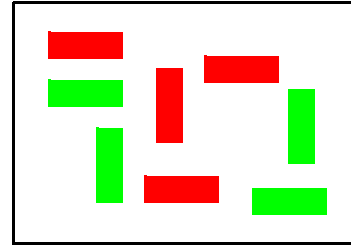


# Guiding Search (Wolfe, Cave, & Franzel, 1989)

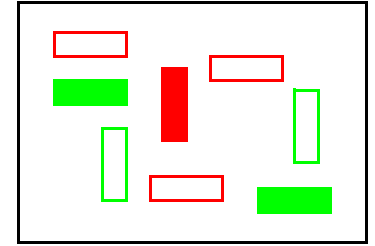
  
target



3:2



2:1

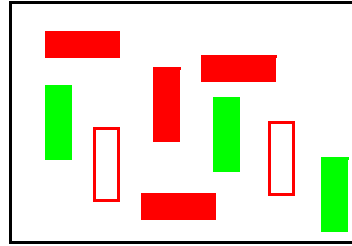


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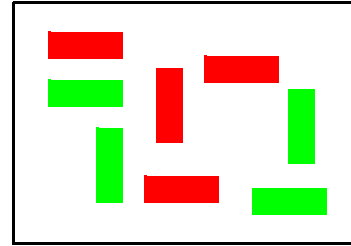
ratio of number  
of features  
defining target  
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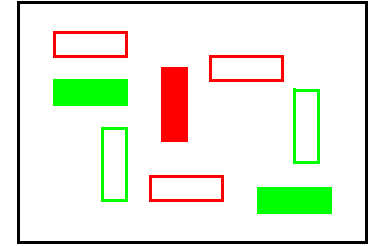
  
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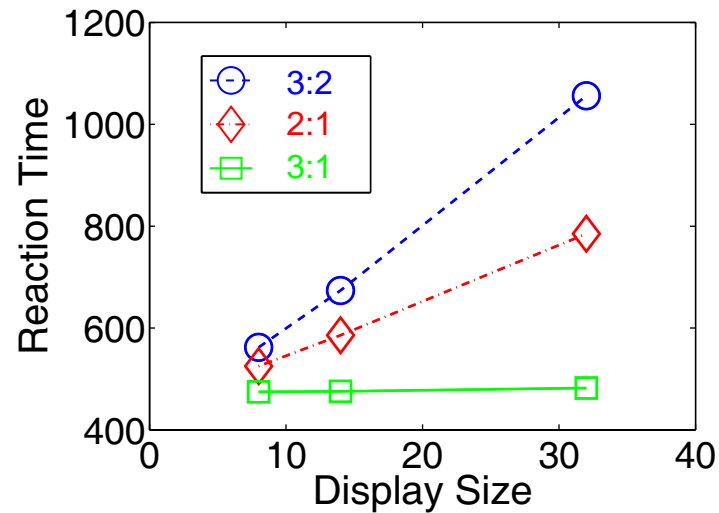


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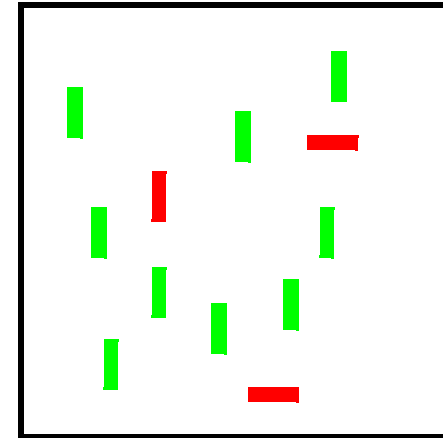
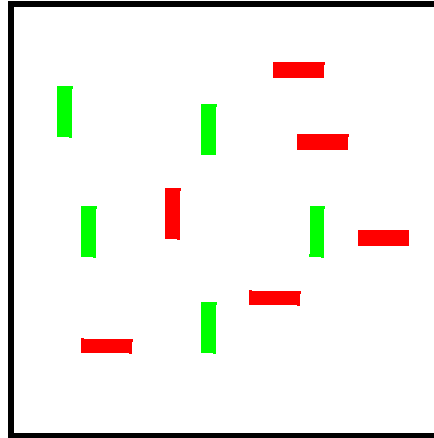
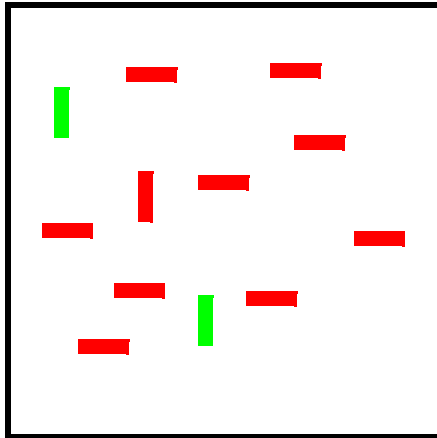
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cf. Shiffrin talk

# Varying Distractor Proportion

(Egeth, Virzi, & Garbart, 1984; Poisson & Wilkinson, 1992; Zohary & Hochstein, 1989)



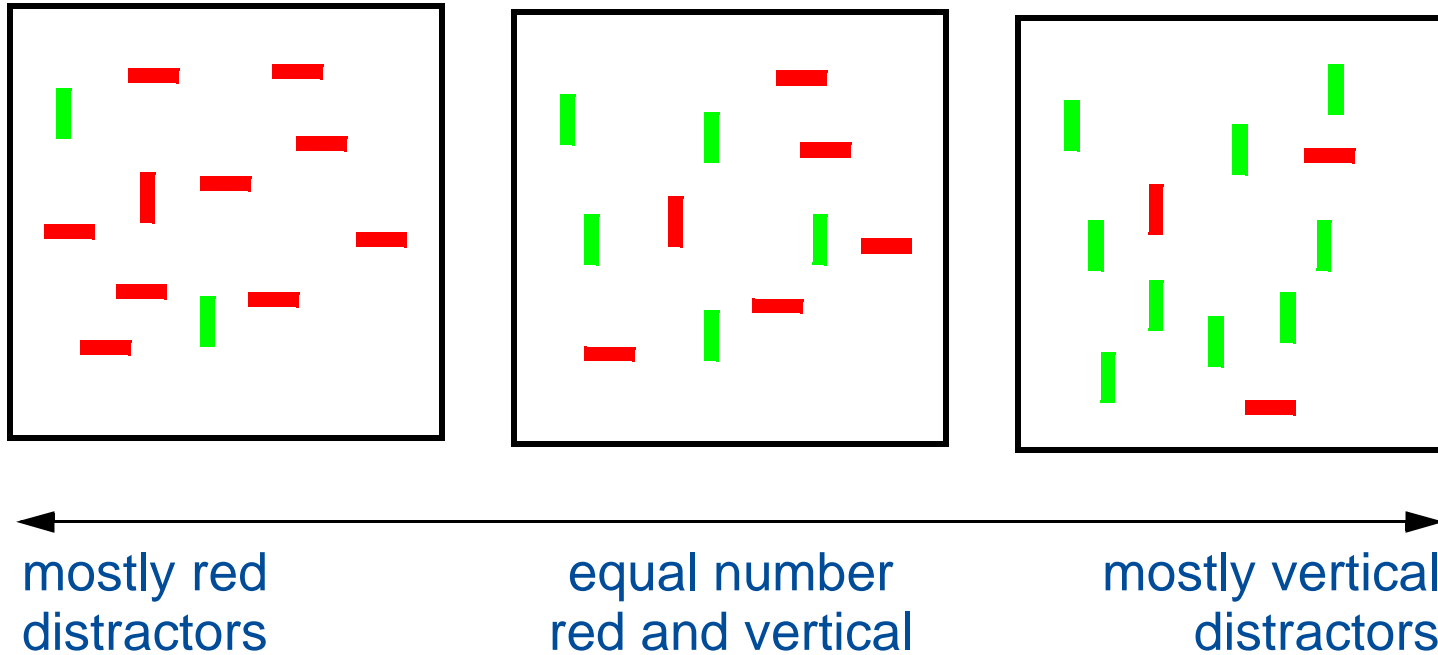
mostly red  
distractors

equal number  
red and vertical

mostly vertical  
distractors

# Varying Distractor Proportion

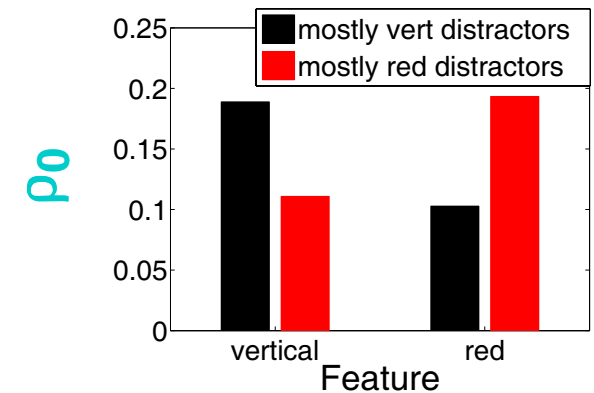
(Egeth, Virzi, & Garbart, 1984; Poisson & Wilkinson, 1992; Zohary & Hochstein, 1989)



## Blocked trials ->

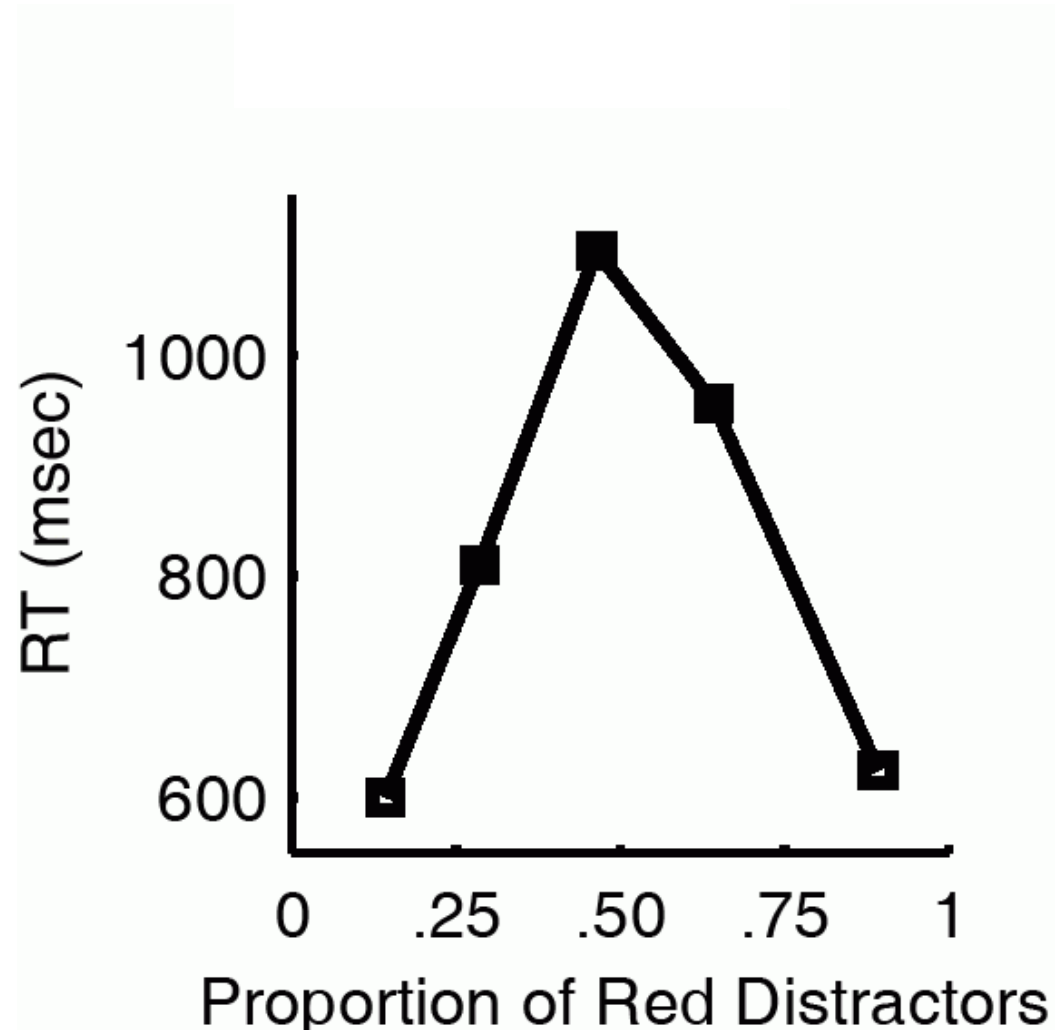
environment statistics make one feature a more discriminable cue ->

EGS gives it greater weighting.

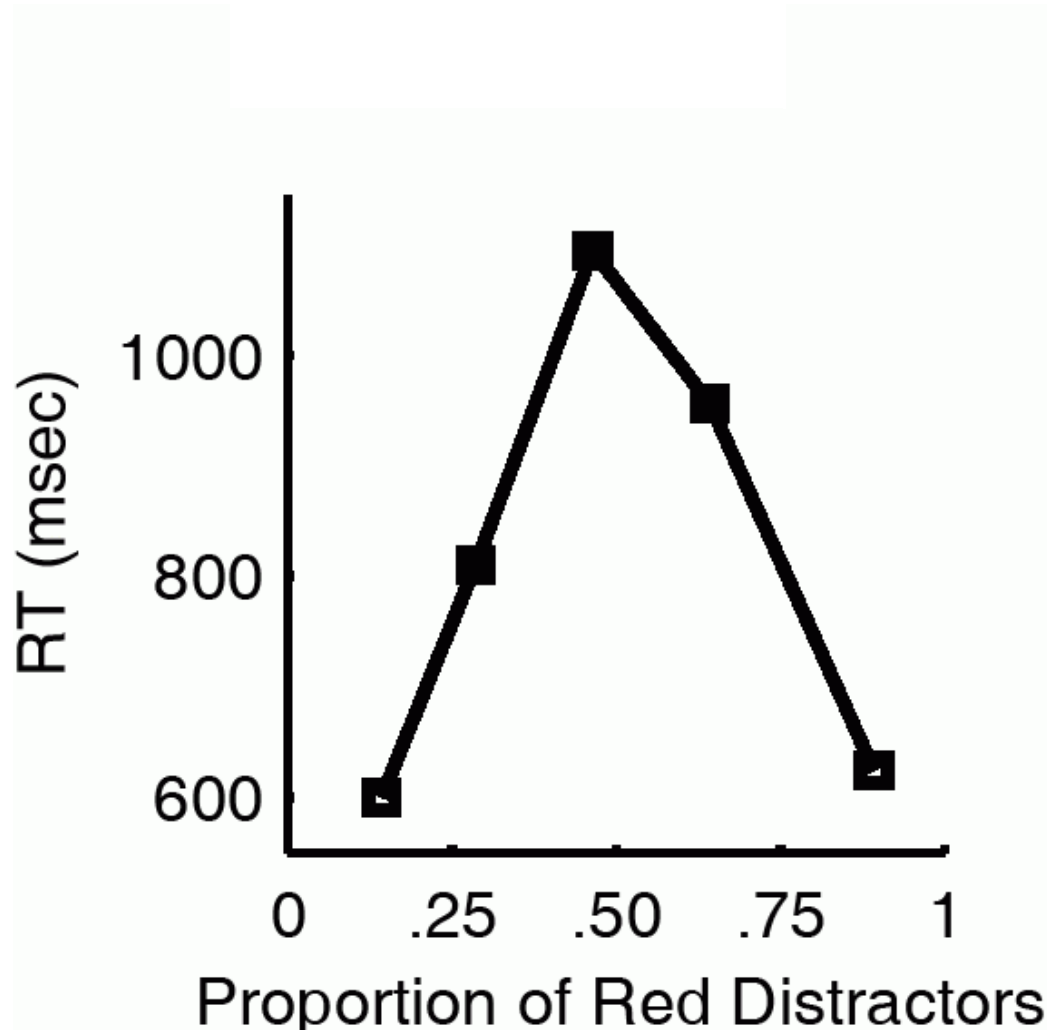




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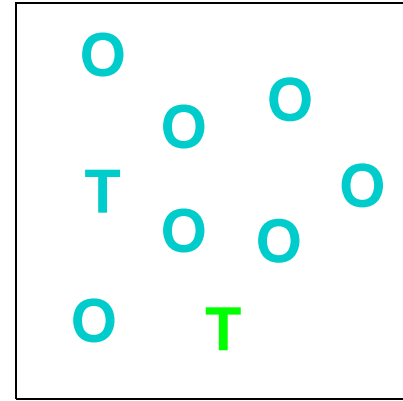
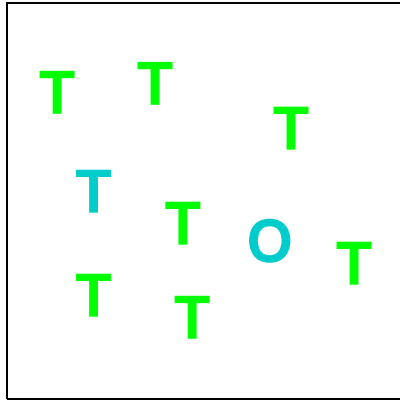
**In these studies, distractor proportion is blocked.**

**Is efficiency achieved when proportion varies within block?**

# Wright and Main (unpublished)

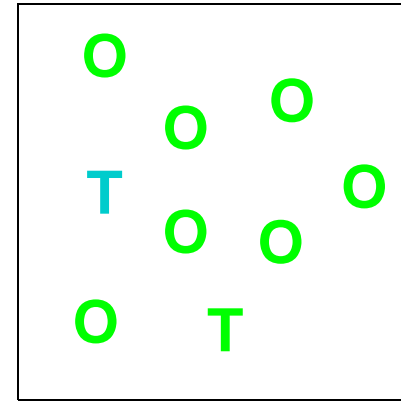
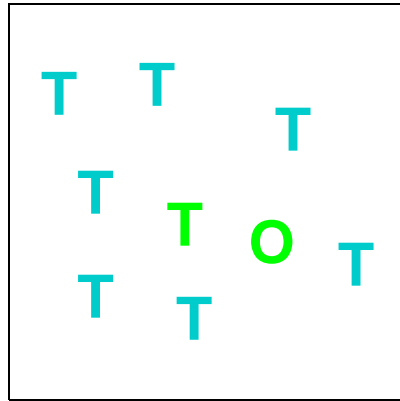
# Wright and Main (unpublished)

Search for **T** among **T** and **O**



# Wright and Main (unpublished)

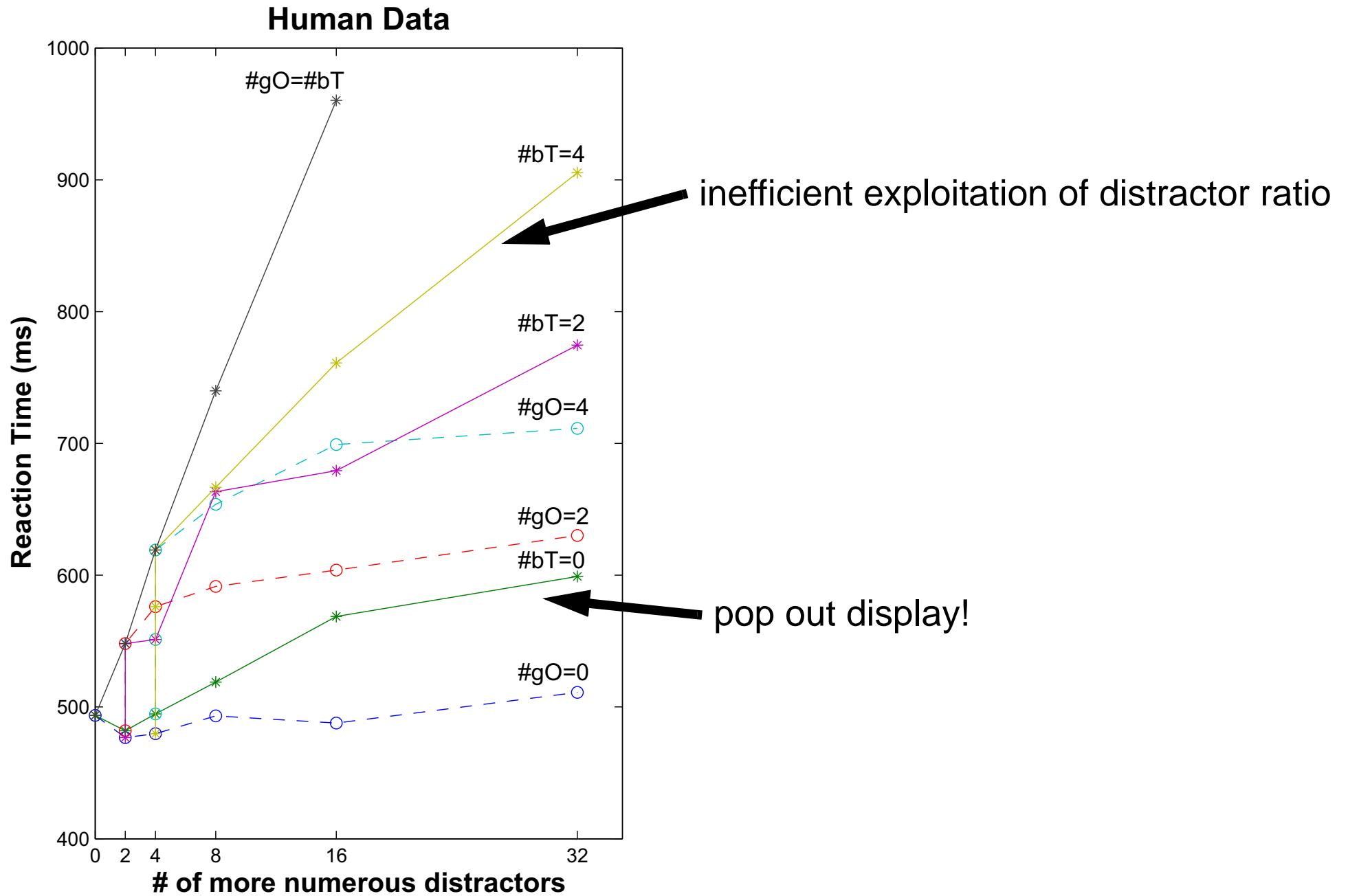
Search for **T** among **T** and **O**



**Number of distractors of each type varies trial to trial**

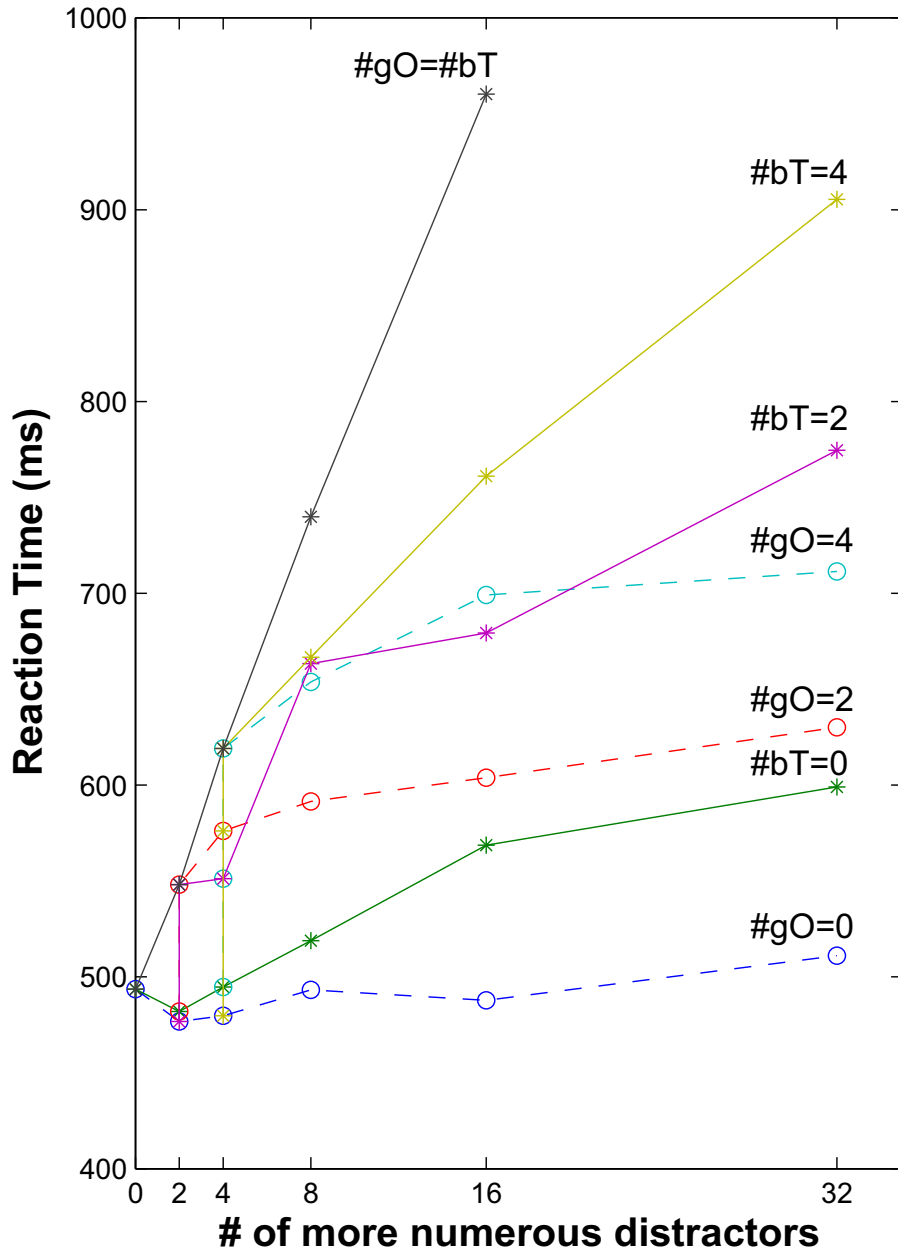
Number of <b>O</b>	Number of <b>T</b>					
	0	2	4	8	16	32
0	X	X	X	X	X	X
2	X	X	X	X	X	X
4	X	X	X	X	X	X
8	X	X	X	X		
16	X	X	X		X	
32	X	X	X			

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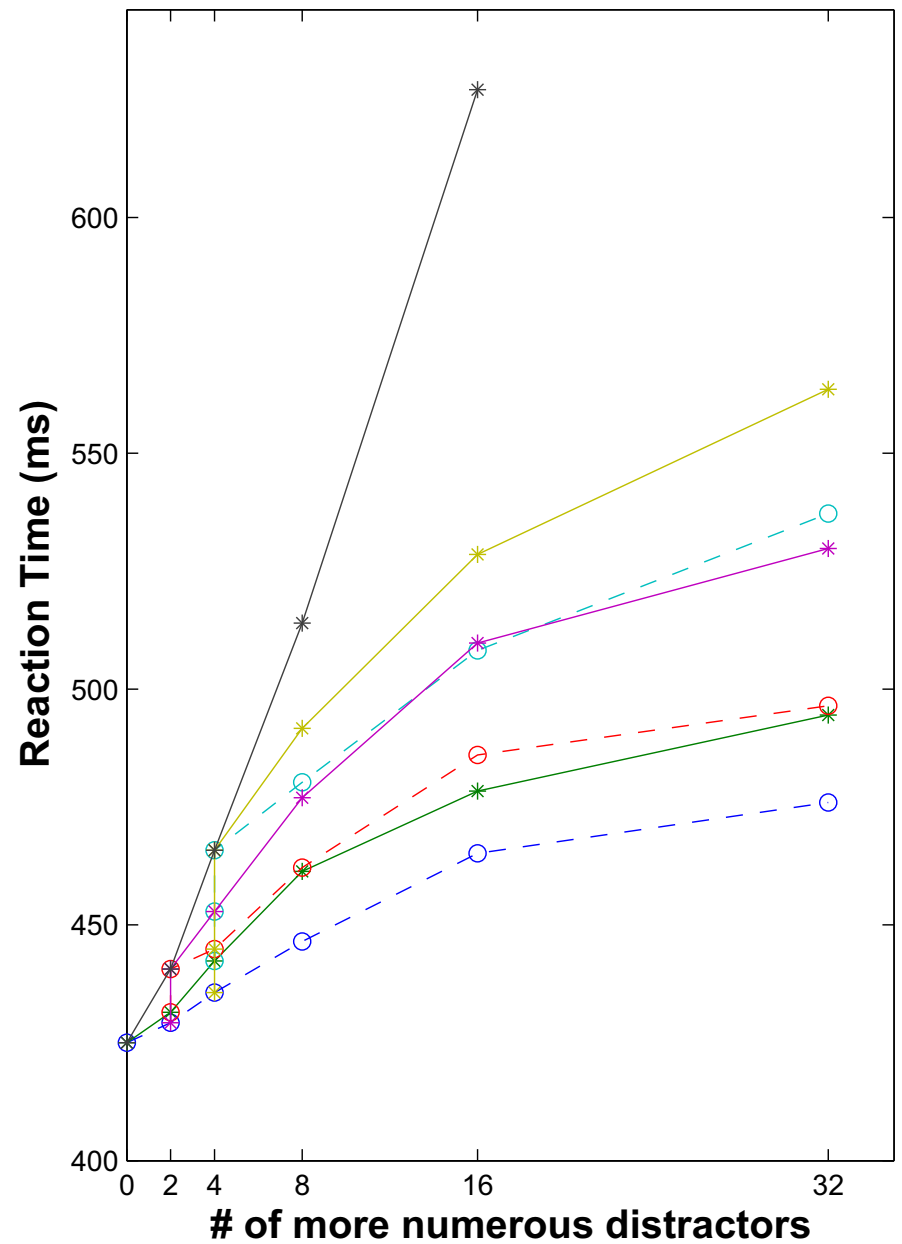


# Wright and Main (unpublished)

## Human Data



## EGS



## **EGS makes strong predictions about intertrial priming.**

Attentional gains on current trial depends on statistics of recent trials.

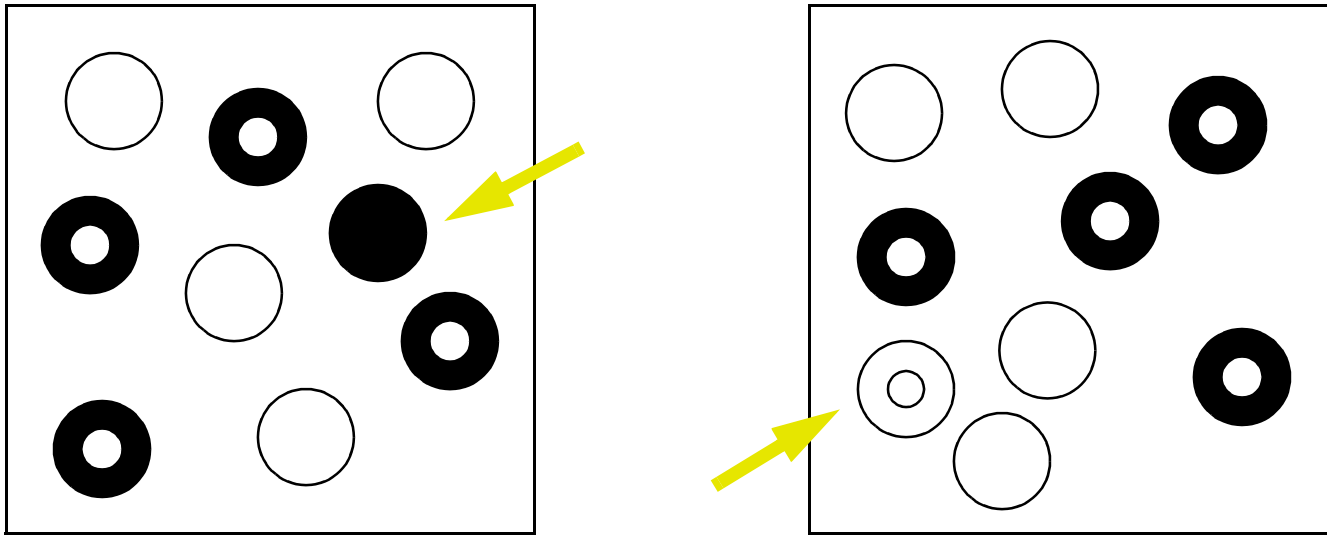
**Had originally hoped to use Wright data to look at how performance on trial  $n$  influenced by trial  $n-1$ .**

Insufficient data



# Kristjánsson and Driver (2008)

## Intertrial priming with oddball conjunction search



target( $t-1$ )  
distractors( $t-1$ )

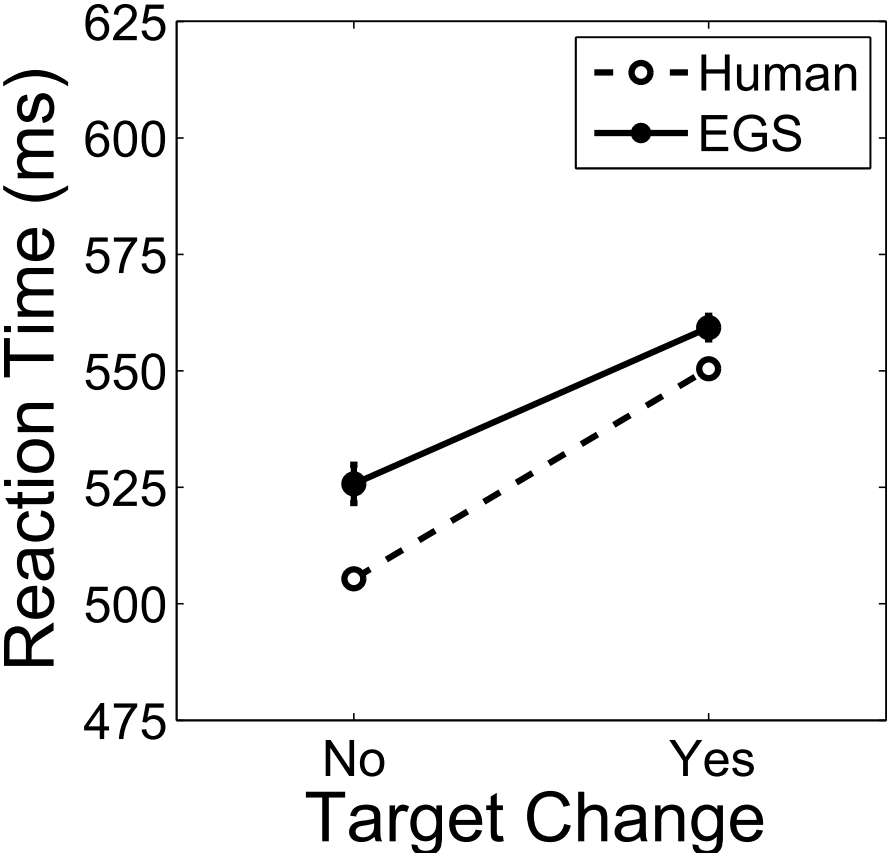


target( $t$ )  
distractors( $t$ )

**Sequential effects in RT based on relationship of target and distractor on trial  $t-1$  to target and distractor on trial  $t$**

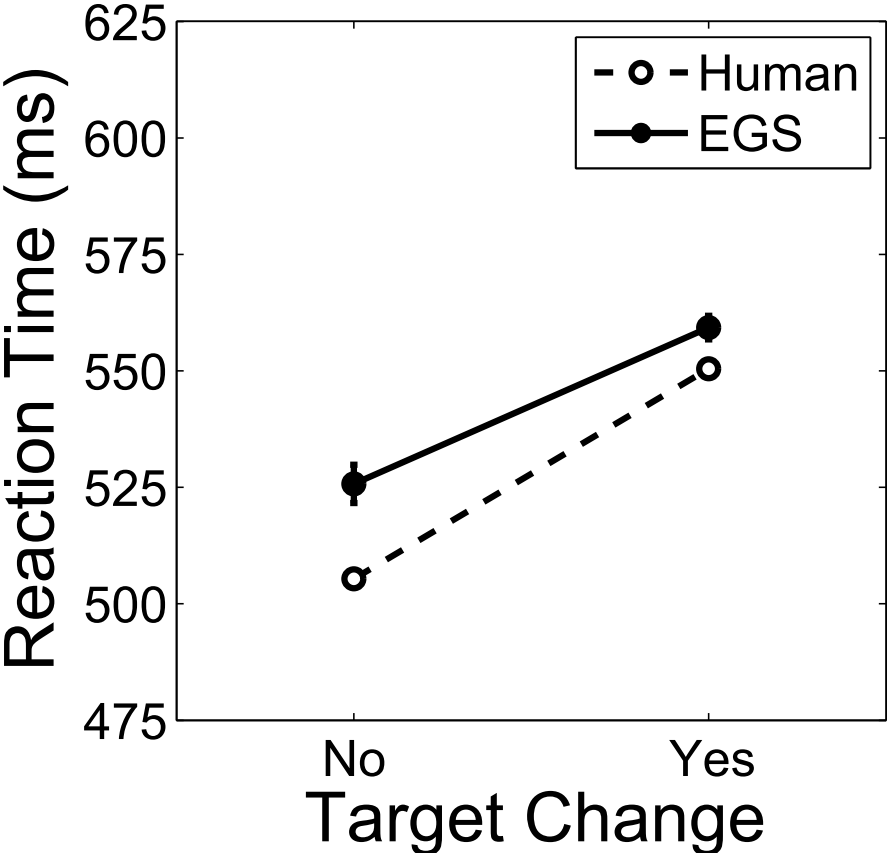
# Kristjánsson and Driver (2008)

## Effect of Target Change

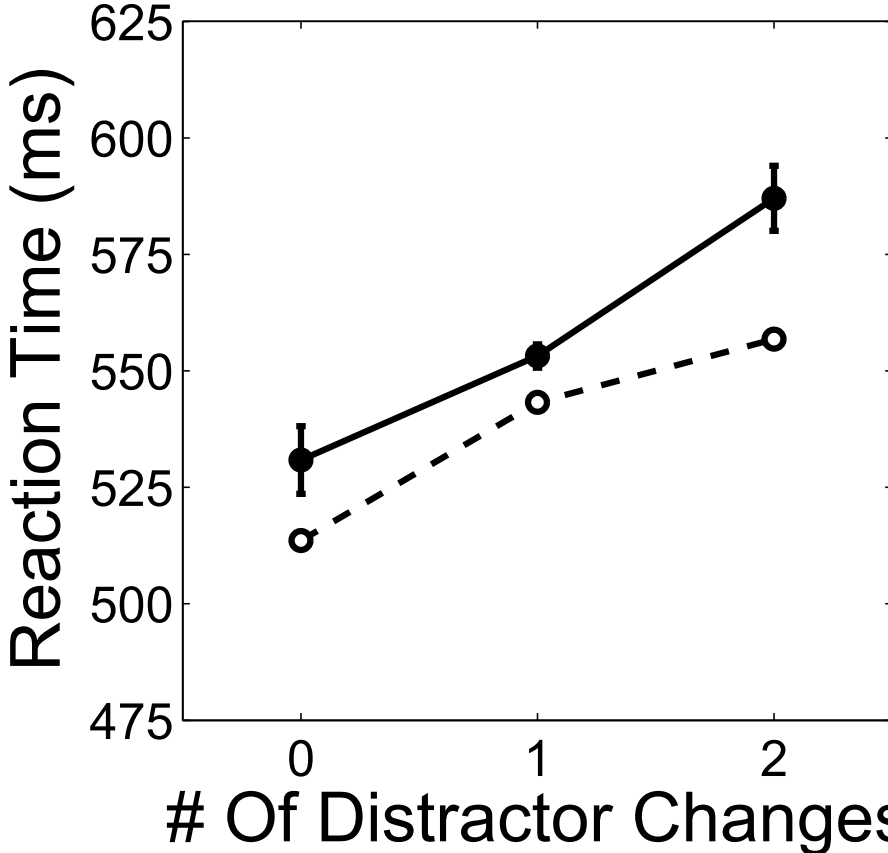


# Kristjánsson and Driver (2008)

### Effect of Target Change

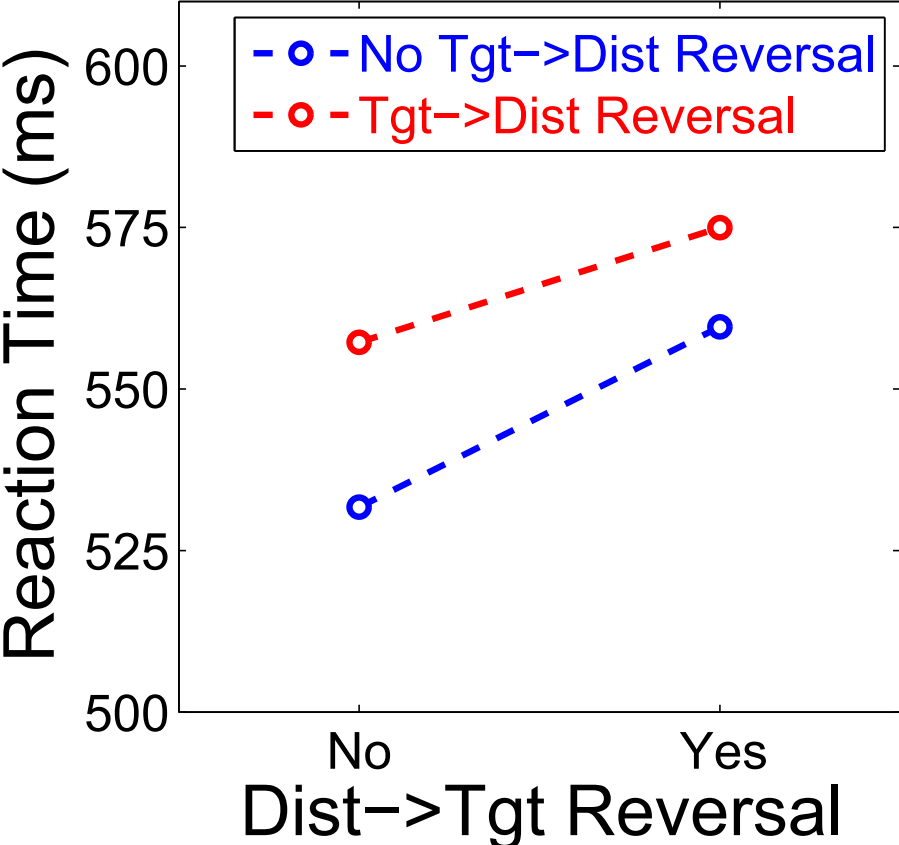


### Effect of Distractor Change

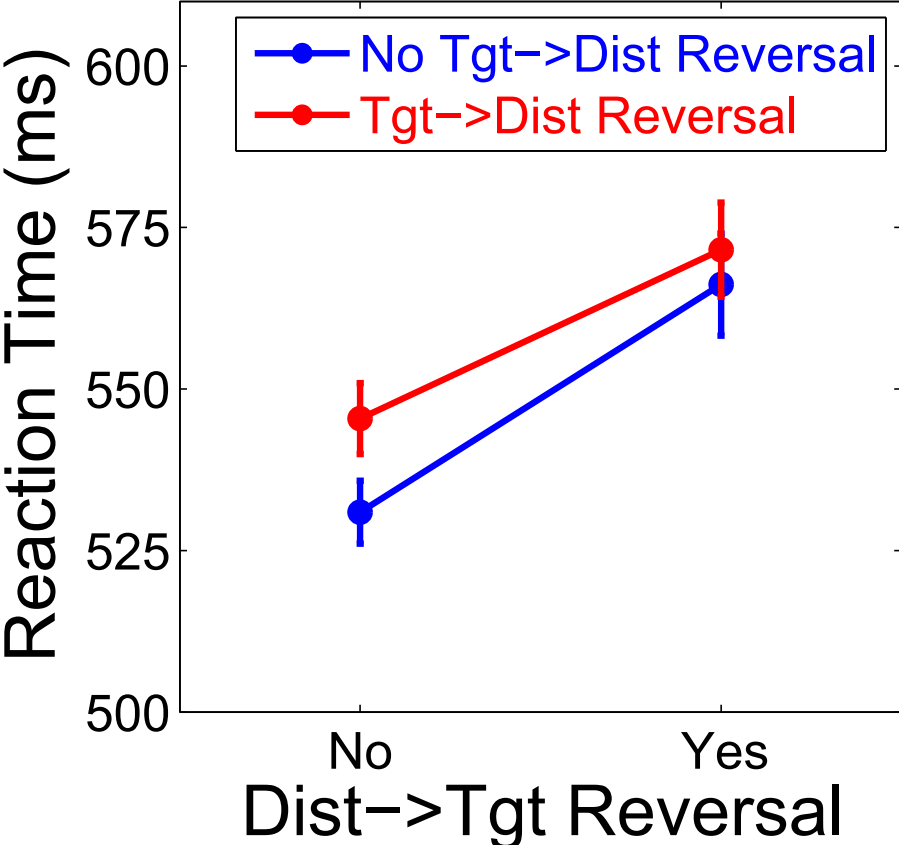


# Kristjánsson and Driver (2008)

## Human

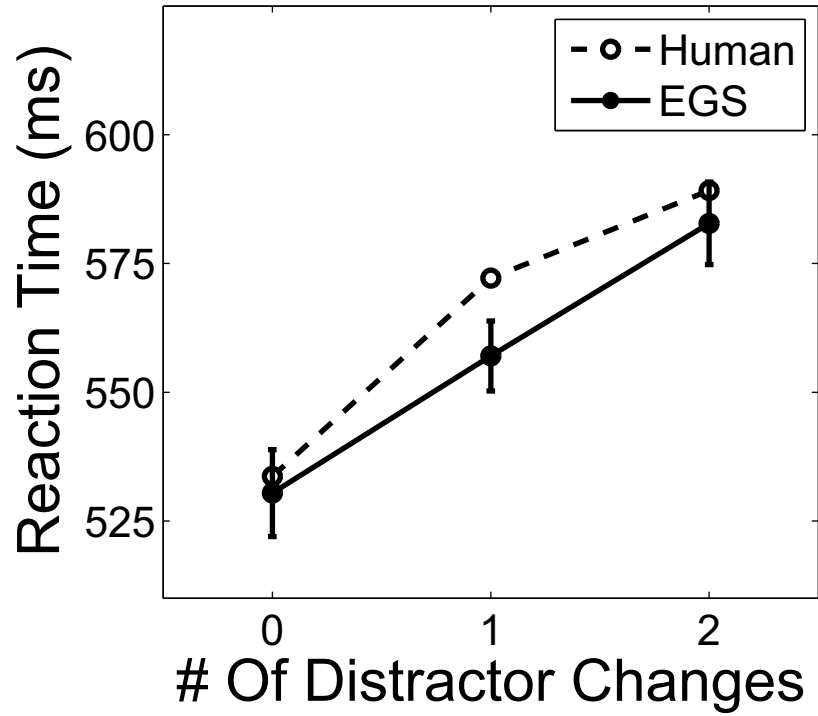


## EGS



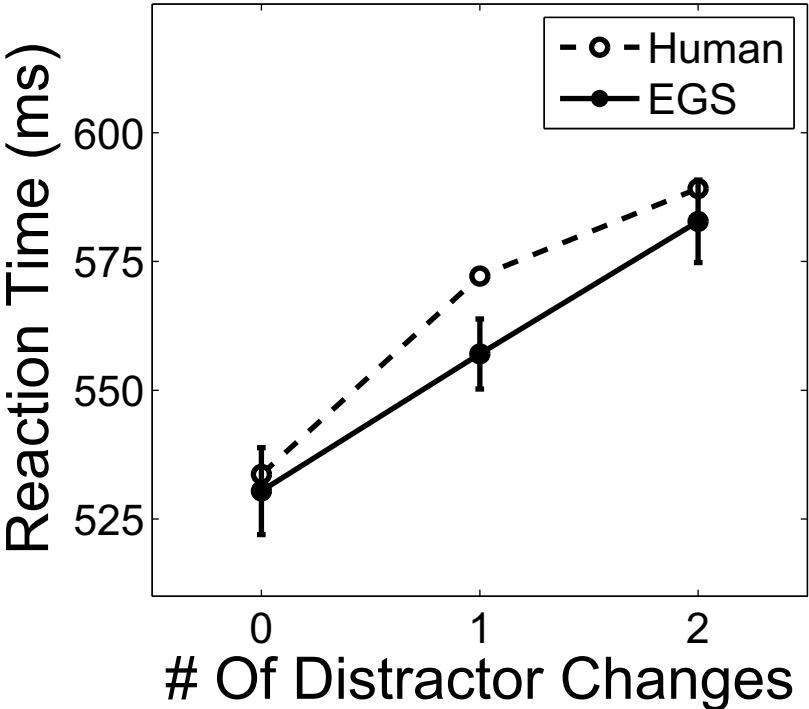
# Kristjánsson and Driver (2008)

Previous Trial Target Absent

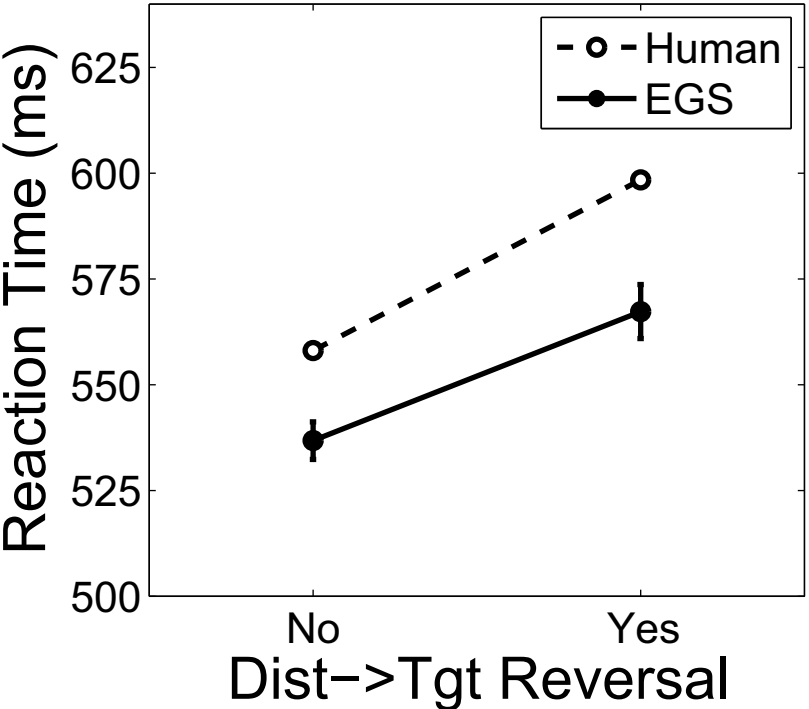


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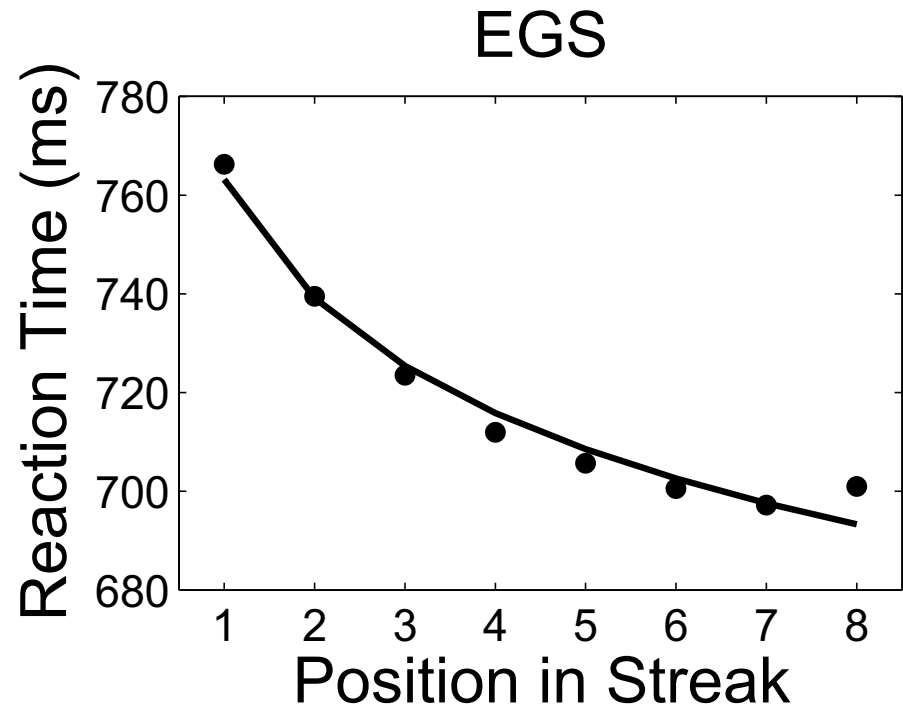
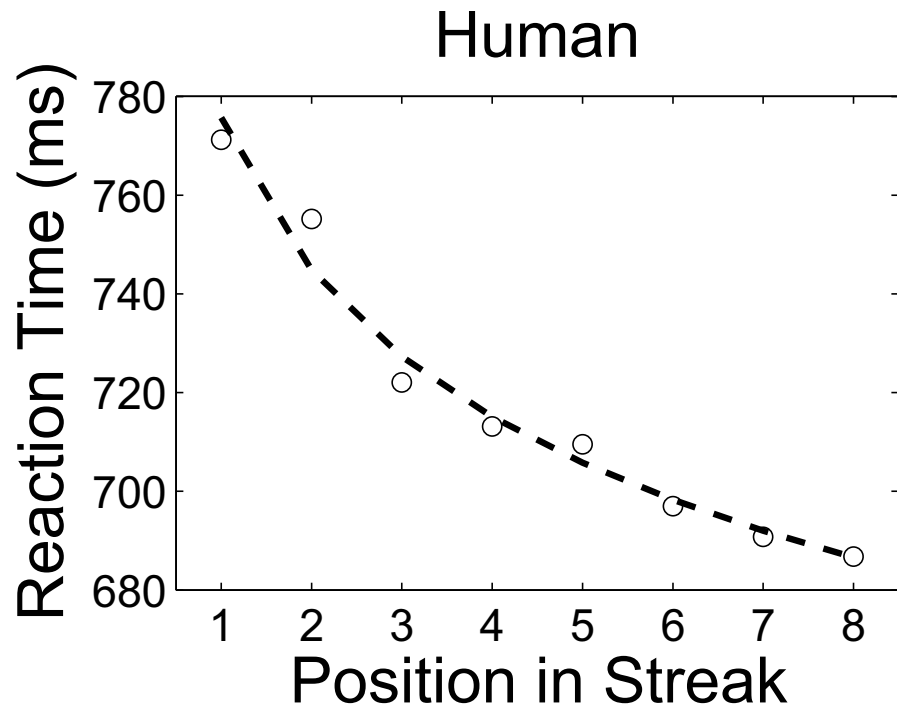


Previous Trial Target Absent



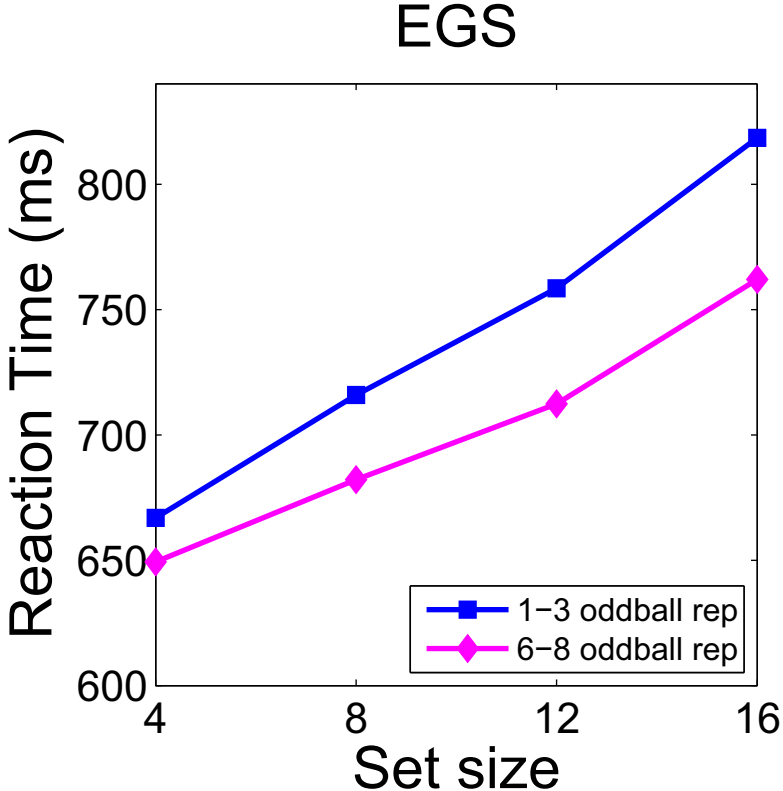
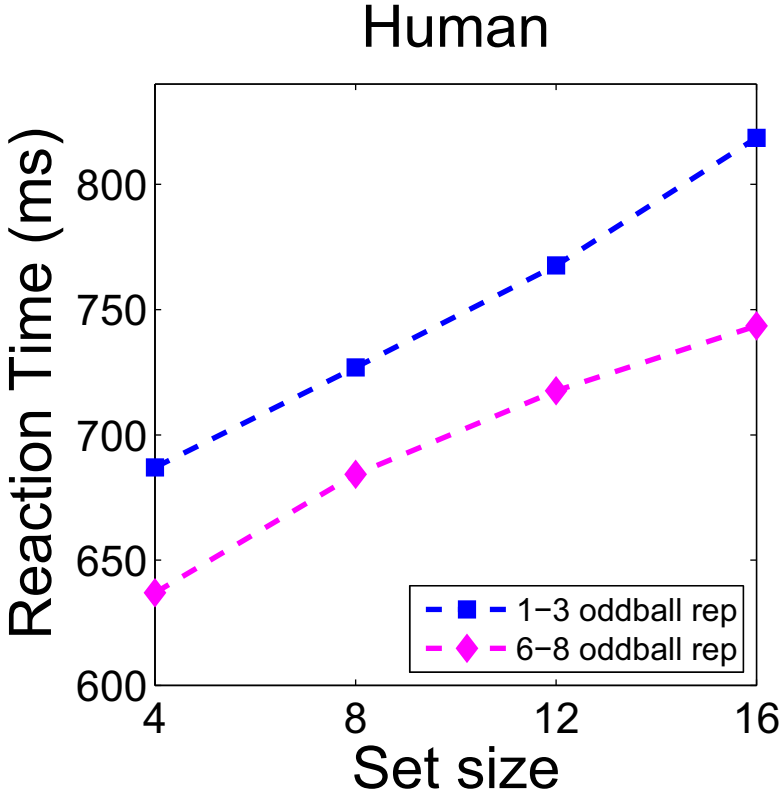
# Kristjánsson, Wang, and Nakayama (2002)

Similar oddball study with streaks of repeated target/distractor combinations

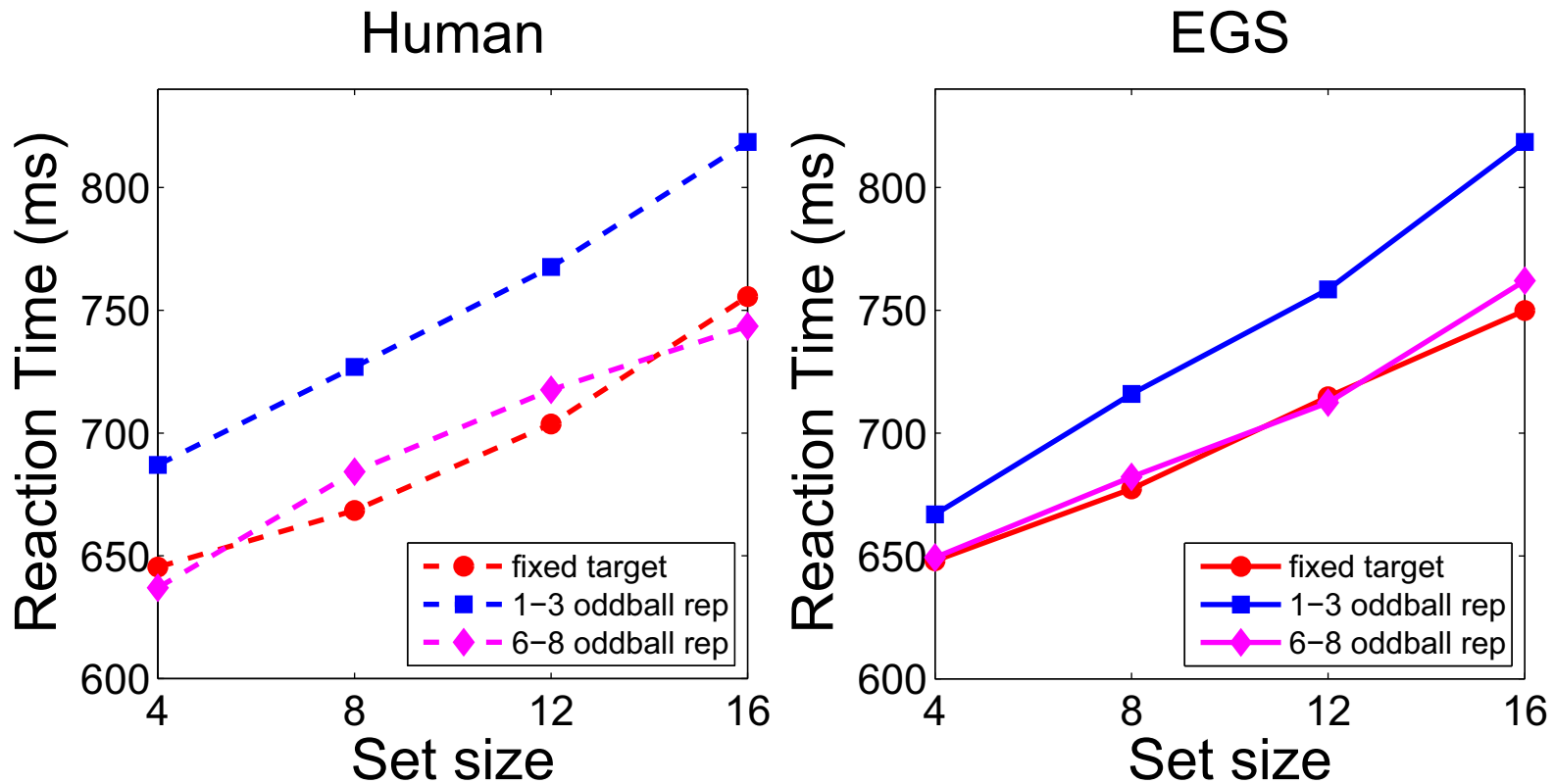




# Kristjánsson, Wang, and Nakayama (2002)

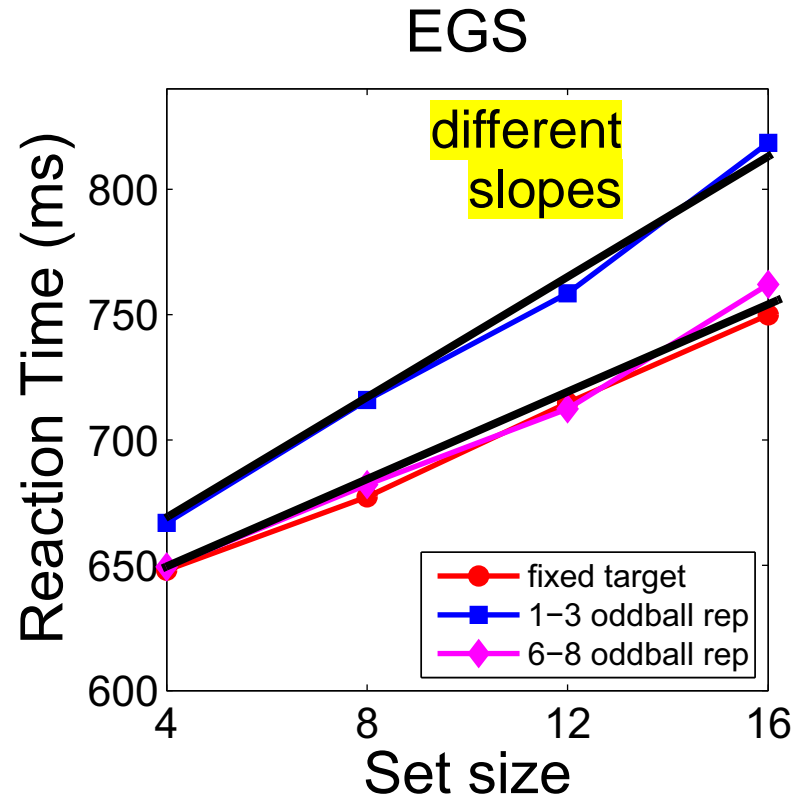
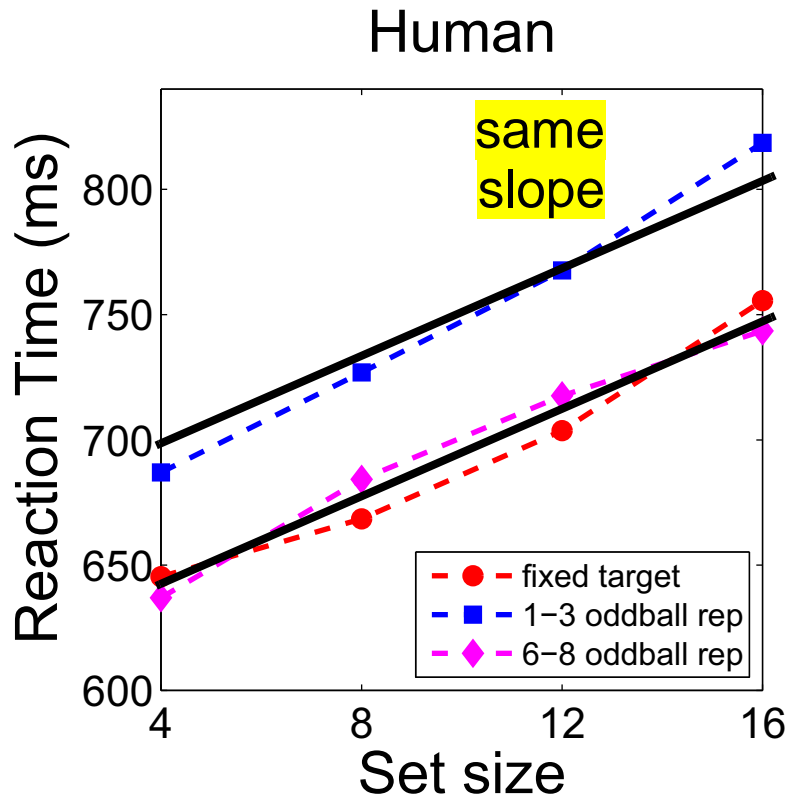


# Kristjánsson, Wang, and Nakayama (2002)



**Control processes involved in specifying fixed target are qualitatively the same as those involved in sequential effects.**

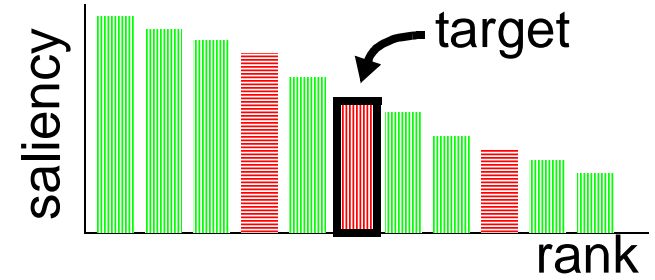
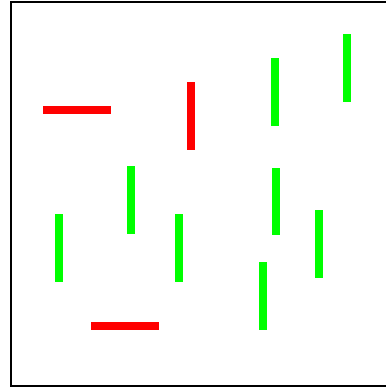
# Modeler Sleaze



**Other findings in literature that priming does not affect search slopes (Kunar et al., 2007; Lamy, 2012; Mozer & Roads, in preparation).**

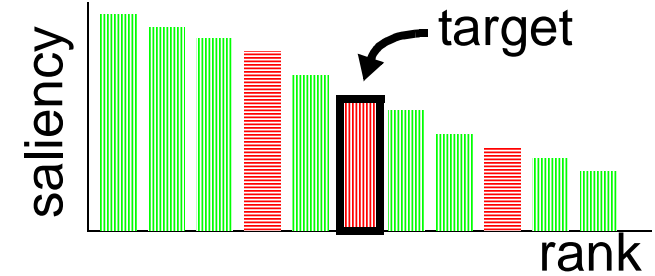
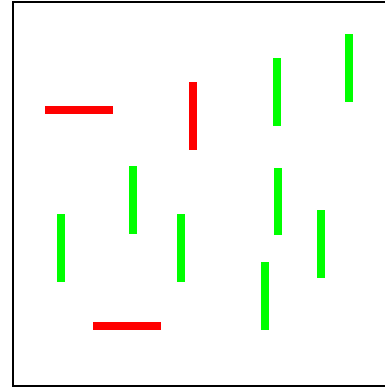
# Long Story Short

**EGS with serial search  
prioritized by saliency rank  
⇒ search-slope reduction.**



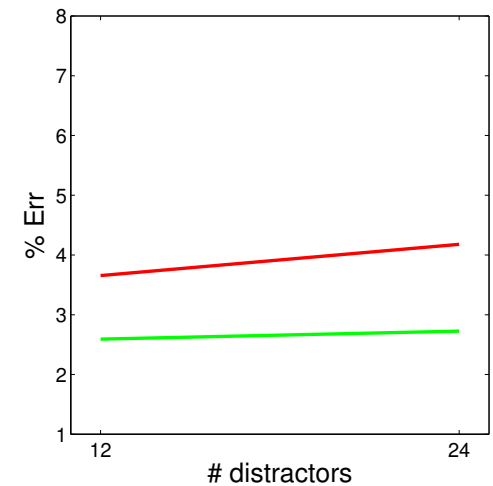
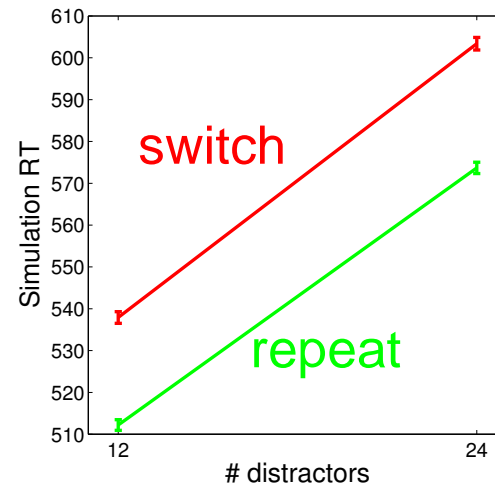
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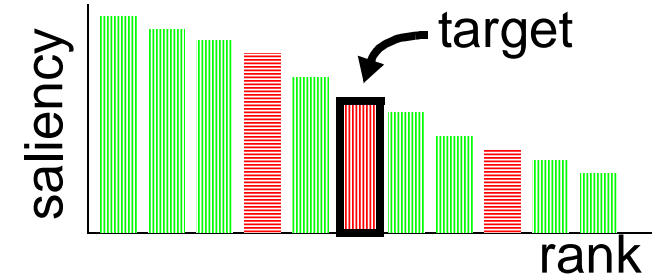
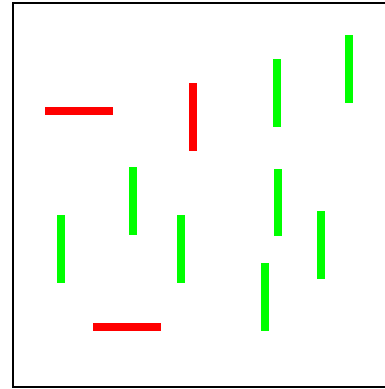
EGS with stochastic selection  
in which elements compete  
proportional to saliency  
⇒ no search-slope reduction.

e.g., one-step accumulator model



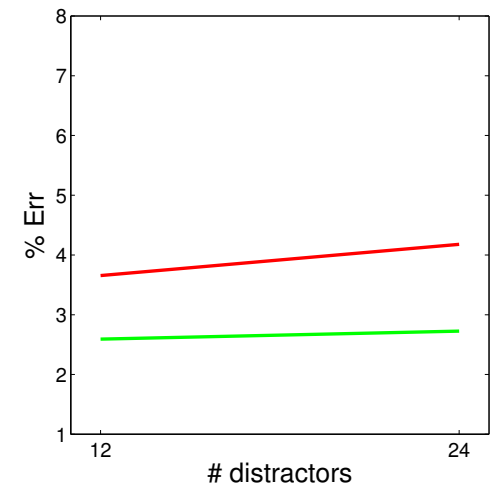
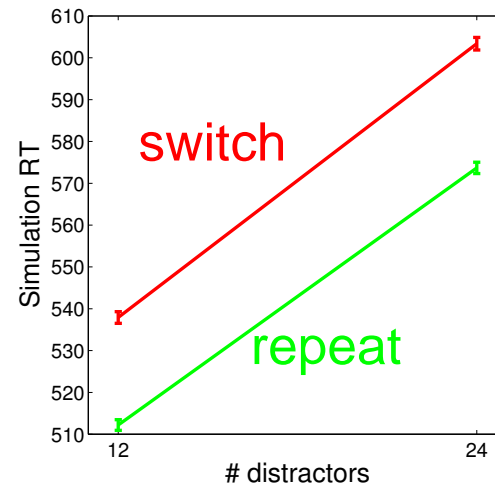
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e.g., one-step accumulator model



## Lamy talk

“no interaction ⇒ intertrial priming does not affect attentional prioritization”

Modeling provides existence proof otherwise.

# Can We Use EGS To Understand Learning At Longer Time Scales?

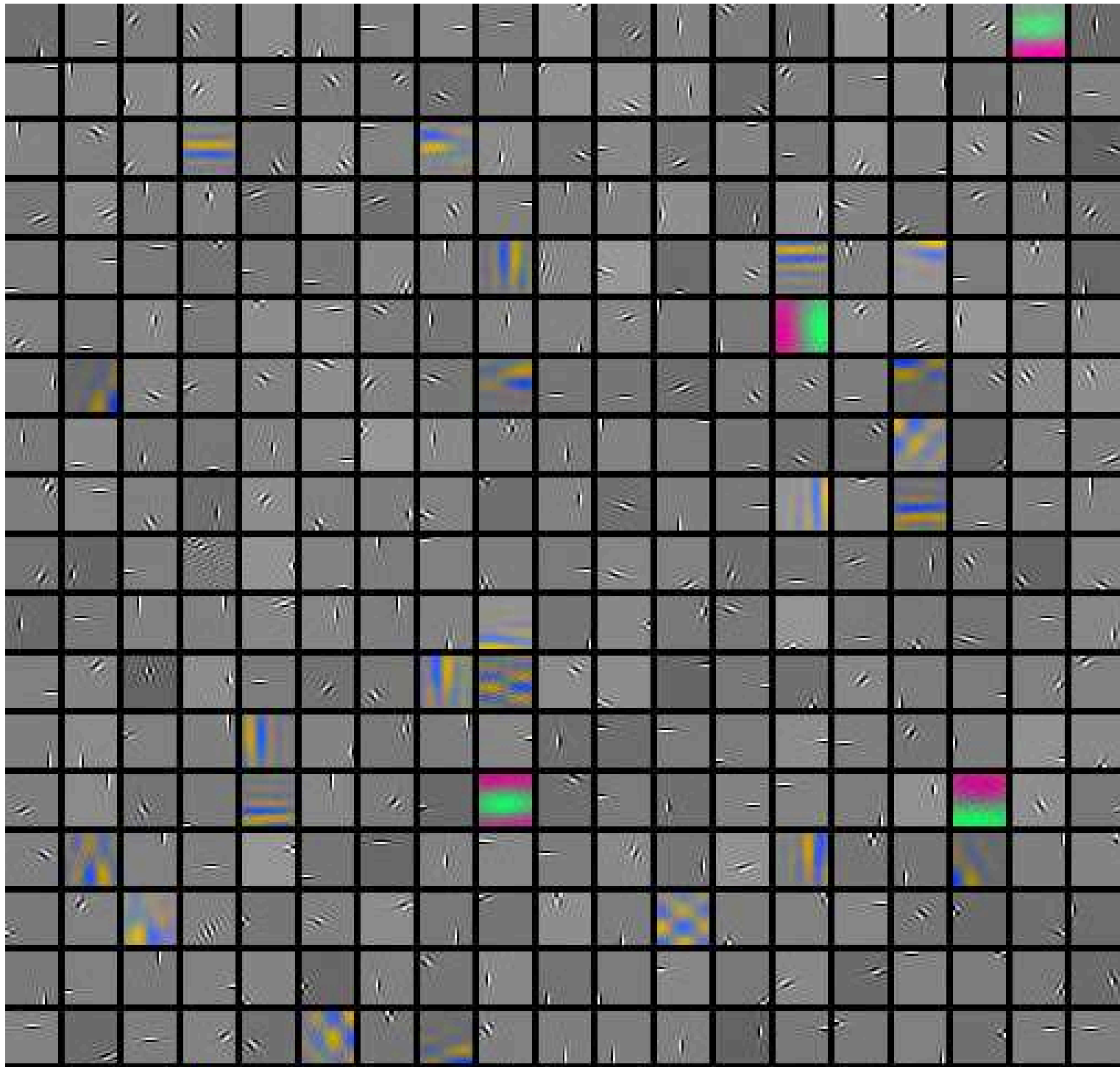
## Initial effort with

- real-world images (MIT LabelMe data base)
- long-term learning to find objects in images



## Exactly same EGS model with

- much lower environment switch probability ( $\sim$  memory decay)
- image based features (ICA/PCA)

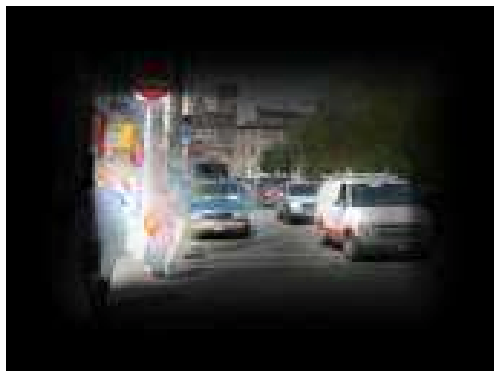
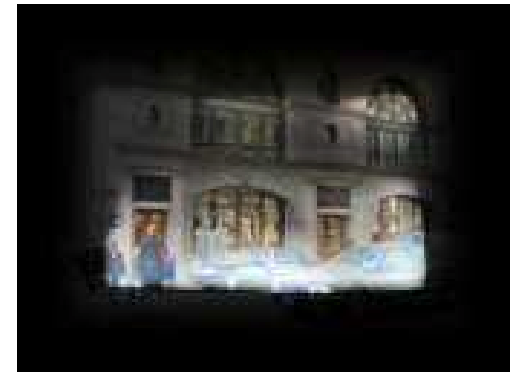
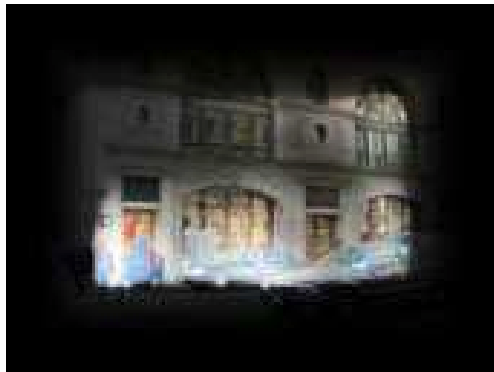
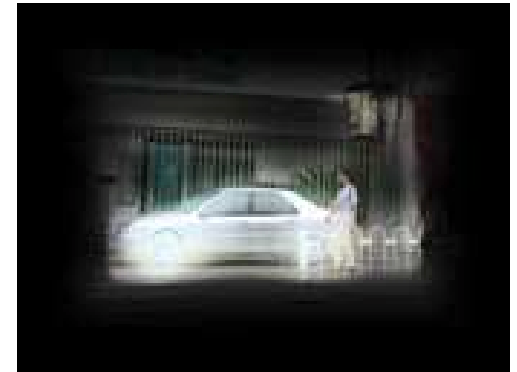
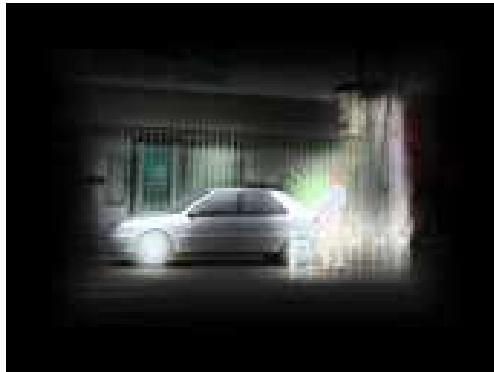




Saliency-Weighted Image: Person

Test Image

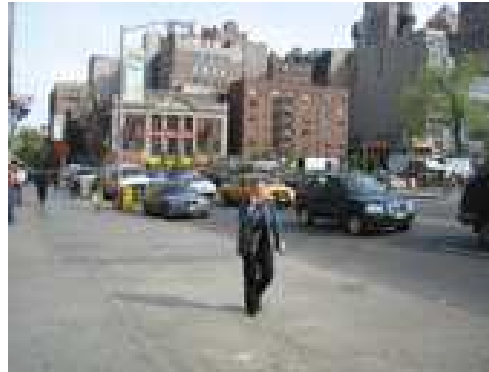
Saliency-Weighted Image; Car



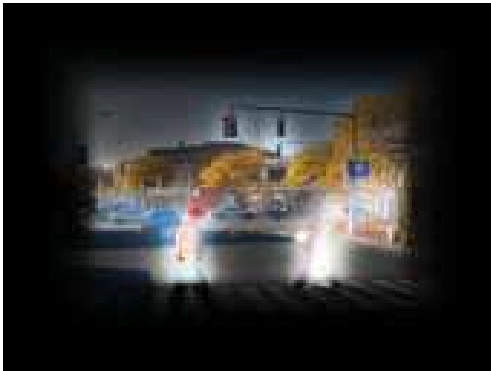
Saliency-Weighted Image; Person



Test Image



Saliency-Weighted Image; Car



## Wolfe's theme

“Beyond feature guidance”

## Mozer's theme

How far can feature guidance take us—can we avoid “syntax”, “semantics”, “scene gist”, etc.? (Similar philosophy as Zelinsky)

In progress: still need to compare to human fixation data, other models

# What Does EGS Offer?

**Formalization of the intuition we share about attentional gains**

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Formalization of the intuition we share about attentional gains

## A computational-level theory of visual search

Goal of attention: target detection  $\Rightarrow$  saliency: target probability

Control emerges as a consequence of statistical inference on the task environment in which an individual operates.

Adaptation (at every time scale) results from updating beliefs about task environment.

# What Does EGS Offer?

Formalization of the intuition we share about attentional gains

A computational-level theory of visual search

## A perspective on attentional control

Intertrial priming effects have been viewed as either

- a passive, bottom up influence on behavior (e.g., Maljkovic & Nakayama, 1994; Pinto, Olivers, & Theeuwes, 2005)
- driven by or modulated by implicit top-down control processes (e.g., Guyer & Müller, 2009; Wolfe, Butcher, Lee, & Hyle, 2003)

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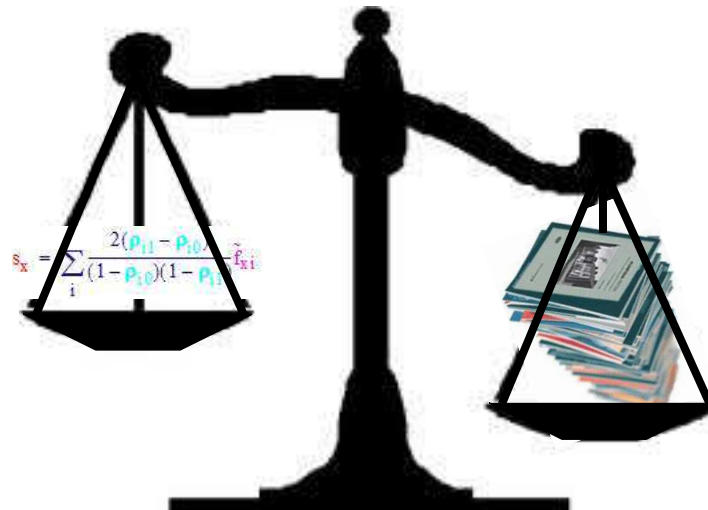
Formalization of the intuition we share about attentional gains

A computational-level theory of visual search

A perspective on attentional control

**Learning is the key to a unified, elegant theory of search.**

⇒ parsimonious explanations of data from many different paradigms and tasks with the same principles.



**STOP HERE MIKE!**



# Search For Die Bohne

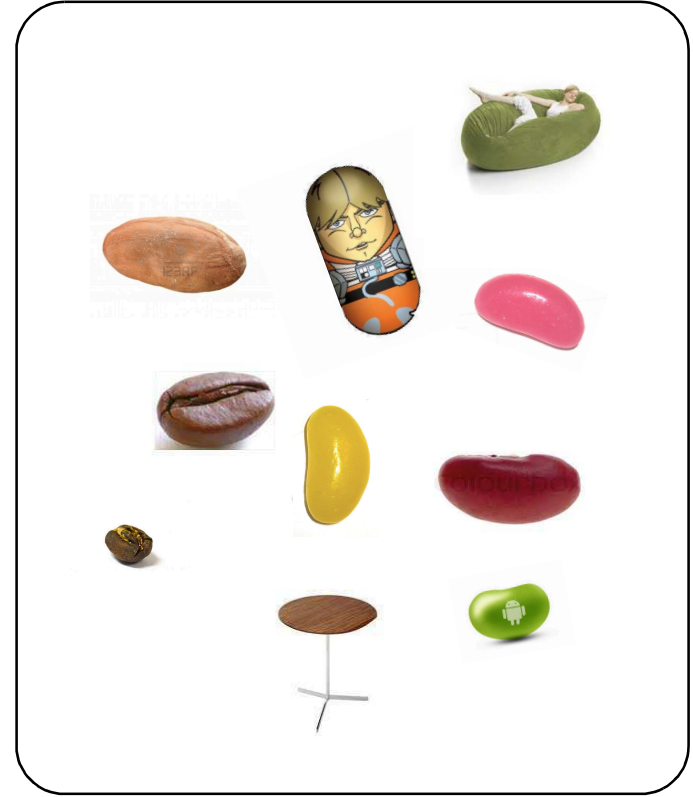


**Even when you recognize the object name...**



**there may be variability in the object's appearance.**

And even when you have a good visual representation of the target...



distractor statistics matter.

# Control Of Attention Depends On Learning Statistical Regularities On Multiple Time Scales

Time Scale	What Is Learned	Example
Coarse	Visual invariants	word $\Rightarrow$ visual features
Intermediate	Structure of environments	reward-associated features (Anderson); object categories (Shiffrin)
Fine	Properties of the immediate environment	white coffee cups; intertrial priming (Müller, Lamy, Becker)

## My hunch

If we successfully understand adaptation on the finest time scale, we'll be able to handle coarser time scales as well.

(Same learning mechanisms, different decay constants)

# Assumptions Of Environment Model

1. Feature detectors are conditionally independent of one another.

$$P(\mathbf{F}_x | \mathbf{T}_x, \rho) = \prod_i P(F_{xi} | \mathbf{T}_x, \rho)$$

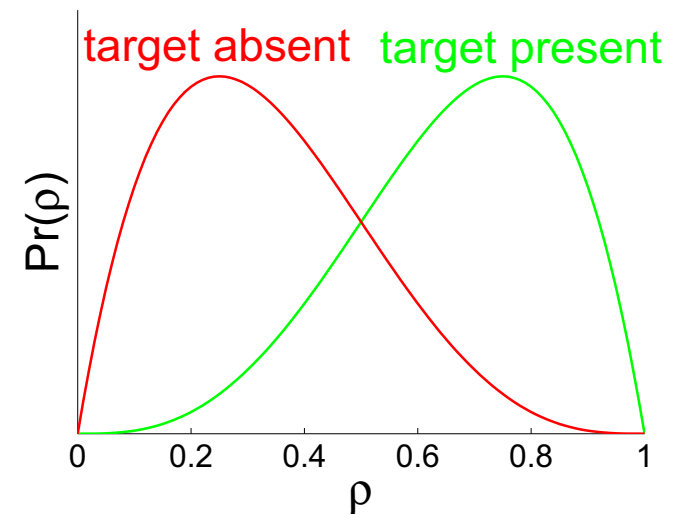
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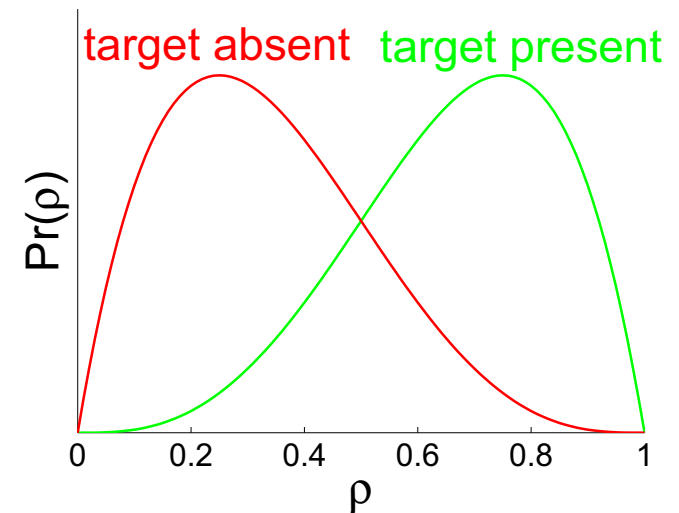
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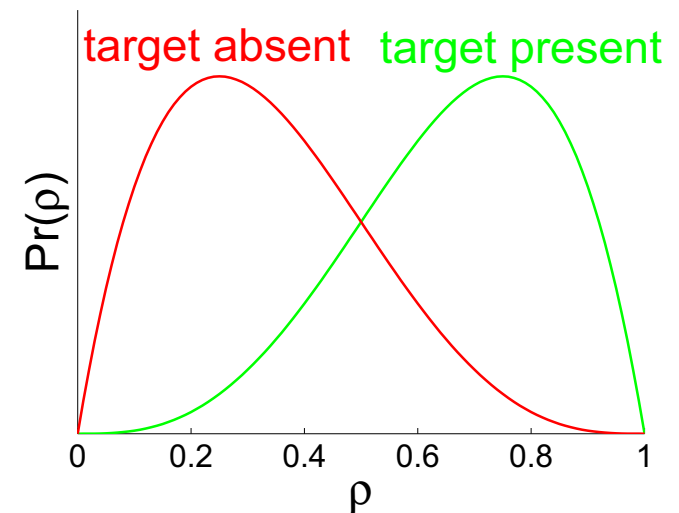
$\rho_{it}$  is unknown but learned through experience.

3. Prior to experience, all features are considered relevant.

$\rho_{i1}$  initialized to be greater than  $\rho_{i0}$

4. Environment is nonstationary.

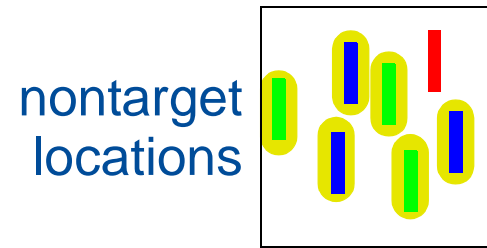
With probability  $\lambda$ , environment and/or task can change.





# How Are Environment Statistics ( $\rho$ ) Estimated?

As locations are inspected during a trial, a supervisory process labels each element as target or nontarget.



Given these observations, update the  $\rho_{it}$  via Bayesian inference.

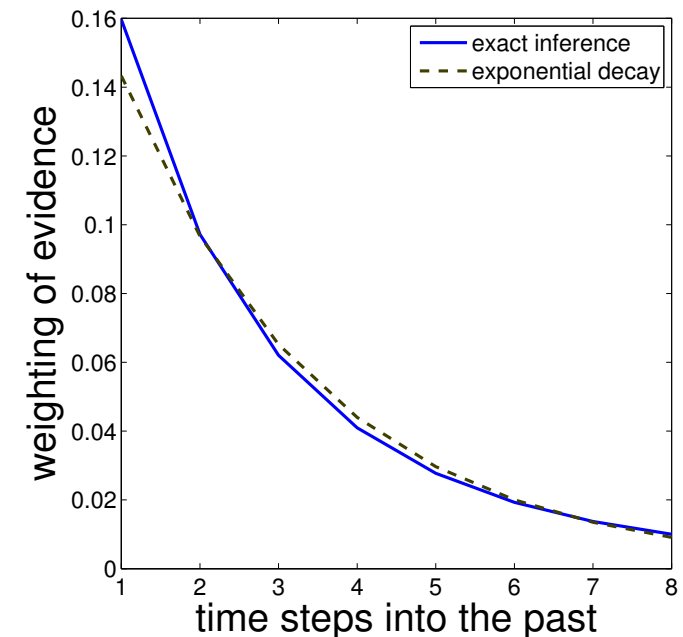
## Exact inference

$\rho_{it}$  is a mixture-of-Betas RV with mixture length linear in number time steps

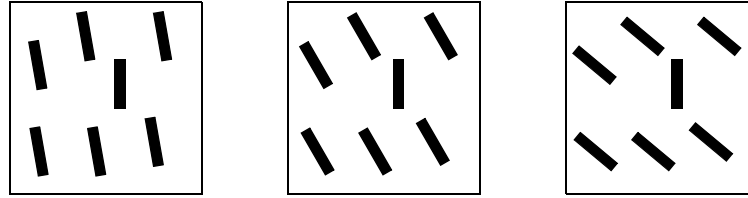
easy to simulate, hard to conceive of as biologically plausible

## Approximately...

update a decaying average of each  $\rho_{it}$  based on the new observations

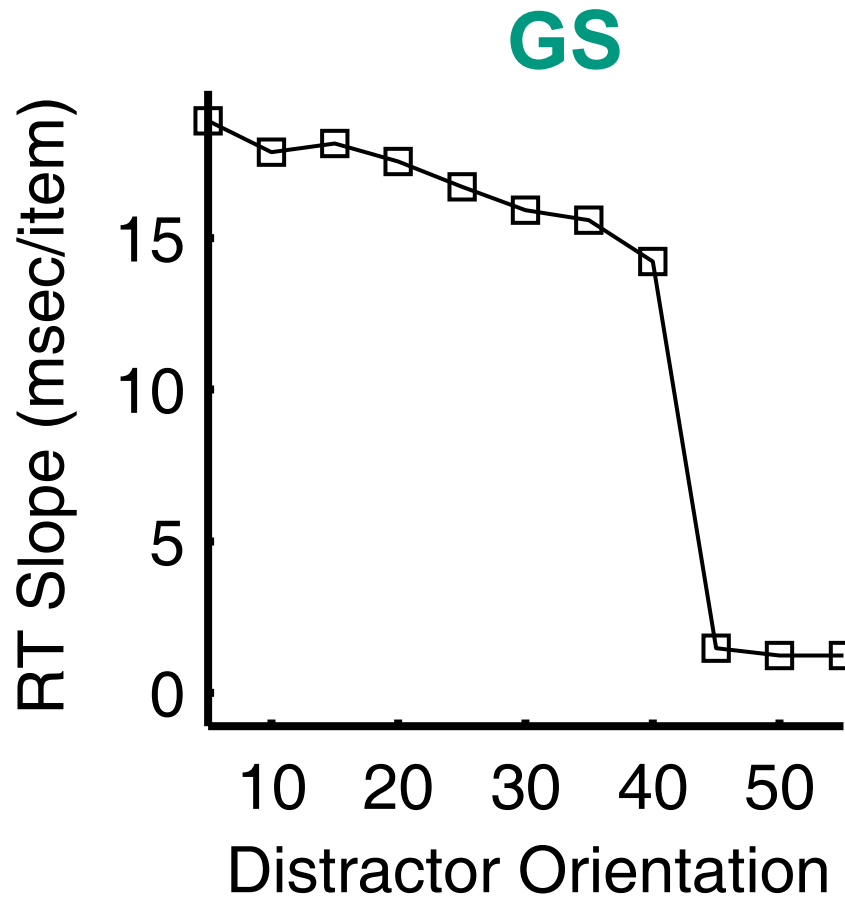
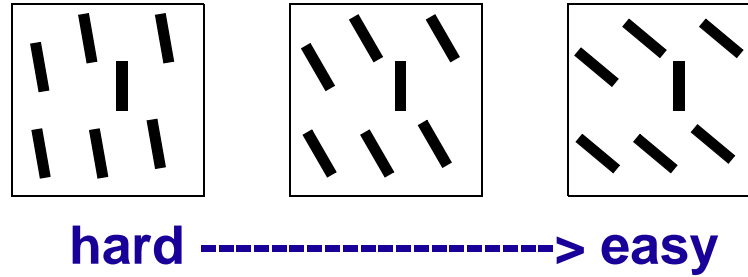


# Target-Distractor Similarity Effects 1

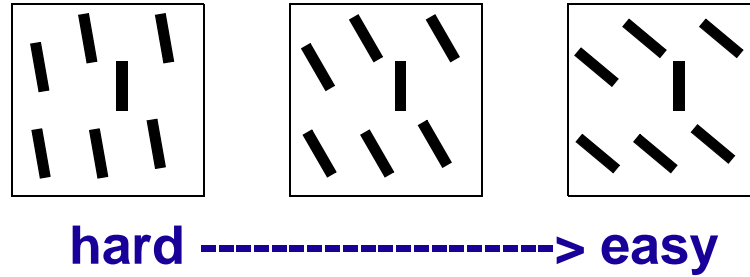


hard -----> easy

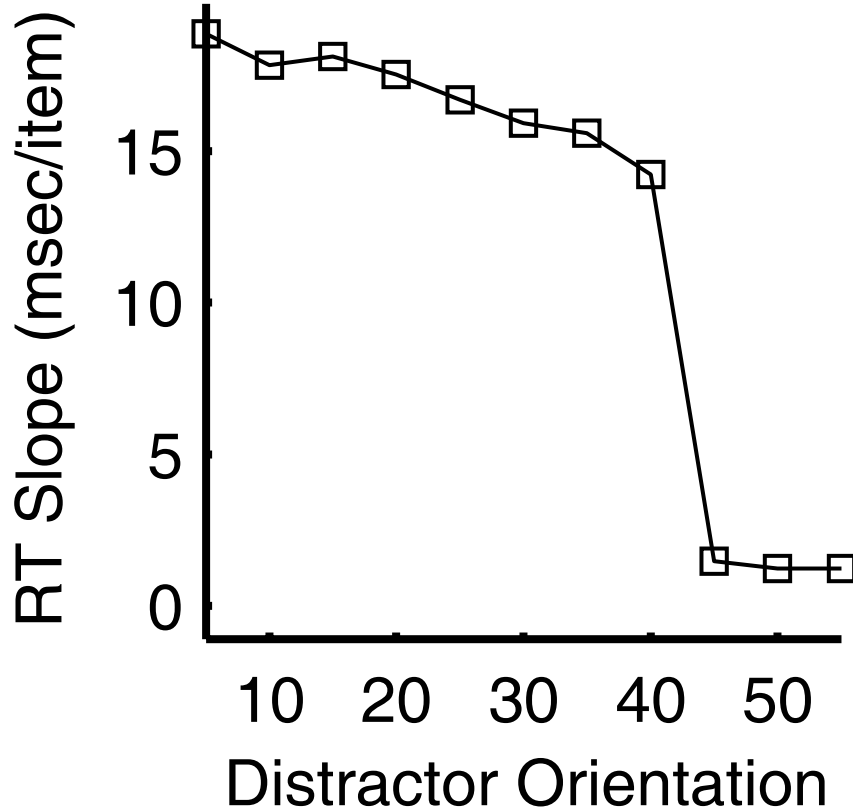
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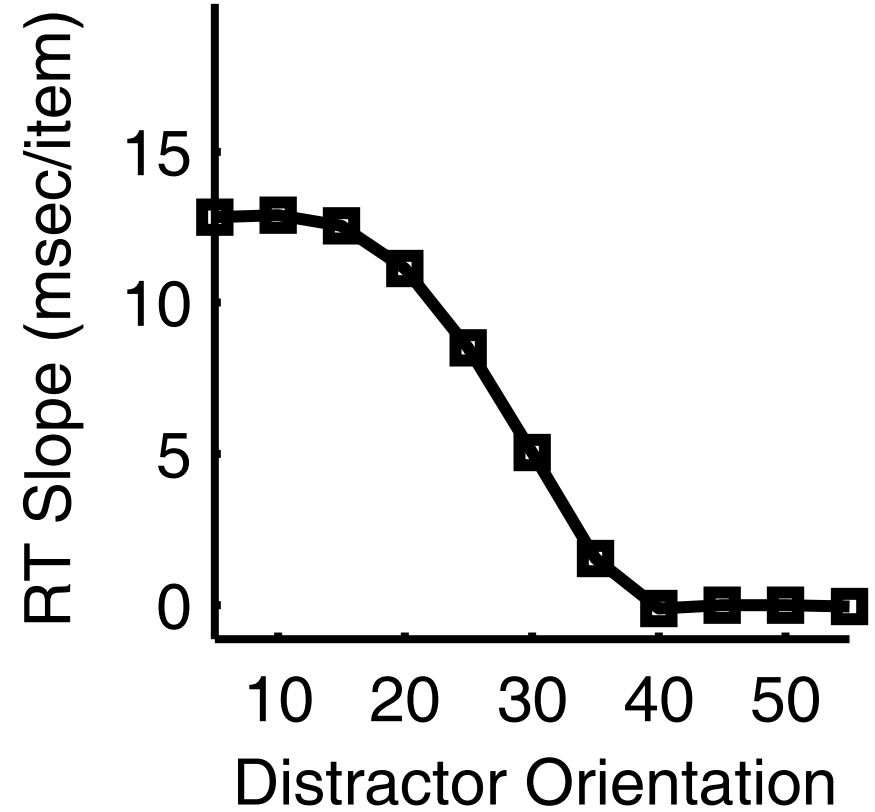
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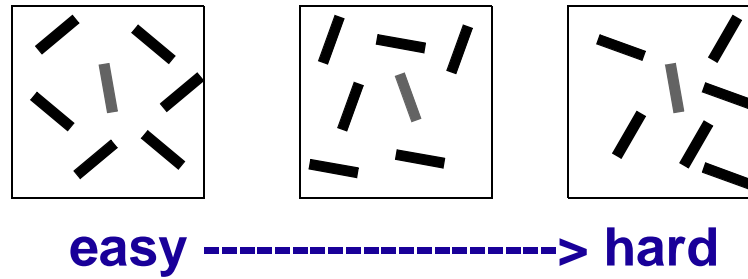
**GS**



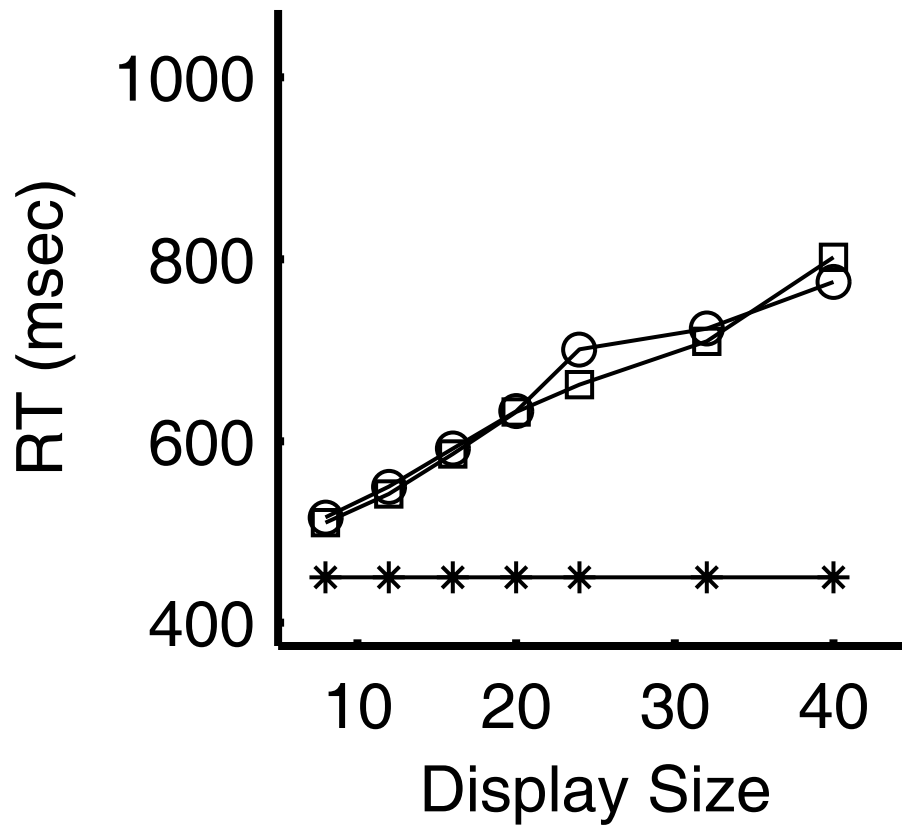
**EGS**



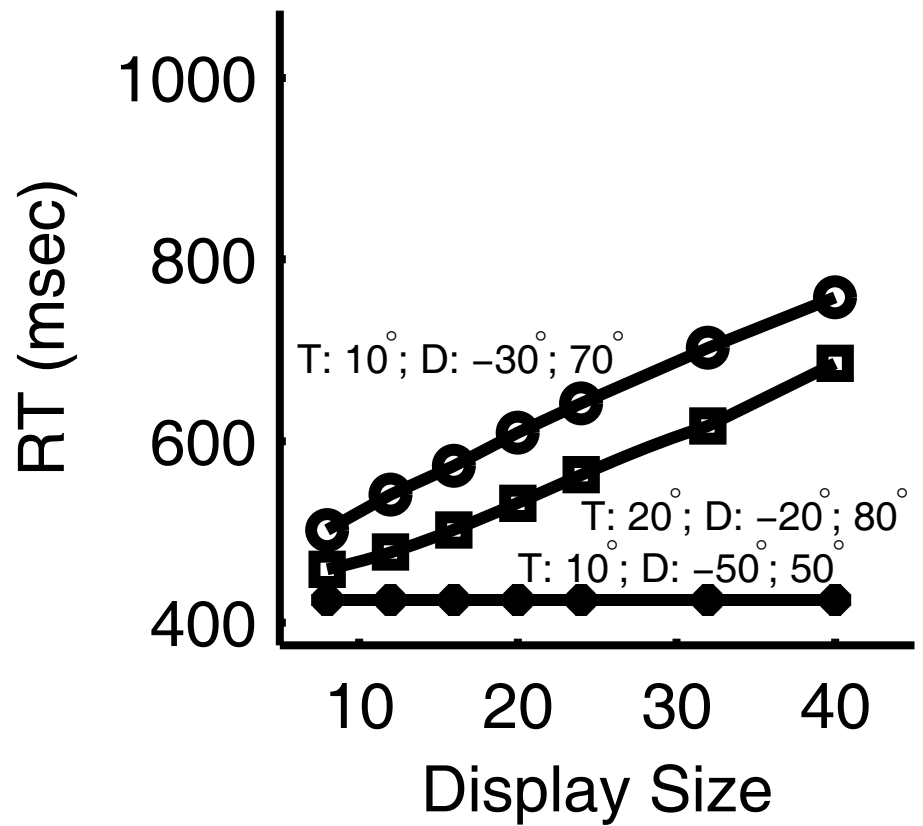
# Target-Distractor Similarity Effects 2



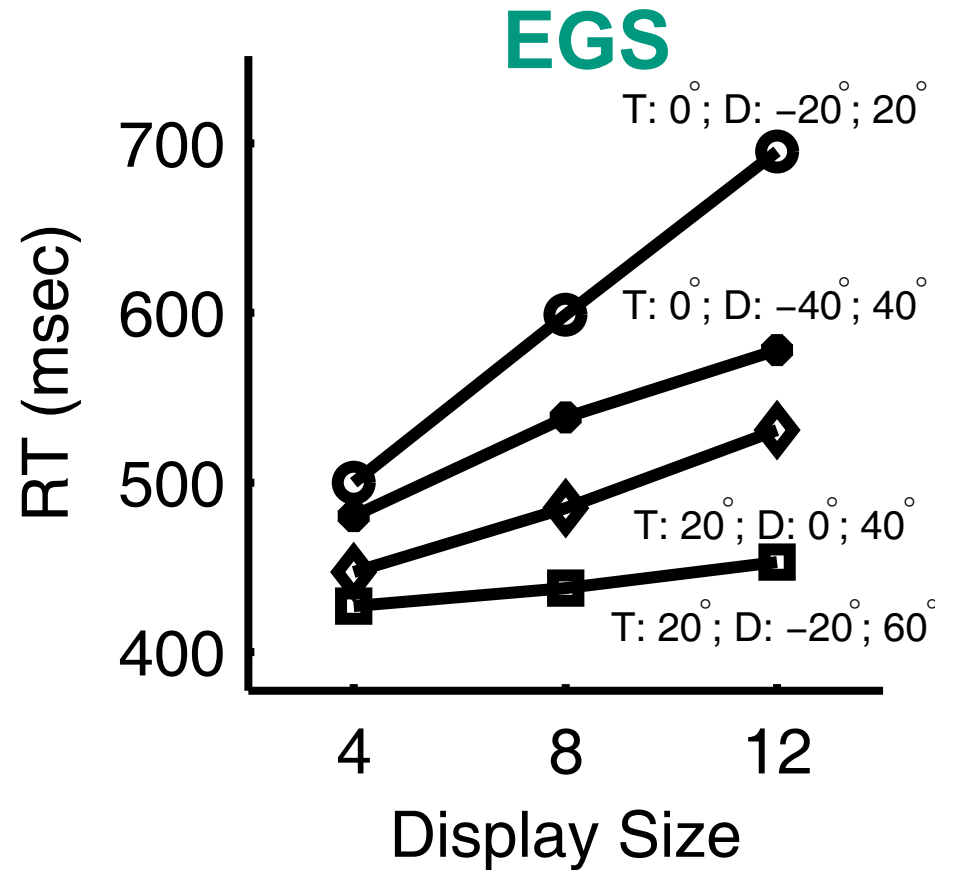
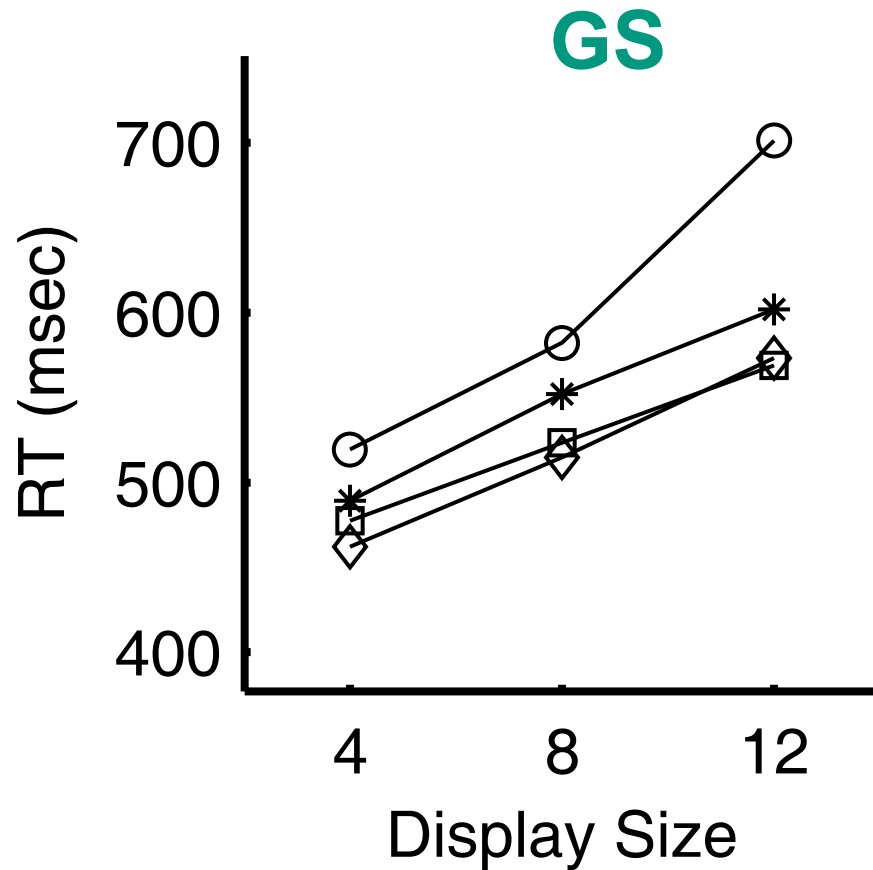
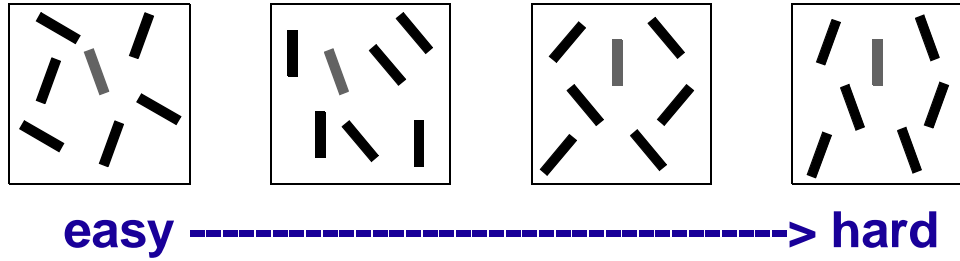
GS



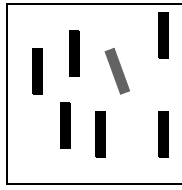
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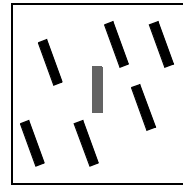
# Target-Distractor Similarity Effects 3



# Target-Distractor Similarity Effects 4

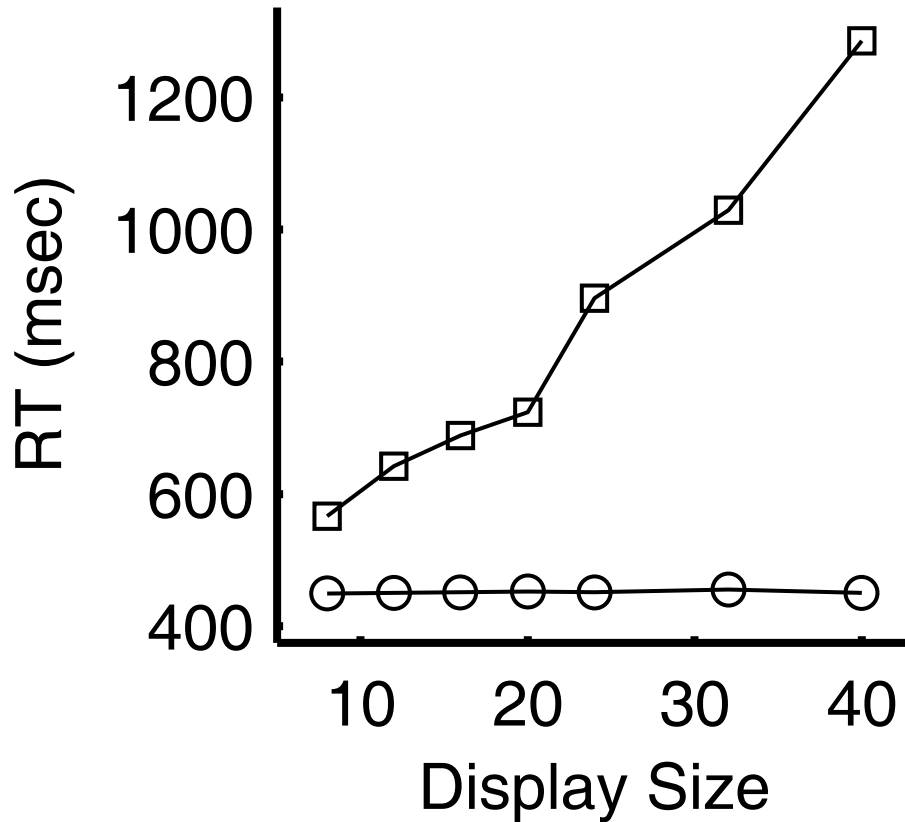


easy

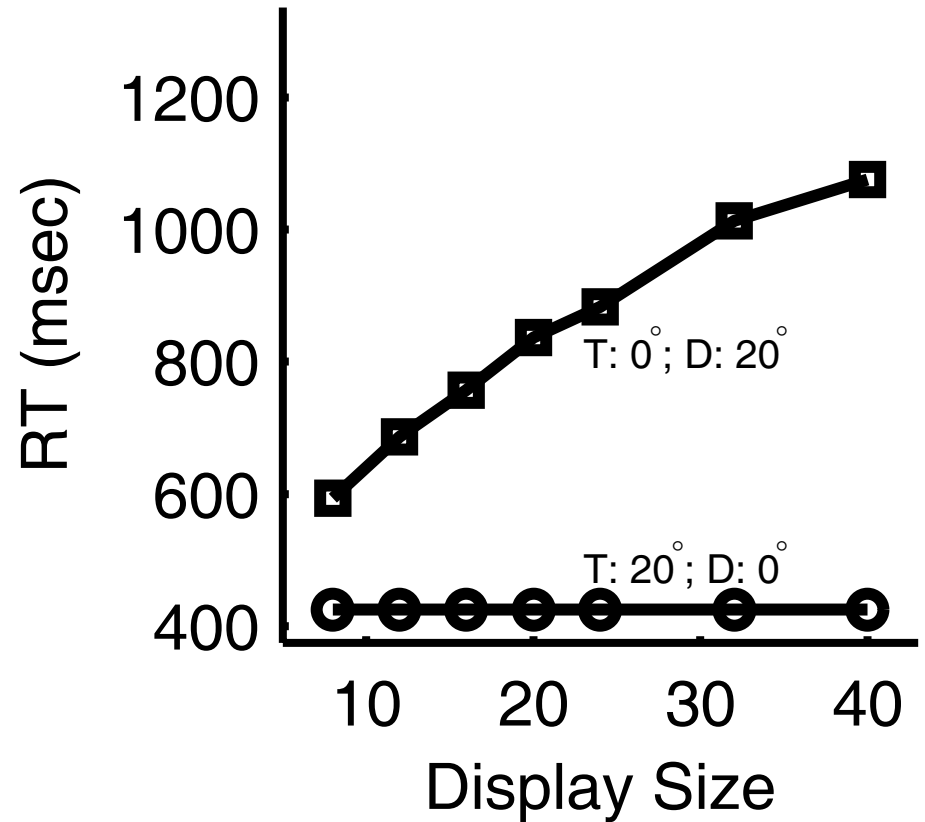


hard

GS



EGS



# What It Boils Down To





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- **Generate stimulus sequence corresponding to experiment.**

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- **Initialize task statistics**

$$p_{i0} \sim \theta, p_{i1} \sim \varphi, \text{ such that } \theta < \varphi$$

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- **On each trial, perform feature extraction on display.**

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- **Compute saliency at each location  $x$**

$$s_x = \sum_i \sum_{t=0}^1 \frac{1-2t}{\rho_{it}(1-\rho_{it})} (f_{xi} - \rho_{it})^2$$

# What It Boils Down To

- **Generate stimulus sequence corresponding to experiment.**
- **Initialize task statistics**
- **On each trial, perform feature extraction on display.**
- **Compute saliency at each location  $x$**
- **Determine response time based on ranking**

$$\text{Response\_time} = \mu_0 + \mu_1 \text{ saliency\_ranking\_of\_target}$$

# What It Boils Down To

- **Generate stimulus sequence corresponding to experiment.**
- **Initialize task statistics**
- **On each trial, perform feature extraction on display.**
- **Compute saliency at each location  $x$**
- **Determine response time based on ranking**
- **Update task statistics based on current trial feature activity**

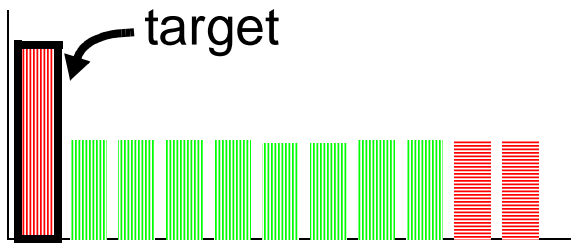
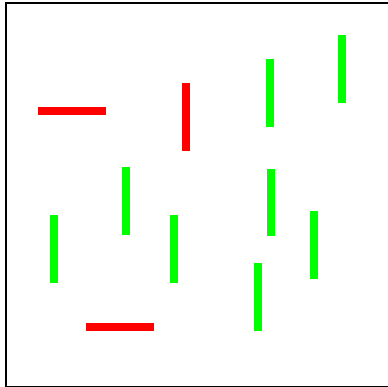
$$\alpha_{it} \leftarrow \lambda \alpha_{it}^0 + (1 - \lambda) \left( \alpha_{it} + \sum_{x \in \chi_t} f_{xi} \right)$$

$$\beta_{it} \leftarrow \lambda \beta_{it}^0 + (1 - \lambda) \left( \beta_{it} + \sum_{x \in \chi_t} 1 - f_{xi} \right)$$

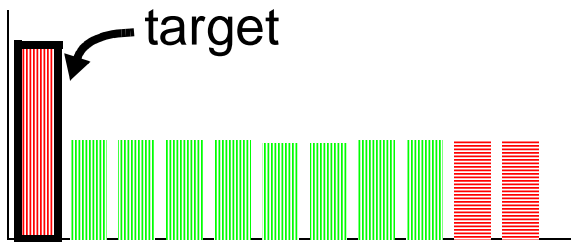
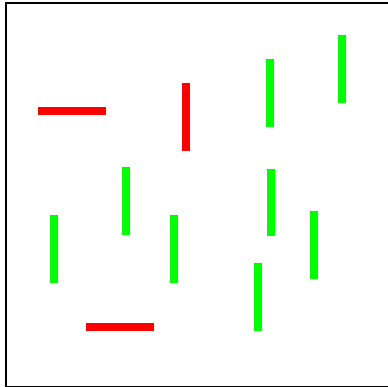
where  $\rho_{it} = \frac{\alpha_{it}}{(\alpha_{it} + \beta_{it})}$

Basically, compute mean activity of feature when target is present and when target is absent.

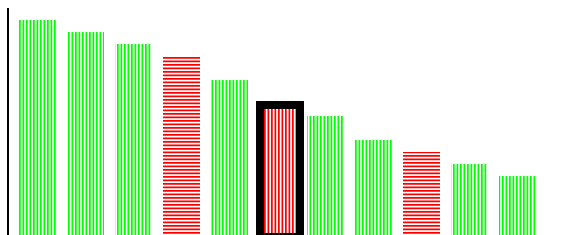
# TRIAL 1



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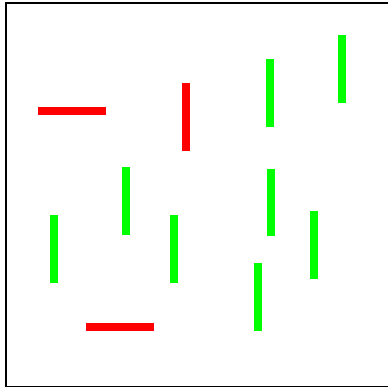


Noisy rank-based  
prioritization of  
display elements



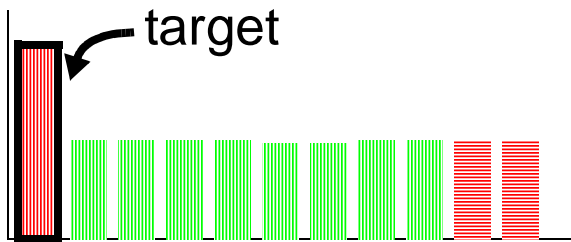
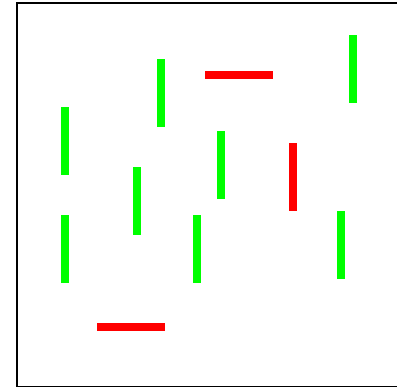


TRIAL 1

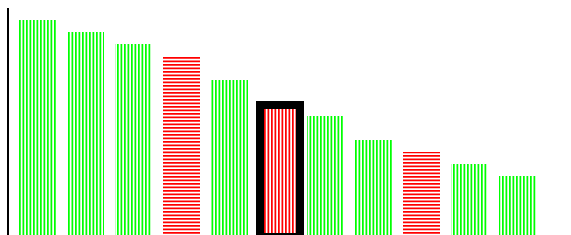


↑↑ red gain  
↓↓ vertical gain

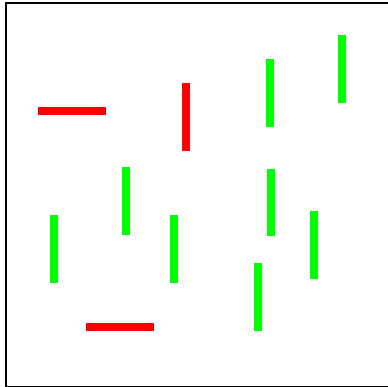
TRIAL 2



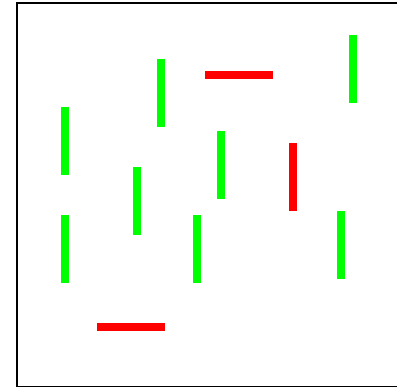
Noisy rank-based  
prioritization of  
display elements



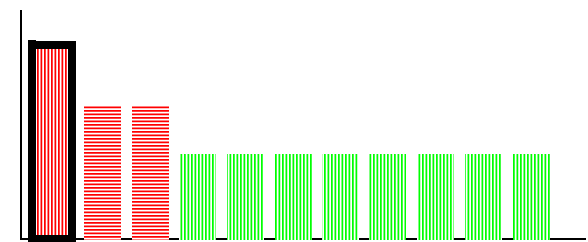
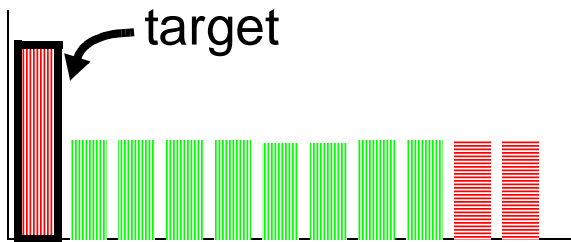
TRIAL 1



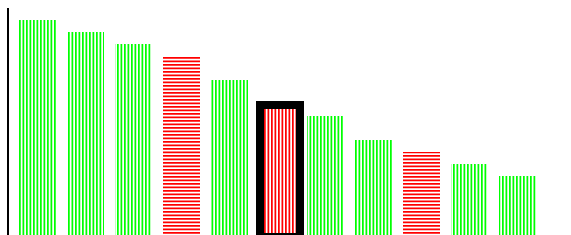
TRIAL 2



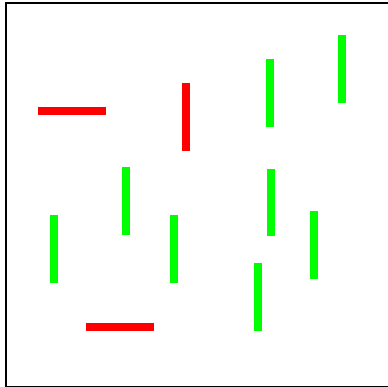
↑↑ red gain  
 ↓↓ vertical gain



Noisy rank-based  
 prioritization of  
 display elements

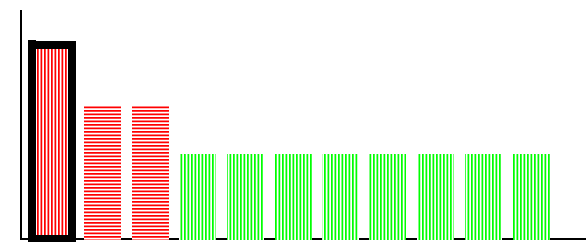
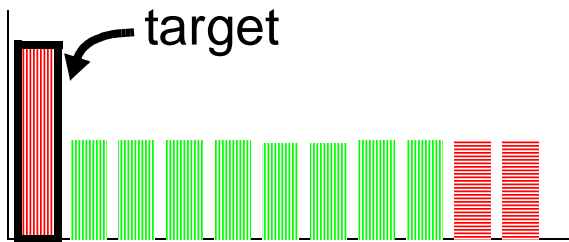
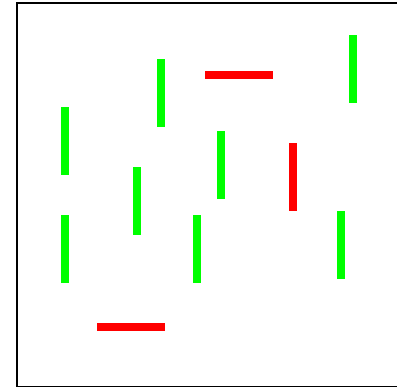


## TRIAL 1

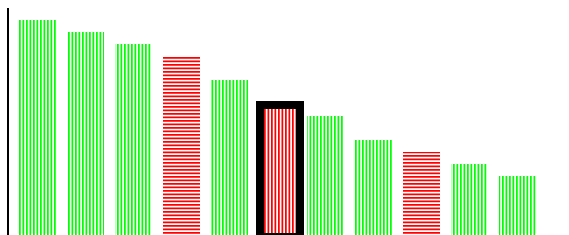


↑ red gain  
↓ vertical gain

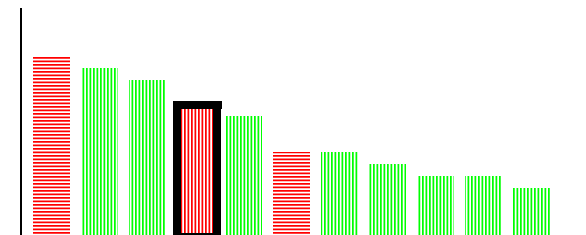
## TRIAL 2



Noisy rank-based  
prioritization of  
display elements



*A proportion of  
green verticals  
become less active  
than target; excluded  
from search*



# Can we blame oddball task?

Task involves not just search, but comparing display elements to determine which is target.

# Can we blame oddball task?

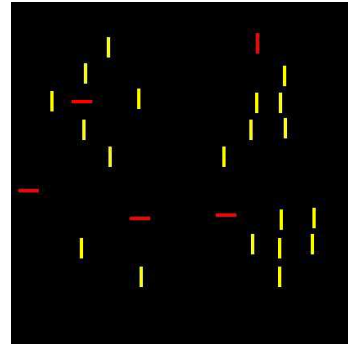
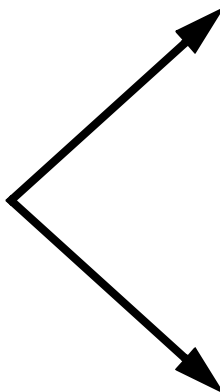
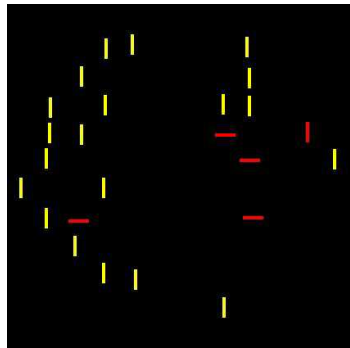
Task involves not just search, but comparing display elements to determine which is target.

## Alternative

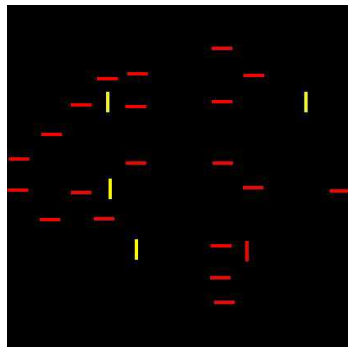
Fixed target search



Trial-to-trial variation in distractor statistics



repeat

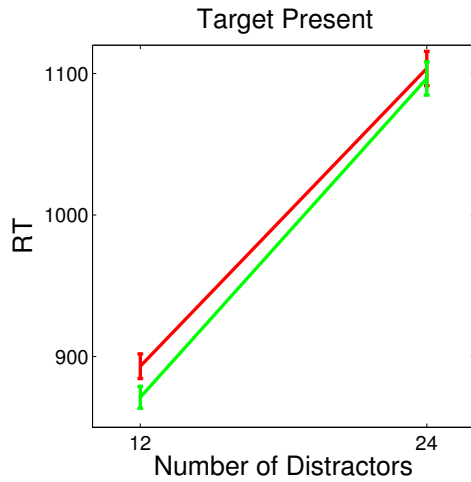


switch

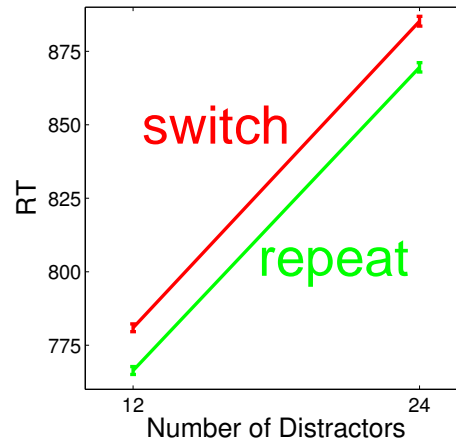
# A Desperate And Failed Effort...

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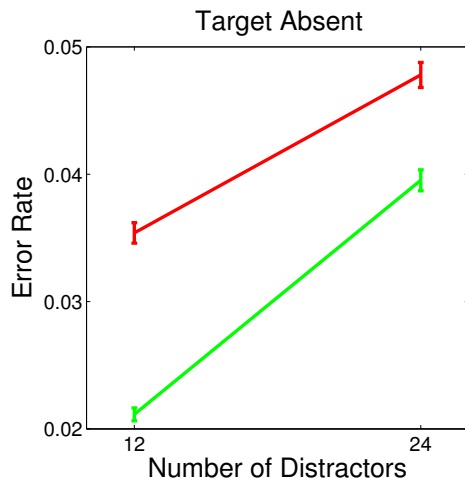
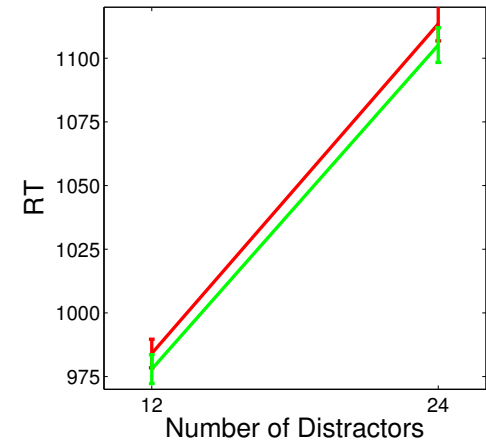
present  
or absent?



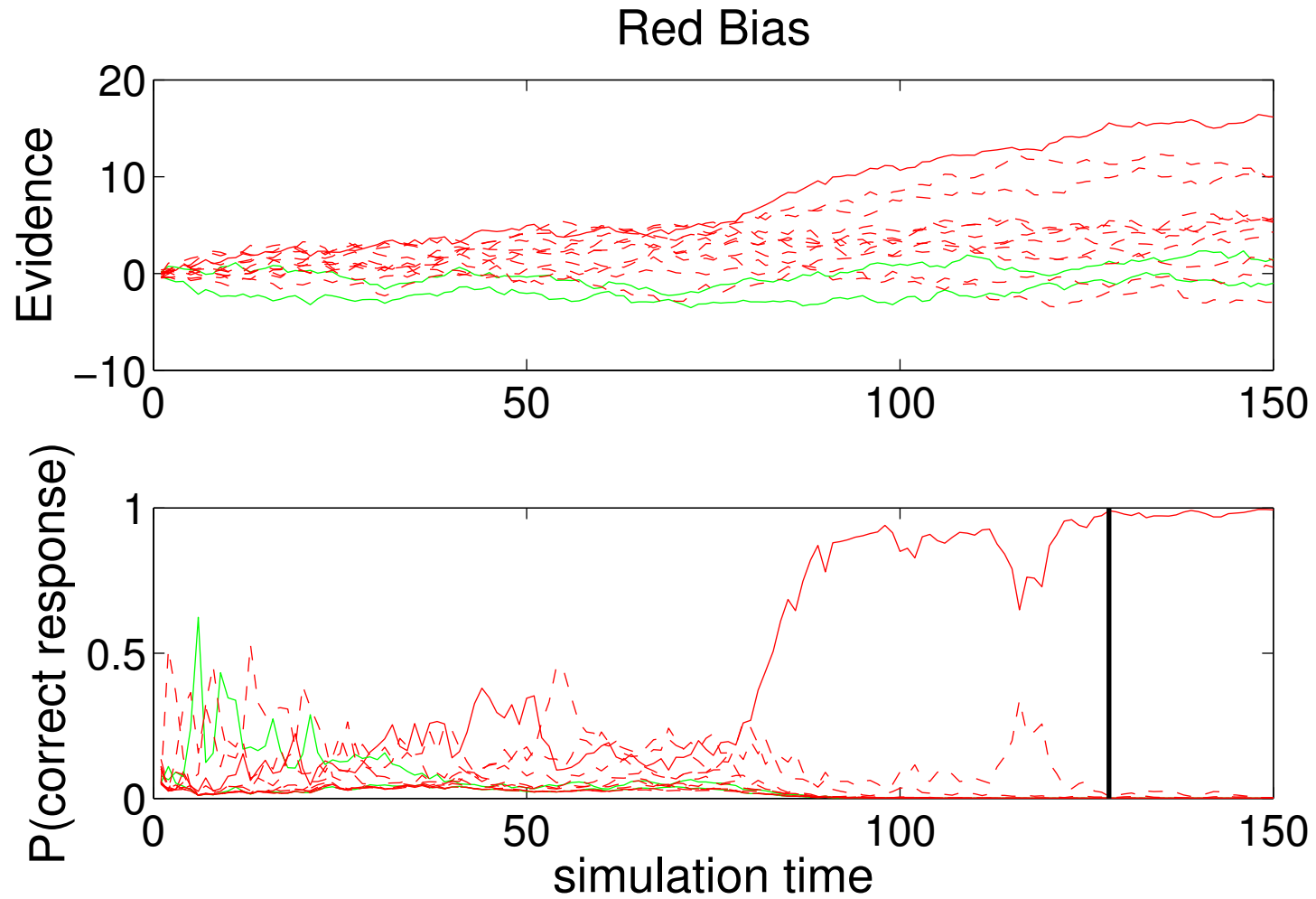
left or  
right side  
of display?



target  
contains  
gap?

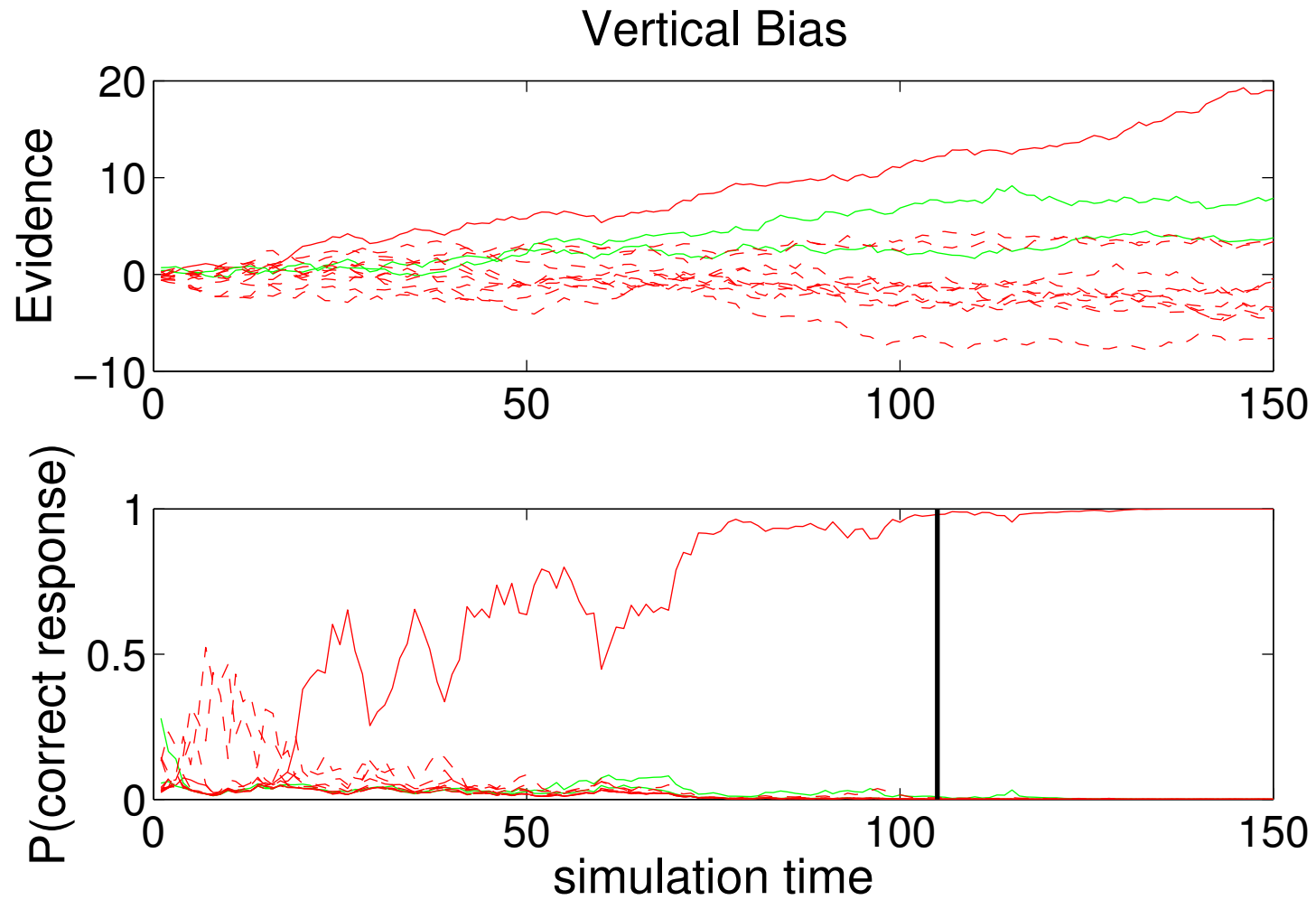


# Accumulator Model

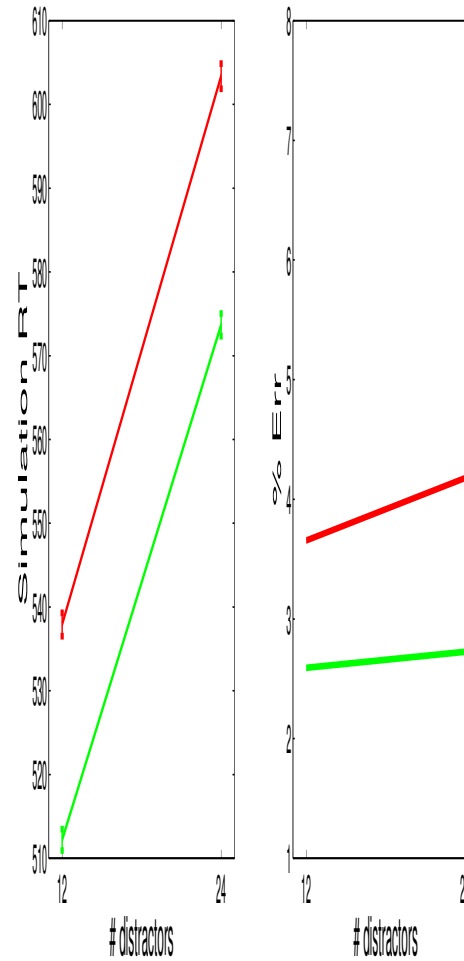




# Accumulator Model



# Accumulator Model



## Lesson for experimentalists

No interaction  $\neq$  priming effects are post- or pre-attentive

**\* BEGIN DIGRESSION \***

# Estimating $\rho$

$\rho$  depends on task environment.

$\rho$  is estimated based on experience performing task.

**E.g., what's the response of a red feature detector for a target ( $\rho_{\text{red},1}$ )?**

Collect responses of red feature detector at locations containing a target.

Suppose we observe: .47, .62, .91, .55, .80

**Could compute maximum likelihood estimate, i.e.,**

$$\hat{\rho}_{\text{red},1} = (.47 + .62 + .91 + .55 + .80) / 5$$

**Instead, model uses Bayesian parameter estimation.**

Consider all possible values of  $\rho$  and determine their plausibility based on how well they fit the data.

# Intuitive Example

Coin with unknown bias,  $\rho$  = probability of heads

Sequence of observations: H T T H T T T H

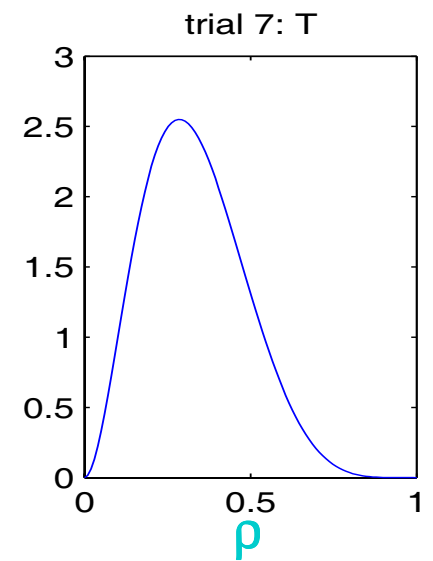
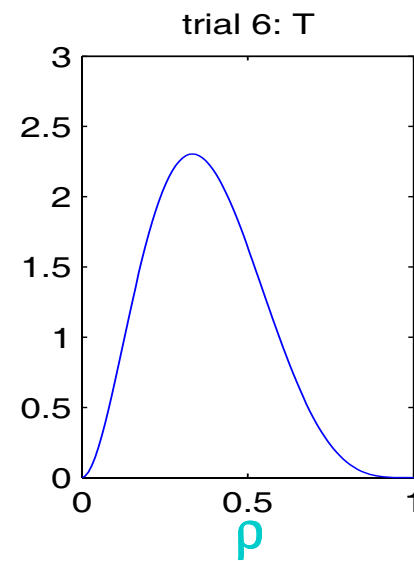
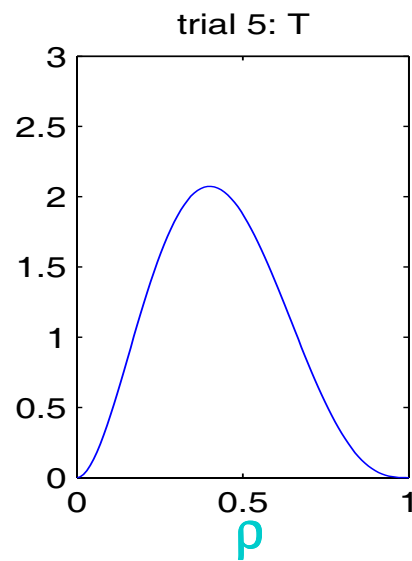
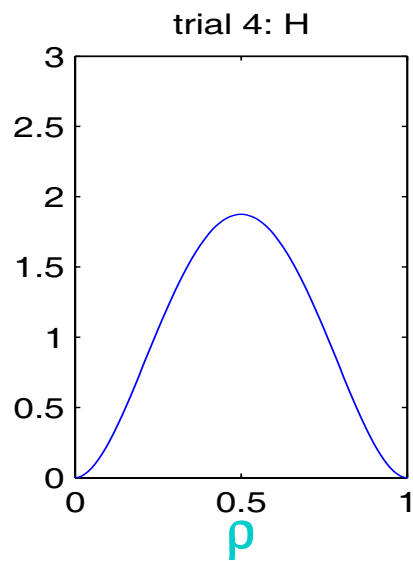
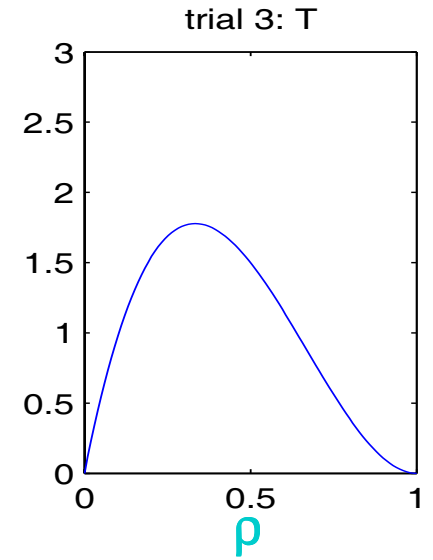
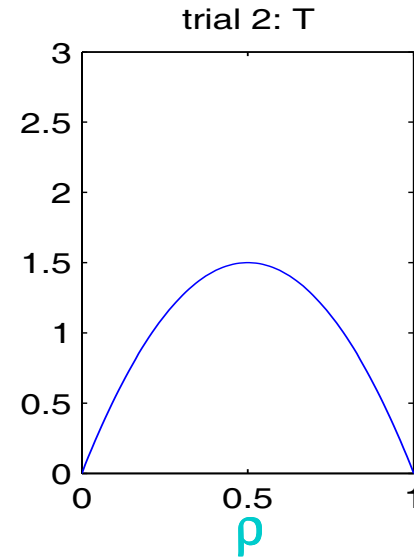
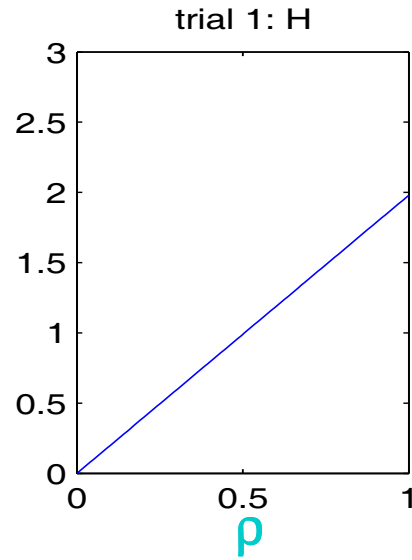
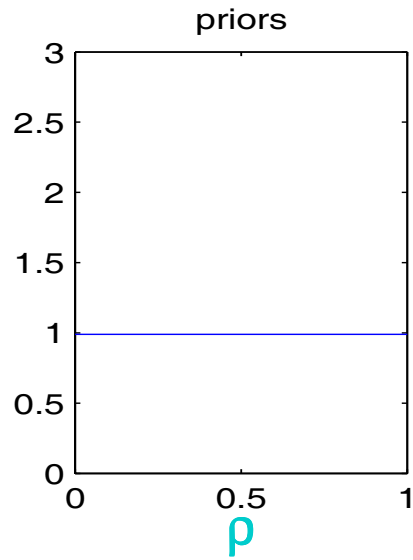
Maximum likelihood approach

$$\rho = 3 / 8$$

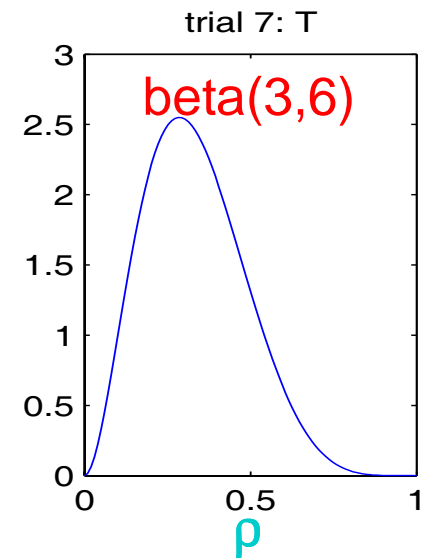
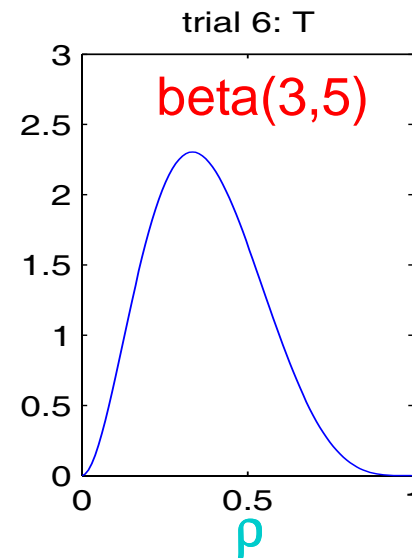
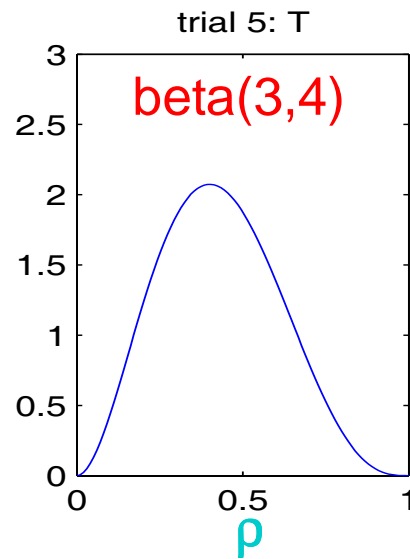
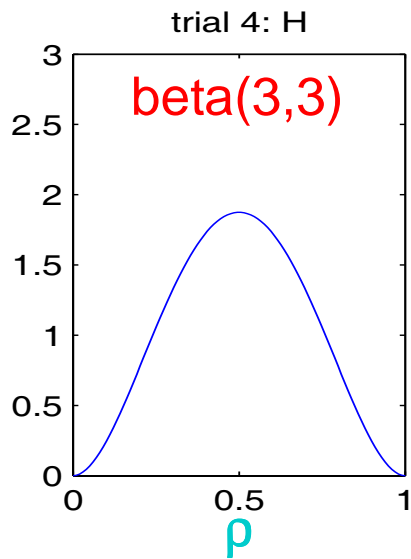
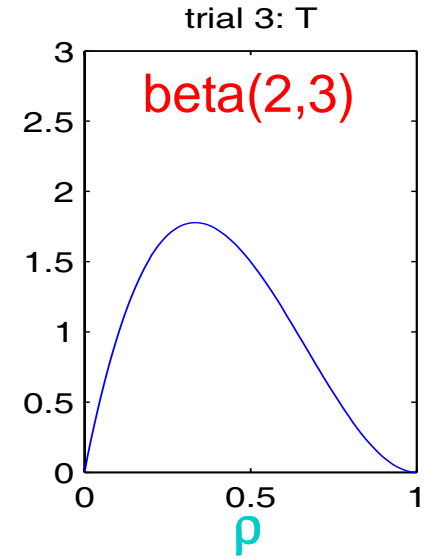
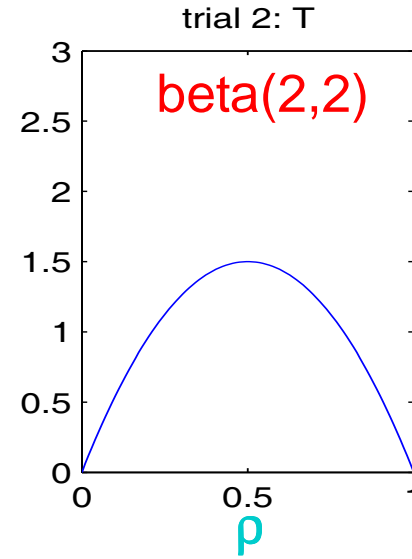
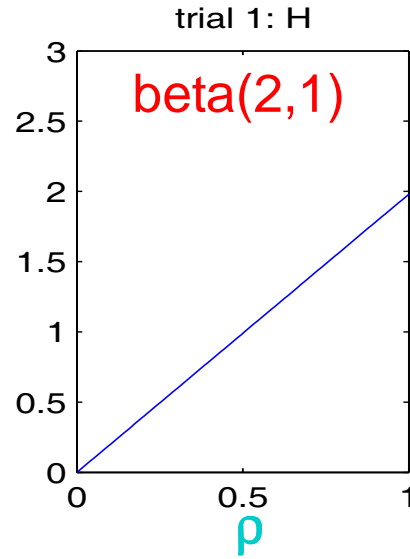
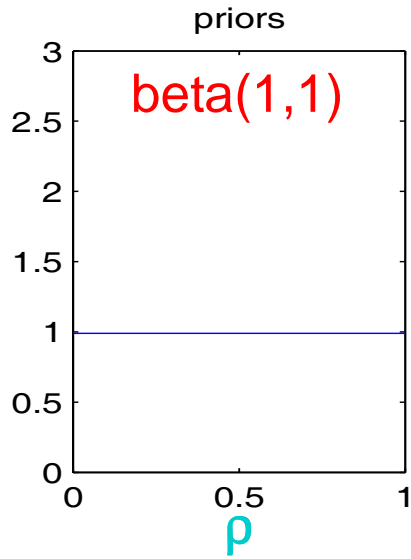
Bayesian approach

set of hypotheses, each associated with a different value of  $\rho$

# Coin Flip Sequence: H T T H T T T

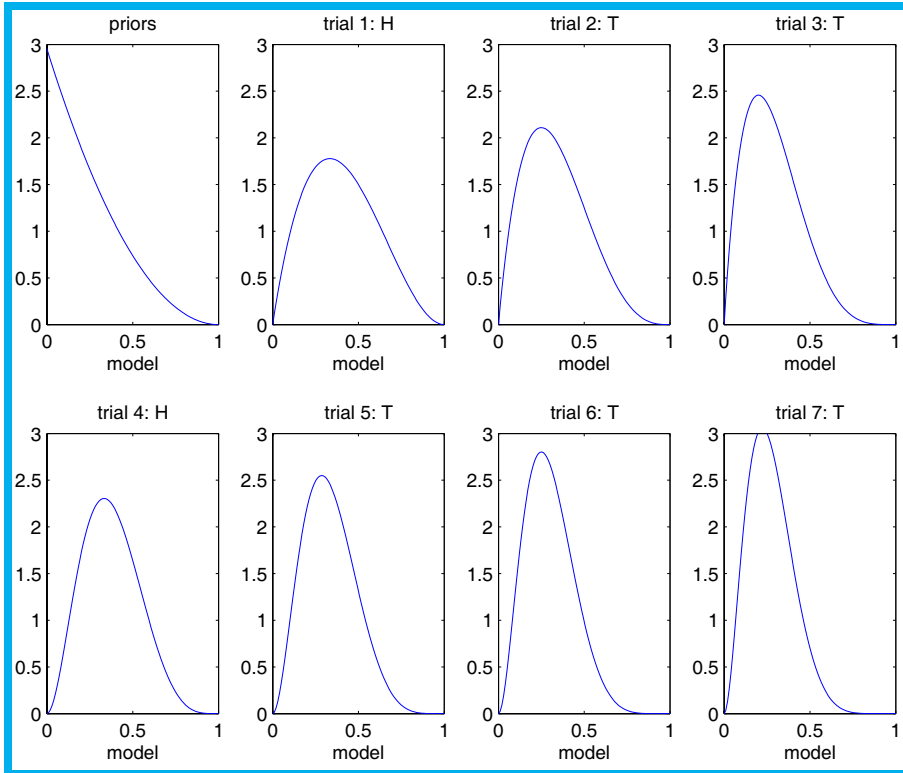


# Coin Flip Sequence: H T T H T T T

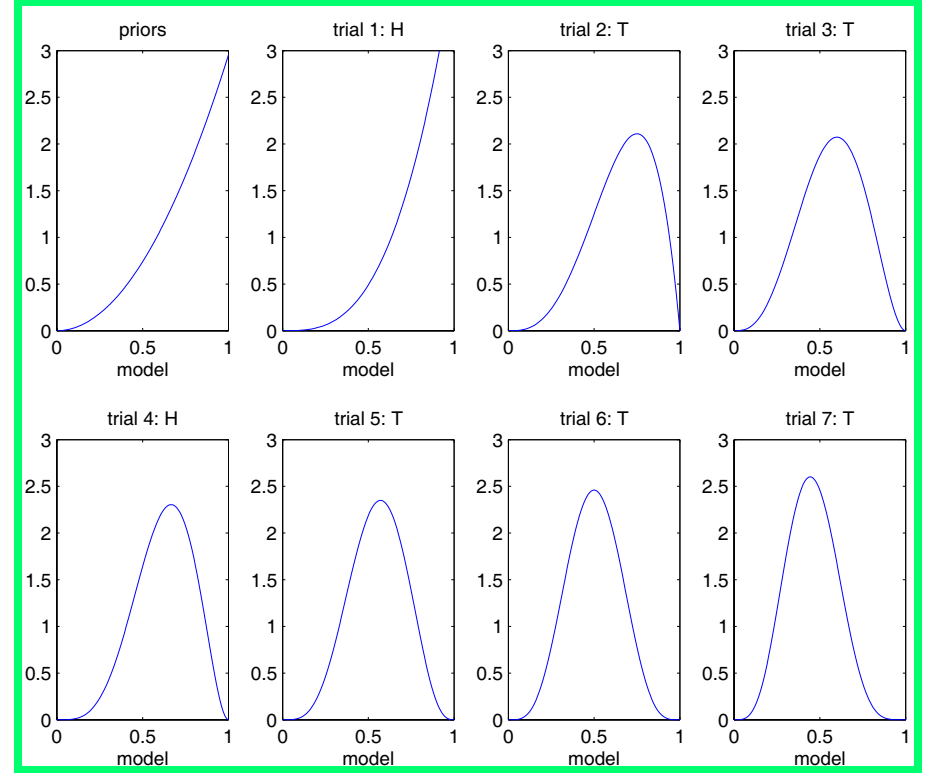


# Effect of Prior Knowledge

## low head-probability bias



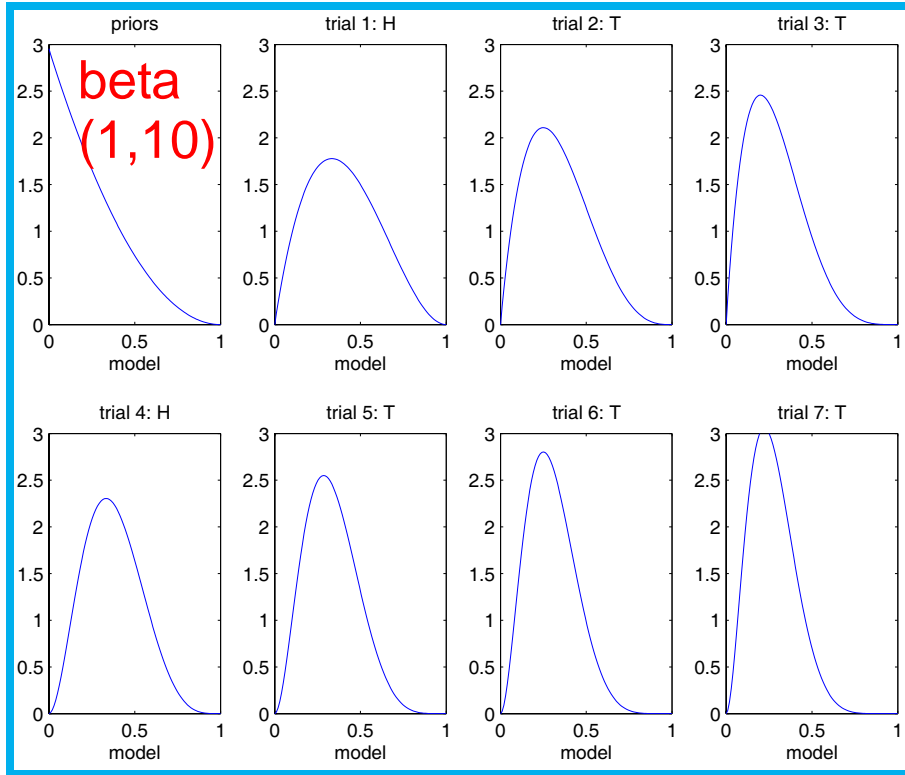
## high head-probability bias



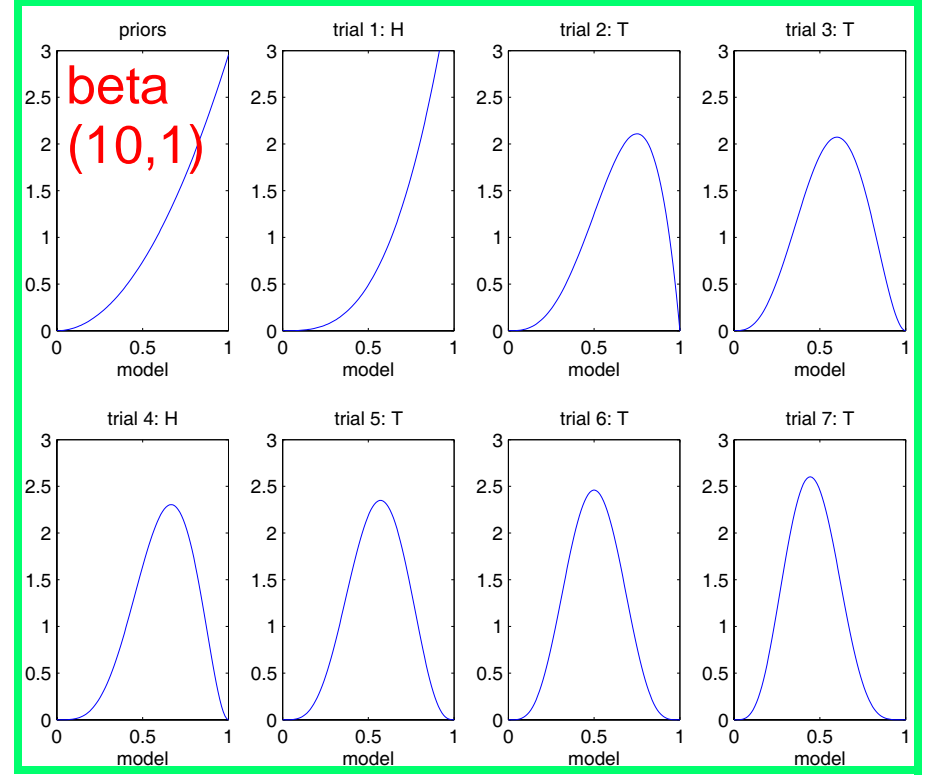


# Effect of Prior Knowledge

## low head-probability bias



## high head-probability bias



**\* END OF DIGRESSION \***

**Saliency**  $\equiv P(T_x | F_x, \rho)$

- task statistics – learned thru experience
- feature activity (vector) at location x
- target at location x? (1=true, 0=false)

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## Bayes' Rule

$$P(T_x | F_x, \rho) = \frac{P(T_x)P(F_x | T_x, \rho)}{\sum_{t=0}^1 P(T_x = t)P(F_x | T_x = t, \rho)}$$

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$P(F_x | T_x, \rho)$  is a task-specific model of the environment.

Indicates visual system response ( $F_x$ ) for targets ( $T_x=1$ ) vs. nontargets ( $T_x=0$ )

We make specific claims about the form of this model that resides in our head.

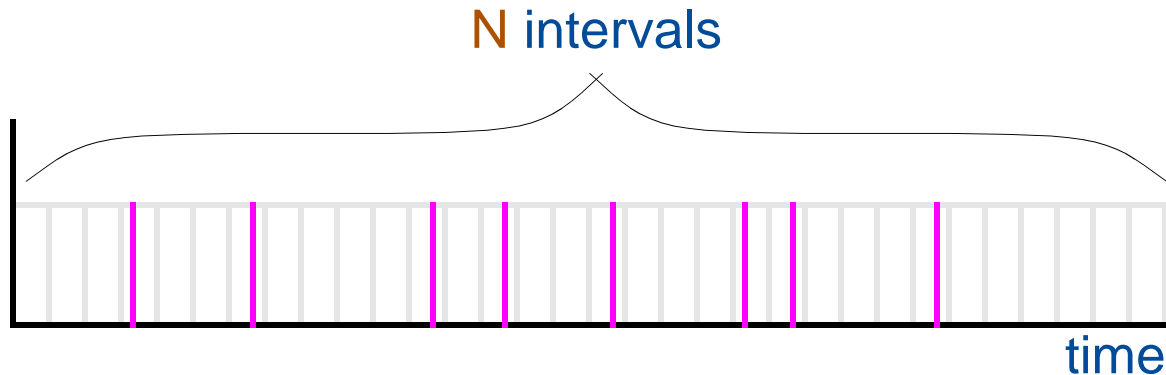
This model does not have to match the actual environment.

Generally, models are simplified to be mathematically tractable.

1. Assume feature responses are conditionally independent of one another, i.e.,

$$P(\mathbf{F}_x | T_x, \rho) = \prod_i P(F_{xi} | T_x, \rho)$$

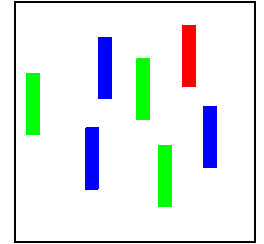
2. Assume feature detection is carried out by a rate-coded spiking neuron



$F_{xi}$ : count of the number of spikes observed for feature  $i$  at location  $x$

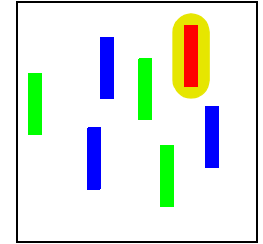
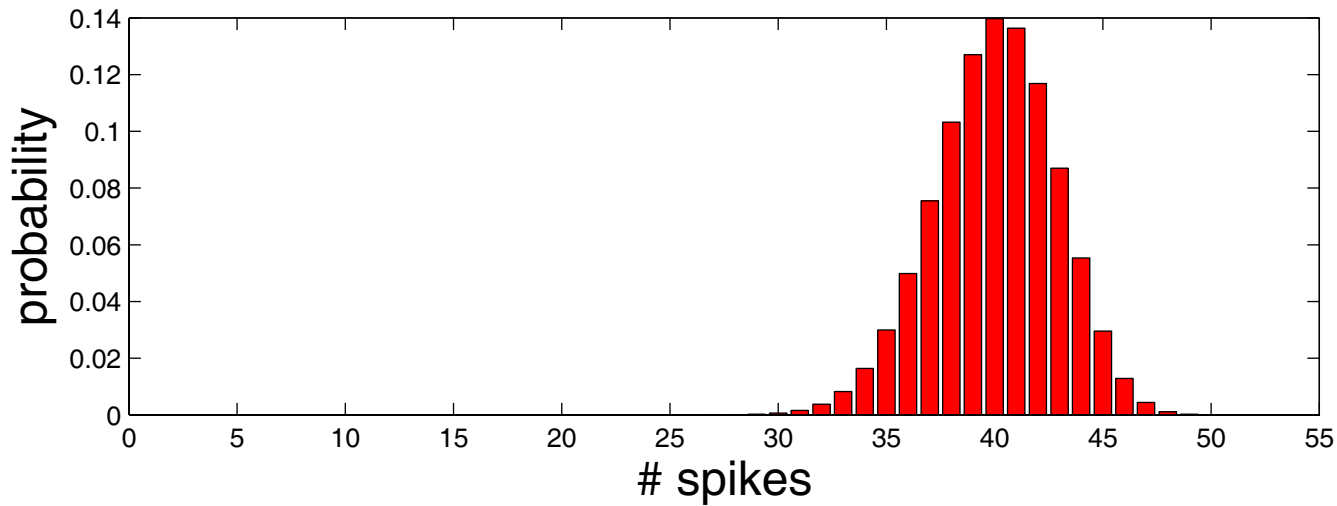
$\rho_{it}$ : spike rate for feature  $i$  if the target is of type  $t$

**Consider search for a red object among non-red.  
What will the response of the red feature detector be?**



Consider search for a red object among non-red.  
What will the response of the red feature detector be?

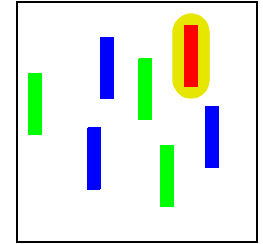
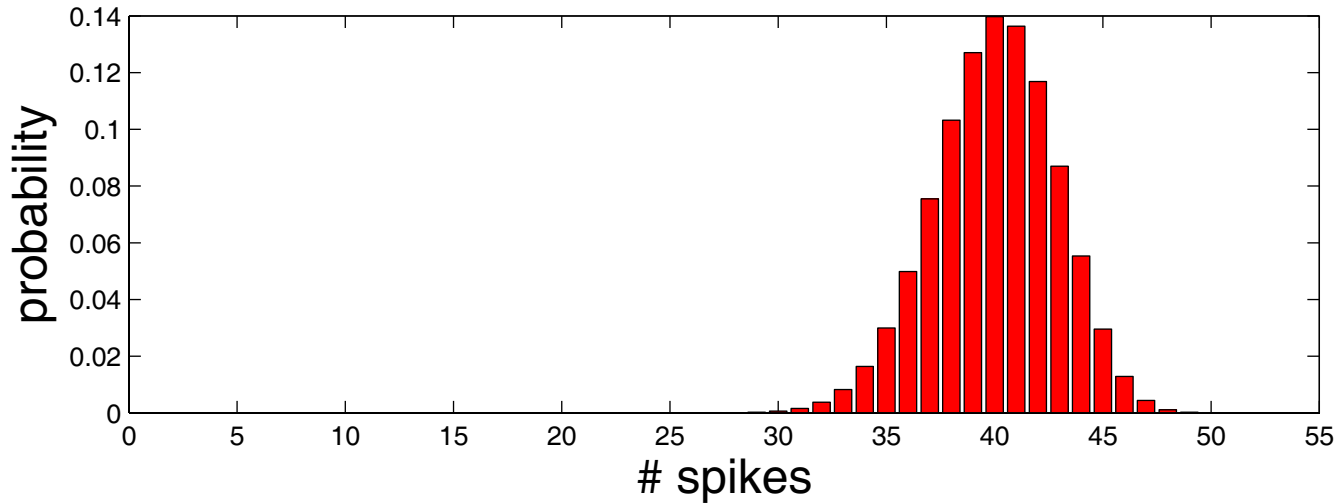
$P(F_{\text{red}} | T = 1, p_{\text{red}, 1} = 0.8)$  for  $N=50$



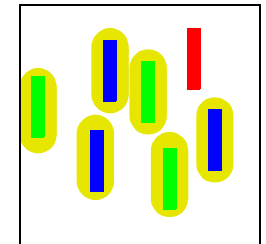
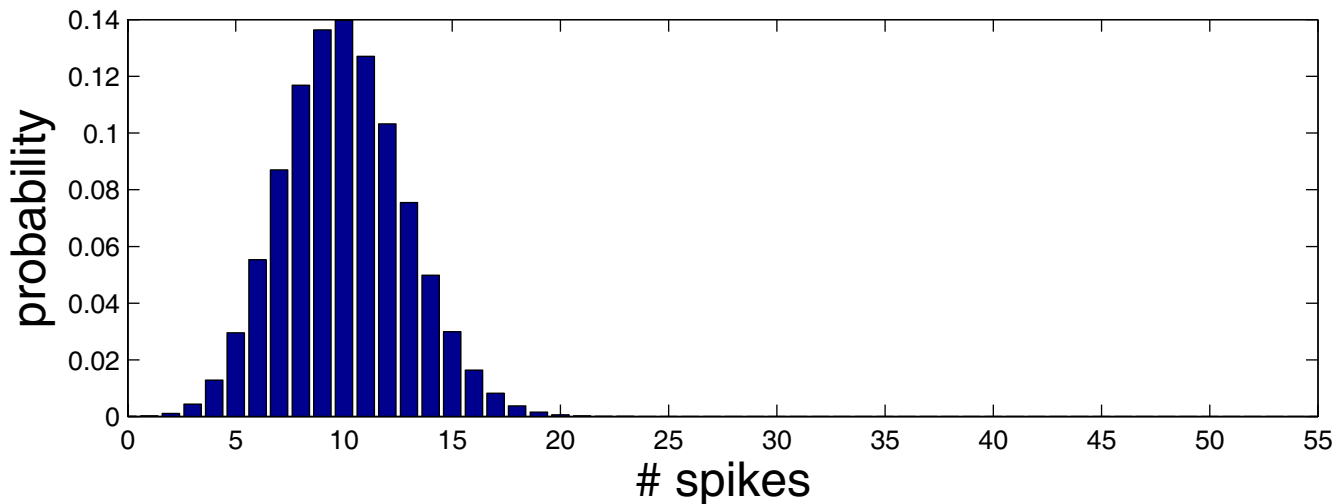


Consider search for a red object among non-red.  
What will the response of the red feature detector be?

$P(F_{\text{red}} | T = 1, p_{\text{red}, 1} = 0.8)$  for  $N=50$



$P(F_{\text{red}} | T = 0, p_{\text{red}, 0} = 0.2)$  for  $N=50$



$$P(F_{xi} | T_x, \rho) \sim \text{Binomial}(\rho_{it}, N)$$

number of time intervals

spiking rate of feature i  
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number of time intervals

spiking rate of feature i  
for target (t=1) or distractor (t=0)

$$\sim \text{Gaussian}(N\rho_{it}, N\rho_{it}(1 - \rho_{it}))$$

# To Recap

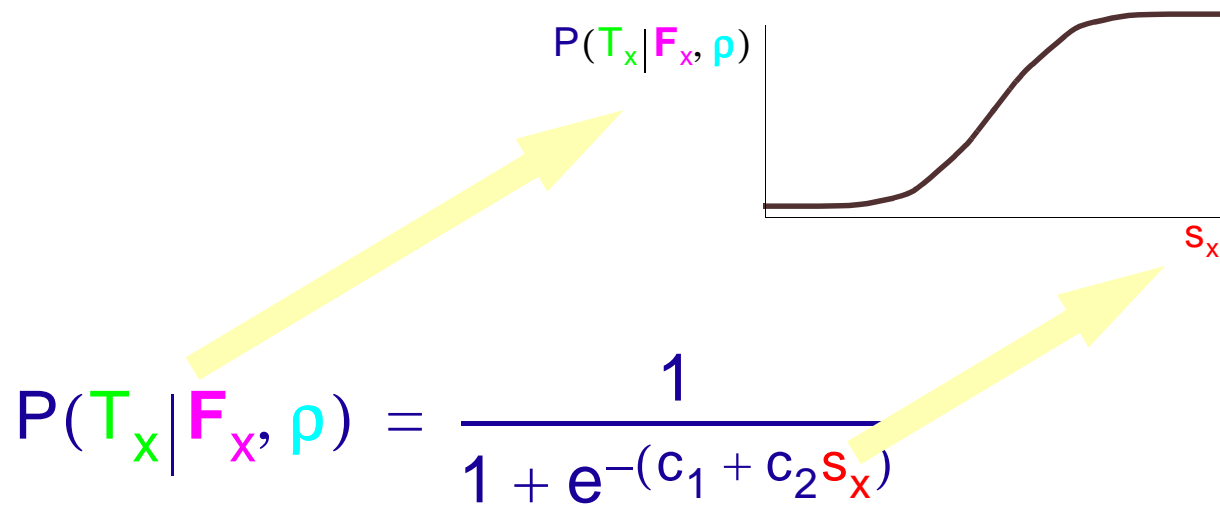
**Saliency is defined to be the probability that a location contains a target,  $P(T_x | F_x, \rho)$**

**Given the following assumptions:**

1. Feature responses are conditionally independent of one another.
2. Feature detection is carried out by neurons that spike at rate  $\rho_{it}$ .
3. The neuron is sampled for a time window that would allow  $>30$  spikes.

**we get...**

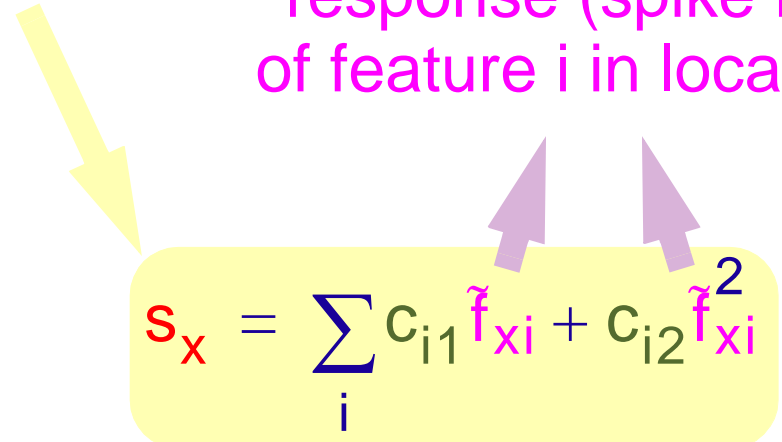
$$P(T_x | F_x, \rho) = \frac{1}{1 + e^{-(c_1 + c_2 s_x)}}$$



Because attentional priority depends on relative saliency, we can substitute  $s_x$  for  $P(T_x | F_x, \rho)$ .

$$P(T_x | F_x, \rho) = \frac{1}{1 + e^{-(c_1 + c_2 s_x)}}$$

response (spike rate)  
of feature  $i$  in location  $x$


$$s_x = \sum_i c_{i1} \tilde{f}_{xi} + c_{i2} \tilde{f}_{xi}^2$$

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$$\frac{2(\rho_{i1} - \rho_{i0})}{(1 - \rho_{i0})(1 - \rho_{i1})}$$

$$\frac{1}{\rho_{i0}(1 - \rho_{i0})} - \frac{1}{\rho_{i1}(1 - \rho_{i1})}$$



**Experience-  
Guided Search**

$$S_x = \sum_i c_{i1} \tilde{f}_{xi} + c_{i2} \tilde{f}_{xi}^2$$

**Guided Search**

$$S_x = \sum_i c_{i1} \tilde{f}_{xi}$$

# Two Further Assumptions of Model

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## 1. Bias that all features are considered relevant in the absence of experience

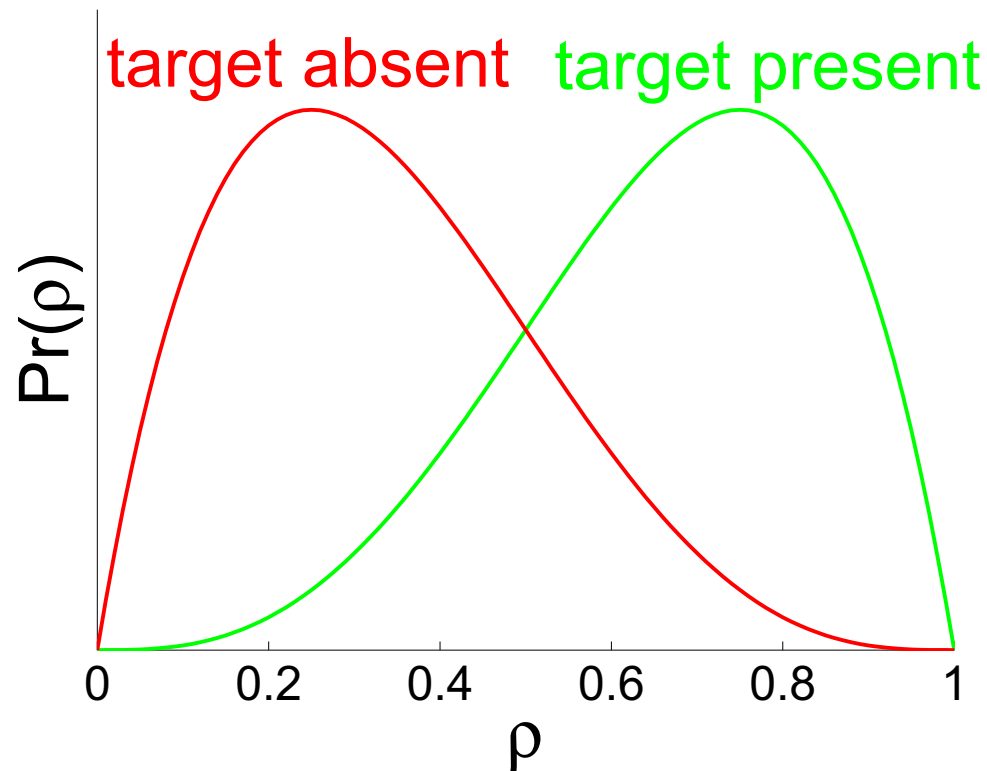
Achieved by treating  $\rho$  as a Beta random variable with imaginary-count prior

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Encode not just the most likely value of  $\rho$ , but uncertainty distribution.



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With probability  $\lambda$ , environment and/or task can change.

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Encode not just the most likely value of  $\rho$ , but uncertainty distribution.

## 2. Environment is nonstationary

With probability  $\lambda$ , environment and/or task can change.

**From these two claims, we have *three* free parameters total.**

Qualitative performance does not depend on parameters as long as  $\lambda > 0$  and  $E[\rho_{i0}] < E[\rho_{i1}]$