Journal of Experimental Psychology:
Human Perception and Performance
1977, Vol. 3, No. 1, 92–104

Sequential Effects in Judgments of Loudness

Walt Jesteadt, R. Duncan Luce, and David M. Green
Harvard University

A multiple regression analysis of sequential effects in magnitude estimation and absolute identification is presented as an alternative to the approach used by Lockhead and his students. The new analysis indicates that sequential effects do not extend over more than one trial. This is in agreement with the response ratio hypothesis. A more detailed multiple regression analysis of these sequential effects indicates that the magnitude of the correlation between successive responses is heavily dependent on the decibel difference between successive signals. This is not in agreement with the response ratio hypothesis, and the hypothesis is reformulated to take account of this finding. This modification of the model is tested by comparing distributions of normalized responses to theoretical distributions suggested by the model and to a possible alternative distribution.

Once one tries to examine more than the central tendency (mean, median, or geometric mean, as the case may be) of responses in absolute identification, category, or magnitude estimation procedures, one must not ignore the possibility that the responses have been contaminated by factors other than simple variability in the internal representations of the stimuli. Sequential effects and response drift are two such factors. We were led to the present study and analysis of these factors not because we were interested in them per se, but because they impeded our attempts to compare the distributions of responses in magnitude estimation with predictions derived from timing and counting models (Green & Luce, 1974; Luce & Green, 1972).

The existence of substantial sequential effects in absolute identification of loudness has been known for some time (Helson, 1948; Garner, 1953; Parducci, 1956; Pollack, 1964). More recently, these effects have been studied in detail by Lockhead and his students (Holland & Lockhead, 1968; Ward, 1972; Ward & Lockhead, 1970, 1971). Furthermore, Cross (1973) and Ward (1973) have established comparable results for magnitude estimation.

One striking—and appalling—result of the Lockhead analysis is that sequential effects appear to extend over as many as five trials. This was shown as follows: Suppose stimulus s_i is presented on the current trial of an absolute identification experiment, and that $r_{ij}(k)$ is the average numerical response to stimulus s_i, conditional on stimulus s_j having been presented k trials earlier. Then $r_{ij}(k) - s_i$ represents the average response deviation due to the earlier stimulus. Such loudness data (Ward & Lockhead, 1971, Figure 1), averaged over the 10 stimulus values (i) and grouped into successive pairs of j values ($1 + 2, \ldots, 9 + 10$) for various values of k, are shown in the left panel of Figure 1. An extreme stimulus on the immediately preceding trial "attracts" the response by about half a response category. Stimuli on Trials 2, 3, 4, and 5 before the trial under consideration also affect the response but in the opposite direction. Similar results are also found when the analogous analysis is performed, conditional on the previous responses, as shown in the right panel of Figure 1. The

This research was supported by a grant from the National Science Foundation and by a National Institutes of Health postdoctoral fellowship to the first author.

Walt Jesteadt is now at the Boys Town Institute for Communication Disorders in Children, 8401 West Dodge Road, Omaha, Nebraska 68114.

Requests for reprints should be sent to R. Duncan Luce, Department of Psychology and Social Relations, Harvard University, Cambridge, Massachusetts 02138.

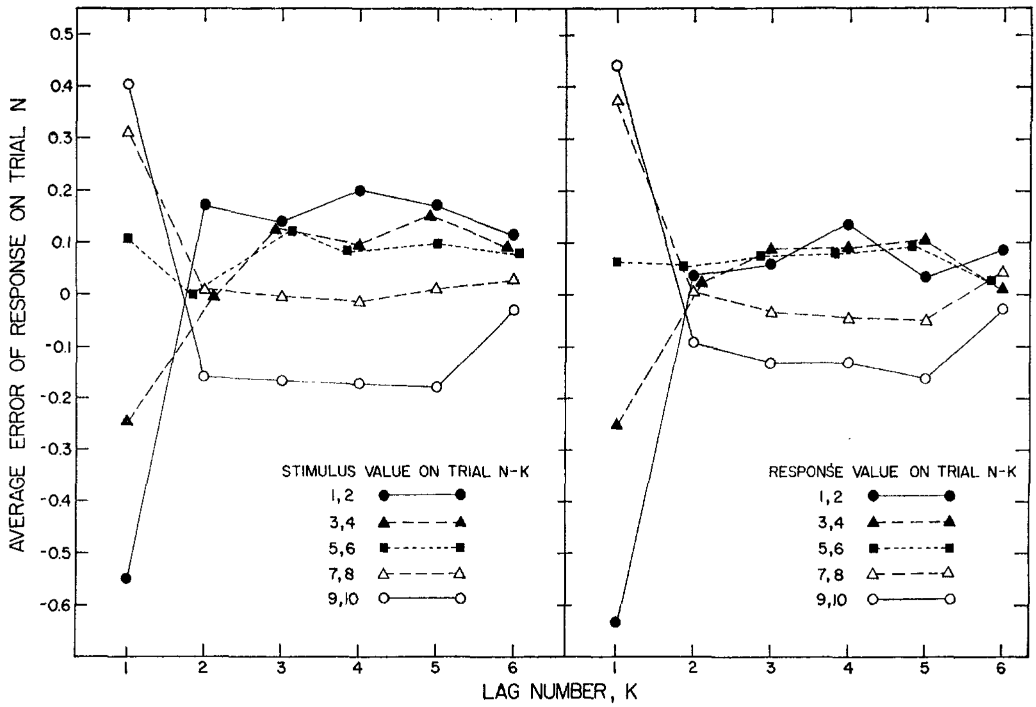

FIGURE 1. Sequential stimulus and response effects in absolute identification observed in the Lockhead analysis. (From "Response System Processes in Absolute Judgment" by L. M. Ward and G. R. Lockhead, *Perception & Psychophysics*, 1971, *9*, Figure 1, p. 73. Copyright 1971 by The Psychonomic Society, Inc. Reprinted by permission.)

major features of these graphs have been replicated a number of times and will also be evident in our data.

If accepted at face value, the depth of the effect is disturbing. Apparently, one must take into account stimuli and responses occurring as many as five trials back in order to correct each response for sequential effects, in which case the correction task is formidable. However, it is by no means obvious that such depth is real. Assume, for example, that there are sequential stimulus and response effects that extend only to the preceding trial. In the Lockhead analysis, these effects would appear to propagate over a number of trials because the stimulus and response on trial $n-2$ would affect the response on trial $n-1$, which would in turn affect the response on trial n. It is impossible to know whether events on trial $n-2$ are having a direct influence on the response on trial n unless the experimenter factors out the effects of the response on trial $n-1$. The method of analysis underlying Figure 1 does not do this. One of the goals of this article is to compare other analyses of sequential effects—for example, linear multiple regression—and to determine if such long-term effects still appear to exist in these analyses. We shall argue that such effects are largely a result of the mode of analysis and that sequential effects actually extend only to the preceding trial.

The model for magnitude estimation proposed by Cross (1973), which nicely summarizes much of his data, and a generalization of this model, called the response ratio hypothesis by Luce and Green (1974), postulate a dependency only on the events of the preceding trial and no direct dependency on earlier trials. In particular, let $\underset{\sim}{R}_n$ denote the random variable representing the response on trial n, $\underset{\sim}{S}_n$ be the random variable representing the stimulus presented on trial n, and $\underset{\sim}{X}(s)$ and $\underset{\sim}{X}^*(s)$

be hypothetical, independent internal representations of Signal s. Then, the response ratio hypothesis asserts that the ratio of the responses on trials n and $n-1$ preserves the ratio of the corresponding representations of the stimuli presented on these trials:

$$\frac{\underset{\sim}{R}_n}{\underset{\sim}{R}_{n-1}} = C\frac{\underset{\sim}{X}(\underset{\sim}{S}_n)}{\underset{\sim}{X}^*(\underset{\sim}{S}_{n-1})}, \qquad (1)$$

where C is constant. Luce and Green assume that each stimulus $\underset{\sim}{S}_n$ receives two independent representations, $\underset{\sim}{X}$ and $\underset{\sim}{X}^*$, the first of which is used in determining the response on trial n and the second on trial $n+1$. This assumption is crucial, for if the same representation is used on both trials, then no sequential effects arise, since by induction,

$$\underset{\sim}{R}_n = C^{n-1}\underset{\sim}{X}(\underset{\sim}{S}_n)\underset{\sim}{R}_1.$$

Luce and Green were forced by the existence of sequential effects to employ the next simplest assumption, namely, Equation 1.

The response ratio hypothesis simply postulates that the subjects do what they are told to do in a magnitude estimation experiment: preserve in their responses the subjective ratios of the stimuli on successive trials. If so, there may be both response and stimulus sequential effects from the immediately preceding trial but no direct effects from earlier trials.

The analyses we will present in greatest detail are based on magnitude estimations of loudness from a series of three experiments. The initial purpose of the experiments was to develop a method that would reduce long-term sequential effects and thereby allow us to compare the distribution of response ratios with the predictions of the timing model.

Method

In all three experiments, the signals were 1,000-Hz tones of 500-msec duration. They were presented binaurally, in quiet, via SW-2 Superex headphones to observers who were tested individually in a sound-treated room. The observers responded by typing integer numbers on a Video Systems terminal (essentially a typewriter keyboard and small TV screen) connected to a PDP-15 computer. The procedure was self-paced, with each trial initiated by the response to the previous trial. Runs consisted of 60 trials. Observers completed between 10 and 15 runs in a 2-hour session. There were approximately 60 trials per signal per observer in each experiment. The same four observers were tested in all three experiments. They were paid $2.25 per hour.

Experiment 1: Magnitude Estimation

The 27 signals ranged from 36–88 dB (SPL) in 2-dB steps. Observers were instructed to assign numbers to the different tones so that the ratios of the numbers were the same as the ratios of the loudnesses. At the beginning of the first session, after the procedure had been explained, they were asked to do several trials of magnitude production for 2:1 ratios within the signal range to be used in the experiment. The first several runs of magnitude estimates were also considered part of the training procedure, and the data were discarded.

Experiment 2: Ratio Estimation with Erase Tones

In this experiment, the 16 signals ranged from 36–50 dB (SPL) and 74–88 dB (SPL) in 2-dB steps. The purpose of the 24-dB gap was to affect the distribution of differences between successive stimuli. The signals were presented in pairs, with a 500-msec interval of quiet between the two members of a pair. The observers were instructed to enter a ratio corresponding to the loudness ratio of the two tones, and they were encouraged to use the same number scale they had used in Experiment 1 when entering the components of the ratio. That is, they entered a magnitude estimate of the first tone as the numerator and an estimate of the second tone as the denominator. In an effort to isolate the tone pairs and to eliminate the influence of preceding tone pairs on present responses, a series of five "erase" tones, randomly selected from the entire signal range, was presented between trials. The individual erase tones were 400 msec in duration and were separated by 100-msec gaps.

Experiment 3: Magnitude Estimation

Experiment 3 replicated Experiment 1. Its purpose was to demonstrate that any change in the magnitude of the sequential effects observed in Experiment 2 resulted from the change in procedure, not from increased practice.

Correlation Results

As we show below, the procedure used in Experiment 2 succeeded in eliminating sequential effects between pairs of tones. However, in the course of a multiple regression analysis of sequential effects in all three experiments, we found that the

TABLE 1

INCREMENT IN MULTIPLE CORRELATION PRODUCED BY ADDING ADDITIONAL VARIABLE

Subject and experiment	Stimulus						
	I_n	I_{n-1}	I_{n-2}	I_{n-3}	I_{n-4}	I_{n-5}	n
Subject 1							
Experiment 1	.739	.030	.002	.000	.000	.000	1,046
Experiment 3	.755	.037	.004	.000	.001	.000	1,369
Subject 2							
Experiment 1	.834	.010	.000	.001	.000	.001	715
Experiment 3	.819	.000	.000	.001	.000	.000	1,232
Subject 3							
Experiment 1	.631	.035	.004	.001	.000	.001	1,353
Experiment 3	.751	.046	.003	.000	.002	.001	1,383
Subject 4							
Experiment 1	.824	.045	.002	.000	.000	.000	1,477
Experiment 3	.766	.030	.001	.001	.000	.000	1,661
M	.765	.029	.002	.001	.000	.000	
σ	.066	.016	.002	.000	.000	.001	
	Response						
	I_n	R_{n-1}	R_{n-2}	R_{n-3}	R_{n-4}	R_{n-5}	
Subject 1							
Experiment 1	.739	.071	.003	.002	.000	.001	1,046
Experiment 3	.755	.058	.002	.000	.001	.001	1,369
Subject 2							
Experiment 1	.834	.023	.001	.000	.000	.000	715
Experiment 3	.819	.003	.001	.000	.000	.000	1,232
Subject 3							
Experiment 1	.631	.135	.005	.001	.000	.001	1,353
Experiment 3	.751	.085	.001	.000	.000	.001	1,383
Subject 4							
Experiment 1	.824	.062	.000	.001	.000	.000	1,477
Experiment 3	.766	.053	.000	.000	.000	.000	1,661
M	.765	.061	.002	.000	.000	.000	
σ	.066	.040	.002	.001	.000	.001	

Note. I = signal intensity; R = response.

depth of the effects in the regular magnitude estimation experiments was much less than we had been led to believe. We therefore shifted our emphasis from Experiment 2 to the analysis of the control Experiments 1 and 3.

Linear Regression Model

A convenient way to explore the depth of sequential effects in magnitude estimation is to employ a simple linear regression model on some function of the responses and the signals. We shall use as the dependent variable the logarithm of the response on the present trial, $\log \underset{\sim}{R}_n$; as independent variables we will use (a) the logarithm of the signal intensity on the present trial, $\log I_n$; (b) the logarithm of the previous signal intensities, $\log \underset{\sim}{I}_{n-i}$ ($i = 1, 2, \ldots$); and (c) the logarithm of the previous responses, $\log R_{n-k}$ ($k = 1, 2, \ldots$). We use the logarithm for three reasons. First, the variability of magnitude estimates is roughly proportional to their expected value. Second, the response ratio hypothesis (Equation 1) strongly suggests using the logarithm of the responses. And third, if, as in the timing model, $1/\underset{\sim}{X}(s)$ is distributed as a gamma of order k and intensity $\mu(s)$, then the variance of $\log \underset{\sim}{X}(s)$ can be shown to be $\zeta(2, k-1)/\mathrm{I}\,(k-1)$,

TABLE 2

Parameters of Regression Equation 3

Subject and experiment	Parameter				Multiple correlation	
	γ	α	β	δ	R	R^2
Subject 1						
Experiment 1	.314	−.077	.451	−.428	.907	.872
Experiment 3	.299	−.025	.305	−.408	.903	.815
Subject 2						
Experiment 1	.265	−.071	.374	−.358	.931	.867
Experiment 3	.289	−.058	.219	−.476	.910	.828
Subject 3						
Experiment 1	.230	−.083	.607	−.365	.892	.796
Experiment 3	.253	−.045	.427	−.480	.918	.842
Subject 4						
Experiment 1	.298	−.027	.331	−.276	.942	.887
Experiment 3	.235	−.032	.344	.336	.907	.822
M	.273	−.052	.382	−.307	.914	.841
σ	.032	.023	.116	.268	.016	.032
Results from Logue (1976; $n = 26$)						
M	.250	−.060	.407	−.553		
σ	.072	.088	.289	.639		

Note. γ = regression coefficient associated with stimulus intensity on the present trial; α = regression coefficient associated with stimulus intensity on the previous trial; β = regression coefficient associated with the logarithm of the response on the previous trial; δ = the additive constant.

where ζ is the Riemann zeta function and Γ is the gamma function. The important point is that the variance depends only on the order of the gamma, not on μ, which in turn is a direct function of signal intensity.

The basic linear regression equation can be written as follows:

$$\log \underset{\sim}{R}_n = \gamma \log \underset{\sim}{I}_n + \sum_{i=1}^{M} \alpha_i \log \underset{\sim}{I}_{n-i} + \sum_{k=1}^{N} \beta_k \log \underset{\sim}{R}_{n-k} + \delta + \underset{\sim}{\epsilon}, \quad (2)$$

where γ, α_i, and β_k are the regression coefficients, δ is a constant related to the average response magnitude used by the subject, and $\underset{\sim}{\epsilon}$ is the usual Gaussian error term.

Multiple correlations were computed by varying either the number of previous stimuli used in the regression equation (varying M in Equation 2 and eliminating all of the $\log \underset{\sim}{R}_{n-k}$ terms) or by varying the number of previous responses (varying N in Equation 2 and eliminating all of the $\log \underset{\sim}{I}_{n-i}$ terms). Table 1 presents these multiple correlations for different numbers of previous stimuli and of previous responses by showing the increment added to the correlation with each additional variable. In summary, adding the immediately previous signal or response produces a small but significant increment in the multiple correlation, except for the second observer. Adding stimuli or responses more distant than the previous trial adds virtually nothing for any observer.

Parameters of Regression Equation

Events on the present trial and on the immediately preceding trial seem to encompass all the useful information we can obtain for predicting the present response. Thus, we may reduce Equation 2 to

$$\log \underset{\sim}{R}_n = \gamma \log \underset{\sim}{I}_n + \alpha \log \underset{\sim}{I}_{n-1} + \beta \log \underset{\sim}{R}_{n-1} + \delta + \underset{\sim}{\epsilon}. \quad (3)$$

Table 2 presents the parameters for the various observers. Although the observers exhibit some variability, the average values are representative.

Cross (1973) has proposed a similar regression analysis, but he omits the $\log \underset{\sim}{R}_{n-1}$ term in Equation 3. He finds that α, the coefficient of $\log \underset{\sim}{I}_{n-1}$, is positive with a value of .055. This contrasts with our negative value of $-.052$. When we omit the $\log \underset{\sim}{R}_{n-1}$ term in analyzing our data we also obtain a positive α, with a mean value of .050. Thus, if we use the equation suggested by Cross, we find an assimilation effect for $\underset{\sim}{I}_{n-1}$, whereas if we use Equation 3, we find a contrast effect. Cross urges writing the following:

$$\underset{\sim}{R}_n = \underset{\sim}{I}_n^{\gamma'}(I_{n-1}/I_n)^{\alpha},$$

in which case he calls γ' the "true exponent." Clearly, $\gamma' = \gamma + \alpha$, so for our data $\gamma' = .333$.

These observers listened to many more stimulus presentations than is typical of magnitude estimation experiments. Therefore, it is conceivable that they might have developed unusual or idiosyncratic methods of dealing with the task—that producing 600–900 responses a day might have led to a peculiar pattern of results. Fortunately, at about the time these data were collected, a more orthodox magnitude estimation experiment was in progress nearby (Logue, 1976). In that experiment, 26 observers estimated the magnitudes of 13 tones spaced 5-dB apart from 30–90 dB (SPL). They estimated each tone twice, as is common practice. Logue kindly provided us with her data, which we analyzed using Equation 3. The means and standard deviations of the parameters over observers are listed in the last two rows of Table 2. The pattern resembles ours. This correspondence, as well as that between our results and results obtained in other laboratories, leads us to believe that there is a basic pattern to the sequential effects in magnitude estimation.

We conducted similar analyses of the data from Experiment 2 for ratio estimation with erase tones. The relations among the two signals and two responses constituting a single trial (separated from the previous trial by the erase tones) were in agreement with Tables 1 and 2. The responses within a trial were correlated ($r = .27$), as were the magnitude of the first signal and the response to the second signal ($r = .21$), but no significant correlation was observed between the magnitude of the second signal and the response to the first. Also, no significant correlation existed between the response to the first signal on a given trial and either the intensity of the immediately preceding signal (presented on the previous trial) or the response to that signal. Thus, the erase tones had the desired effect of eliminating sequential effects due to both signals and responses.

In our view, however, the ratio procedure is difficult from the observer's point of view as well as inefficient compared with the simpler magnitude estimation procedure using a single stimulus and a single response on each trial. If all sequential effects stem only from events on the previous trial, then estimating them by these simple regression techniques and correcting for them probably is to be preferred to using the ratio procedure.

Lockhead's Analysis Reconsidered

Our regression analysis provides a much simpler picture of the structure of sequential effects than does the analysis described by Lockhead and his students. The obvious question is whether there are differences in the two sets of data or whether it is simply a difference in the analyses. This is difficult to answer because our experiments are not really comparable to the magnitude estimation experiment described by Ward (1973). Ward used 10 signals covering a 36-dB range, whereas we used 27 spanning a 52-dB range. We reanalyzed the magnitude estimation data from Experiments 1 and 3 following the procedure described by Ward. This procedure called for computing $r_{\cdot j}(k)$, the geometric mean of all responses to all stimuli on the current trial, conditional on stimulus s_j having occurred k trials earlier. To normalize this matrix for plotting, we divided all values of $r_{\cdot j}(k)$ by $r_{\cdot\cdot}(k)$.

Our data and those from Ward (1973) are presented in Figure 2. To make the number

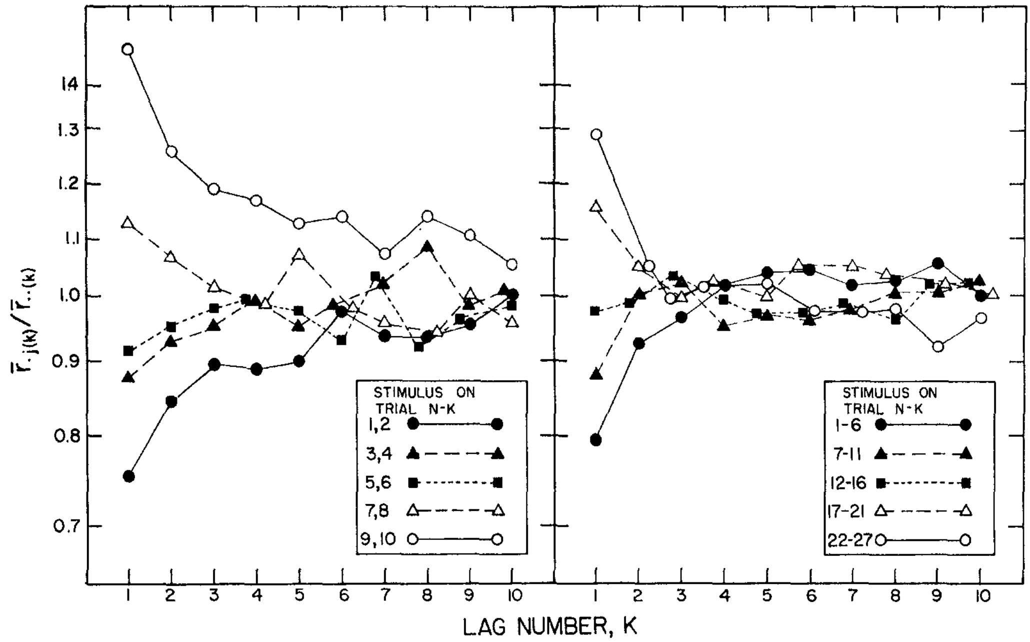

FIGURE 2. Sequential stimulus effects observed in a Lockhead analysis of magnitude estimation data collected by Ward (1973, Table 1). (A geometric mean response, $r._j(k)$, is computed for the responses to all signals, s_i, that were preceded K trials earlier by signal s_j. These mean responses are then normalized by dividing by $r..(k)$, which is the overall mean response. To reduce the number of points, geometric means of these ratios have been computed for adjacent stimulus categories. These have been labeled $\bar{r}._j(k)/\bar{r}..(k)$.

of curves manageable and the data comparable, we have averaged (a) successive pairs of s_j values (1 + 2, ..., 9 + 10) for Ward's data (see left panel of Figure 2) and (b) successive groups of five or six s_j values (1–6, 7–11, 12–16, 17–21, 22–27) for our data (see right panel of Figure 2). The two panels show a similar pattern, although the sequential effects in our data are not as large as those observed by Ward (1973).

Ward and Lockhead (1971) and Holland and Lockhead (1968) have demonstrated comparable effects in absolute identification data using a similar analysis (see Figure 1). We had absolute identification data available (Luce, Green, & Weber, 1976) collected under various conditions. One of these was 10 1,000-Hz tones, spaced evenly over an 11.25-dB range centered on 60 dB (SPL). This is close to their range of 10 dB. There were four observers, each of whom listened for an average of 1,500 trials. The regression analyses look very similar to those presented above for magnitude estimation data. There are small effects of $\underset{\sim}{R}_{n-1}$ and $\underset{\sim}{I}_{n-1}$ on $\underset{\sim}{R}_n$. By taking these effects into account, the proportion of the variance accounted for in $\underset{\sim}{R}_n$ can be increased from .726 for a prediction based on $\underset{\sim}{S}_n$ alone to .741 for a prediction based on $\underset{\sim}{S}_n$, $\underset{\sim}{S}_{n-1}$, and $\underset{\sim}{R}_{n-1}$. The use of additional stimuli or responses from trials $n - 2$ to $n - 5$ only increases the variance accounted for to .738. When we apply the analysis used by Ward and Lockhead (1971) to our data, however, we observe the same pattern that they saw of apparently extended sequential effects. Our data are presented in Figure 3; the data from Ward and Lockhead are presented in Figure 1. The agreement is striking. We conclude, therefore, that similar sequential effects are operating in magnitude estimation and absolute identification and that in both cases they are confined entirely to the events of the previous trial.

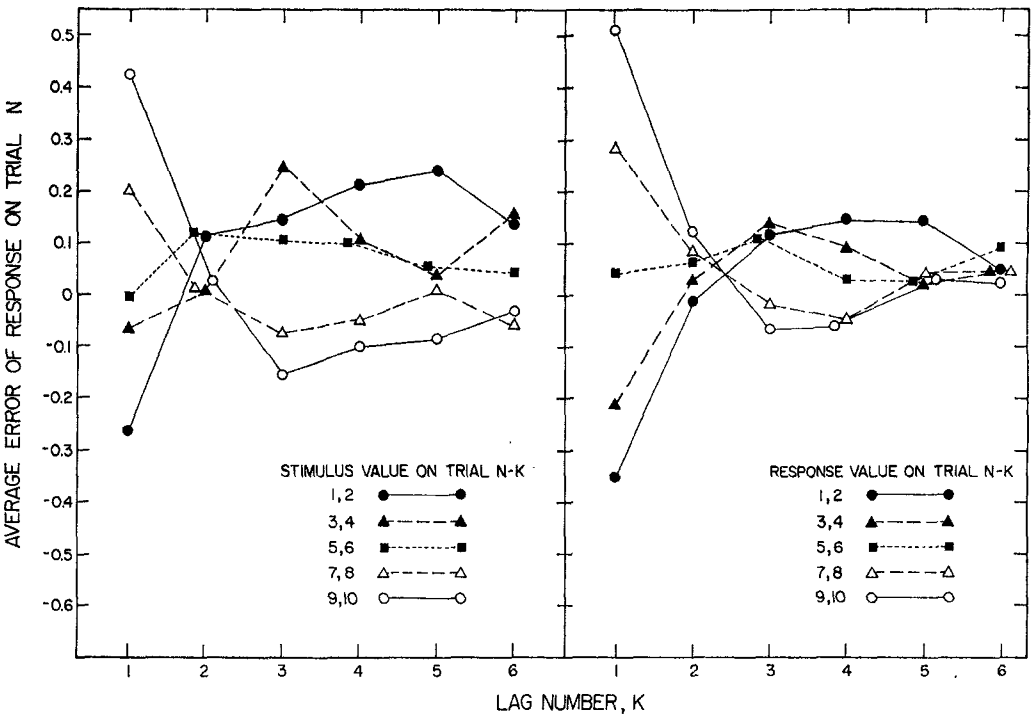

FIGURE 3. Sequential stimulus and response effects in a Lockhead analysis of absolute identification data collected by Luce, Green, and Weber (1976). (The pattern is very similar to that observed by Ward and Lockhead, 1971; see Figure 1.)

Incidentally, the size of the effects seen in Figure 3 depends heavily on the range of stimuli used. Small ranges produce larger effects and larger ranges produce smaller effects.

Correlation Between Successive Responses

We were motivated in much of this analysis by the response ratio hypothesis (Equation 1); however, the linear regression (Equation 3) suggests a serious problem for the response ratio hypothesis because the regression coefficient, β, is not near 1. One cannot be certain of this because the form of the signal terms differs in Equation 1 and Equation 3. Taking logarithms of Equation 1, we obtain the relation

$$\log \underset{\sim}{R}_n = \log \underset{\sim}{R}_{n-1} + \log [\underset{\sim}{X}(\underset{\sim}{S}_n)/\underset{\sim}{X}^*(\underset{\sim}{S}_{n-1})] + \log C,$$

which suggests a regression model of the form

$$\log \underset{\sim}{R}_n = \beta \log \underset{\sim}{R}_{n-1} + \Delta(\underset{\sim}{S}_n, \underset{\sim}{S}_{n-1}) + \epsilon. \quad (4)$$

If the response ratio hypothesis is correct, we should find $\beta = 1$.

There are difficulties in estimating the slope constant, β, in Equation 4, since we should compute it separately for data where $\underset{\sim}{X}(\underset{\sim}{S}_n)/X^*(\underset{\sim}{S}_{n-1})$ is the same. Data based on several different signal pairs in general have different intercepts, and that attenuates the regression coefficient. To avoid this problem, we computed 27^2 regression slopes, 1 for each distinct pair of signal values. The weighted average of β was $.569 \pm .1$ for the four subjects and two experiments.

In the course of this analysis, it became apparent that the regression coefficient depended strongly on the decibel difference of the signal pairs. This finding can be demonstrated most easily if we compute what might be called a normalized regression coefficient. We select successive responses and divide them by $k\underset{\sim}{I}_n^{\gamma'}$, where k and γ' are estimated for each individual subject. (γ' is simply estimated from power

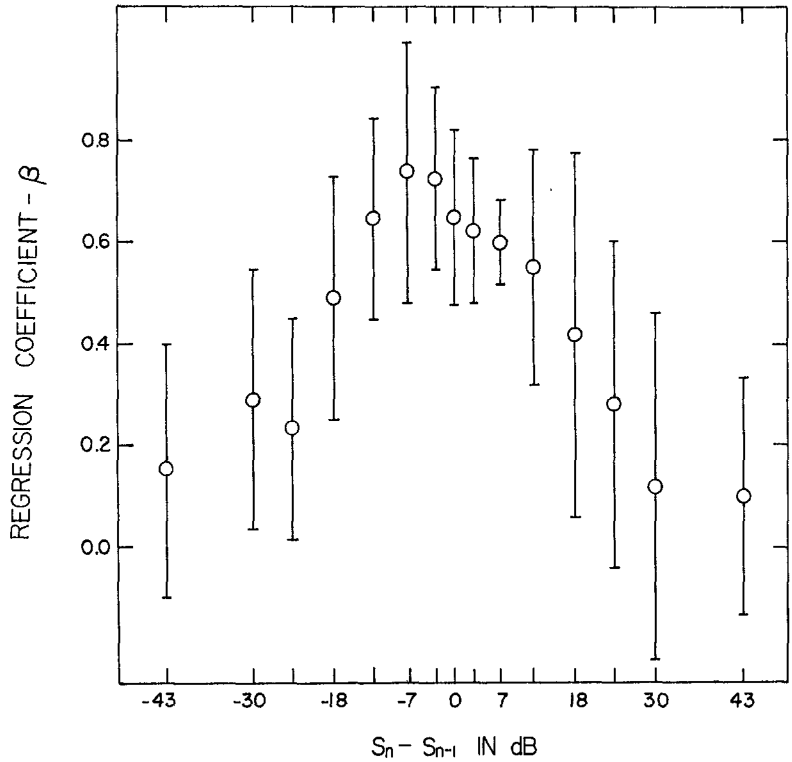

FIGURE 4. Regression coefficient, β, in Equation 5 as a function of $S_n - S_{n-1}$, the decibel difference in the stimuli on successive trials. (Each point is the mean for four subjects in two experiments. The vertical bars indicate the standard deviation of each group of eight values. The 52 possible stimulus differences have been collapsed into 15 categories with approximately equal numbers of trials per category. The categories are: 0, ±2–4, ±6–8, ±10–14, ±16–20, ±22–26, ±28–32, and ±34–52 dB.)

function fits to average data and is not γ of Equation 3.) The logarithm of the ratio $\underset{\sim}{R}_n/k\underset{\sim}{I}_n^{\gamma'}$ has nearly a zero mean and a variance roughly constant and independent of stimulus intensity. We now compute the following regression equation:

$$\log \frac{\underset{\sim}{R}_n}{k\underset{\sim}{I}_n^{\gamma'}} = \beta \log \frac{\underset{\sim}{R}_{n-1}}{k\underset{\sim}{I}_{n-1}^{\gamma'}} + \delta' + \underset{\sim}{\epsilon}, \quad (5)$$

for various ratios, I_{n-1}/I_n. Since the independent and dependent variables in Equation 5 have nearly the same variance, the regression coefficient β is also very nearly the correlation coefficient.

We computed the value of β for each observer in each of the two experiments and for 15 groupings of the signal differences in dB. The means and standard deviations of these values are displayed in Figure 4. As in the previous analysis, the average value of β across all of the categories is clearly less than 1. Furthermore, the deviation of the response from its expected value is highly correlated with the deviation on the previous trial when the change is small, but there is practically no correlation when the change is large. All four observers, in both experiments, showed the same phenomenon. This pattern of results is not consistent with the version of the response ratio hypothesis stated in Equation 1.

Theoretical Discussion

In an attempt to account for various phenomena, including aspects of absolute identification (Luce, Green, & Weber, 1976) and the dependence of $\sigma(\underset{\sim}{R}_n/\underset{\sim}{R}_{n-1})/$

$E(\underset{\sim}{R}_n/\underset{\sim}{R}_{n-1})$ on the ratio of successive signal intensities, we have been led to suppose there is a roving attention band that is about 10–15 dB wide and that has the following property (Luce & Green, 1974). Signals that happen to fall within the band are represented by a sample of neural events (times between pulses, for example) that is from 5–10 times as large as the sample resulting from signals that fall outside the band. Moreover, when the signals are randomly selected, there is evidence that observers tend to locate the band near the intensity of the last signal.

In order to account for our odd pattern of correlations, it is sufficient to suppose that the response ratio hypothesis holds for successive pairs of signals that both lie within the attention interval; but that when at least one lies outside the band, the response to $\underset{\sim}{S}_n$ is based on $\underset{\sim}{X}(\underset{\sim}{S}_n)$, not on $\underset{\sim}{R}_{n-1}\underset{\sim}{X}(\underset{\sim}{S}_n)/\underset{\sim}{X}^*(\underset{\sim}{S}_{n-1})$. This would destroy the correlation between $\underset{\sim}{R}_n$ and $\underset{\sim}{R}_{n-1}$ when $\underset{\sim}{S}_n$ is far from $\underset{\sim}{S}_{n-1}$; of course it would be less than 1 when they are close, since $\underset{\sim}{S}_{n-1}$ may not have been in the attention interval, and it is not known if the tendency to locate the band at the previous signal is any more than a tendency.

We have not yet thought of a sensitive way to test this hypothesis. We will, however, examine our data further on the assumption that it is correct.

Distribution of Magnitude Estimation Responses

Beta Fits

We return now to our original motivation, namely, to consider the distribution of magnitude estimates. If the response ratio hypothesis is true and if $1/\underset{\sim}{X}(s)$ is gamma distributed, as in the timing model, then the ratio of successive responses should be distributed according to a beta distribution of the second kind, with parameters $k(s)$ and $k(s')$ [or an F distribution with $2k(s)$ and $2k(s')$ degrees of freedom; see Green and Luce, 1974]. According to the timing model, $k(s)$ and $k(s')$ are the numbers of interarrival times or sample sizes for the present and previous signals, respectively. As we have just discussed, the pattern of correlation—especially the strong dependence upon the ratio of successive intensities—rules out the response ratio hypothesis in the most general terms. Nevertheless, we have suggested that the response ratio hypothesis may still apply to those successive trials on which the signals both fall within the attention band. The ratios might then be distributed as a beta distribution. Unfortunately, we know of no good way to assure ourselves that this is the case, and so our analysis must be viewed more as a means of obtaining a rough estimate of the parameter values than as a test of the theory. The situation is further complicated by the fact that simulations of beta distributions and attempts to recover their parameters indicate that the functions relating χ^2 to the parameters are very shallow and contain many local minima. To bring order out of this chaos, we considered only a limited range of possibilities for $k(s)$ and $k(s')$.

We proceeded as follows. We first considered all of the data, regardless of the decibel difference between successive signals. We assumed that on the average the two sample sizes would be equal. We found the value of $k(s') = k(s)$ that resulted in a minimum χ^2 when the distribution of response ratios was compared to a beta distribution (we used 26 categories with cut points every 5%, plus cut points at 1%, 2.5%, 7.5%, 92.5%, 97.5%, and 99%). We then considered two categories of trials (a) where the previous signal was within 6 dB of the present one and (b) where the difference exceeded 12 dB. If we assume the attention band is positioned at the intensity corresponding to the last signal, then the present signal should be within the attention band for trials in the first category and outside the attention band for trials in the second. We could not say anything about the previous signal, so we assumed that its sample size was given by the value of $k(s')$ for the unrestricted case.

We then found the value of $k(s) \geq k(s')$ that minimized χ^2 for trials in the first

TABLE 3

Beta Analysis

Subject and experiment	No Sorting			Δ ⩽ 6				Δ > 12			
	$k(s) = k(s')$	χ^2	n	$k(s)$	$k(s')$	χ^2	n	$k(s)$	$k(s')$	χ^2	n
Subject 1											
Experiment 1	5	98.6	1,188	22	5	52.0	299	5	5	82.2	739
Experiment 3	5	108.2	1,473	25	5	54.6	334	3	5	70.0	865
Subject 2											
Experiment 1	12	36.9	767	33	12	26.6	178	7	12	29.5	451
Experiment 3	7	72.3	1,381	28	7	65.8	345	4	7	21.5	788
Subject 3											
Experiment 1	7	56.7	1,572	30	7	39.0	419	6	7	26.9	867
Experiment 3	9	49.9	1,515	17	9	41.4	360	5	9	40.9	887
Subject 4											
Experiment 1	8	56.5	1,658	12	8	53.2	408	5	8	31.8	923
Experiment 3	8	112.2	1,891	14	8	188.4	454	8	8	31.8	1,079

Note. Each χ^2 is based on 25 degrees of freedom. The symbol Δ indicates the absolute difference in decibels between the stimulus on the present trial and the stimulus on the previous trial; $k(s)$ and $k(s')$ are parameters of the beta distribution, estimates of the sample size on the present and previous trials, respectively.

category and the value of $k(s) \leq k(s')$ that minimized χ^2 for trials in the second category. Table 3 presents the results. We see the pattern of a factor of 2–5 in the two sample sizes, not as large as other estimates. This discrepancy is probably due to contamination of the present estimates.

Gamma Fit

Another approach is to attempt to fit the gamma distribution to the distribution of the reciprocals of the individual responses, rather than fitting the beta distribution to the ratios of responses. According to the timing model, the reciprocals of the responses should be gamma distributed with k degrees of freedom, where k is the number of interarrival times. For these fits, we need to correct each response to remove the effects of the previous stimulus and response. We do this using the regression equation (Equation 3) and the parameters indicated in Table 2. The reciprocal of the corrected response is then normalized by dividing the response at any stimulus intensity by the expected value of the response at that intensity. The composite distribution obviously has a mean of unity and should be gamma distributed with a variance of k.

We used the same strategy in fitting gamma distributions as we did in fitting beta, except that it was not necessary to restrict the range of parameter values because there were fewer possibilities and the functions relating χ^2 to the parameter value did not generally have local minima. We first fitted all of the data, regardless of the relation of the present signal intensity to the signal intensity on the previous trial. We then considered the two categories of trials where the previous signal was within 6 dB of the present one and where it differed by more than 12 dB. The values of $k(s)$ and the corresponding minimum values of χ^2 are presented in Table 4. We see the same pattern observed in the beta analysis. The values of χ^2 are generally lower in the cases where the difference in signal levels has been restricted. When the difference is 6 dB or less, the value of $k(s)$ is always greater than in the unrestricted case. This agrees with the assumption that the sample size is larger. When the difference in signal levels is greater than 12 dB, the value of $k(s)$ that minimizes χ^2 is always less than or equal to the value in the unrestricted case. The pattern of these results therefore supports the approach used in fitting the beta distribution as well as our assumptions about the attention band.

Finally, we fitted the same data to a log normal distribution. The log normal has

TABLE 4
GAMMA ANALYSIS

Subject and experiment	No sorting			$\Delta \leqslant 6$			$\Delta > 12$		
	k	χ^2	n	k	χ^2	n	k	χ^2	n
Subject 1									
Experiment 1	6	39.3	1,188	8	22.3	299	5	42.9	739
Experiment 3	7	50.6	1,473	10	27.9	334	7	35.1	865
Subject 2									
Experiment 1	9	44.9	767	15	19.9	178	7	36.6	451
Experiment 3	6	82.8	1,381	11	46.1	345	5	33.0	788
Subject 3									
Experiment 1	5	61.8	1,572	9	21.8	419	4	57.2	867
Experiment 3	7	27.9	1,515	11	18.5	360	7	22.8	887
Subject 4									
Experiment 1	7	51.5	1,658	10	24.2	408	6	51.6	923
Experiment 3	8	201.0	1,891	16	114.9	454	14	220.6	812

Note. Each χ^2 is based on 25 degrees of freedom. The symbol Δ indicates the absolute difference in decibels between the stimulus on the present trial and the stimulus on the previous trial; k is the parameter of the gamma distribution, an estimate of the sample size on the present trial.

many similarities to the gamma and preserves an important and salient property of the data, that is, that the mean-to-sigma ratio is roughly constant and independent of signal intensity. The fits to the log normal were in every case poorer than the fits to the gamma. The average χ^2 associated with the log normal distribution is about a factor of two larger than the χ^2s shown in Table 4. Although the log normal is only one of many alternative hypotheses, we are encouraged that the gamma appears to fit better than this reasonable alternative distribution.

SUMMARY

This article reports data collected in three magnitude estimation experiments; it especially emphasizes the nature of the sequential effects and how to estimate parameter values associated with them. Our findings are these: A simple linear regression model in the logarithm of the response and in the signal level in decibels provides a quite adequate characterization of the sequential dependencies. These effects are almost exclusively due to the signal and response on the previous trial; there do not seem to be any sequential effects resulting from still earlier trials. A similar pattern is demonstrated in absolute identification data. We also present evidence that the magnitude of the correlation between successive responses depends heavily upon the decibel difference between the corresponding signals. This is a new finding and poses a considerable theoretical obstacle to the response ratio hypothesis. Finally, we compare the distribution of normalized responses with two theoretical distributions. The parameters that were estimated using some restricted searches to minimize χ^2 are in rough agreement with our modified response ratio model. The gamma fits to the reciprocal responses are, on an absolute scale, relatively good, and the parameters estimated from these fits are in qualitative accord with the expectations of the model.

REFERENCES

Cross, D. V. Sequential dependencies and regression in psychophysical judgments. *Perception & Psychophysics*, 1973, *14*, 547–552.

Garner, W. R. An informational analysis of absolute judgments of loudness. *Journal of Experimental Psychology*, 1953, *46*, 373–380.

Green, D. M., & Luce, R. D. Variability of magnitude estimates: A timing theory analysis. *Perception & Psychophysics*, 1974, *15*, 291–300.

Helson, H. Adaptation-level as a basis for a quantitative theory of frames of reference. *Psychological Review*, 1948, *55*, 297–313.

Holland, M. K., & Lockhead, G. R. Sequential effects in absolute judgments of loudness. *Perception & Psychophysics*, 1968, *3*, 409–414.

Logue, A. W. Individual differences in magnitude estimation of loudness. *Perception & Psychophysics*, 1976, *19*, 279–280.

Luce, R. D., & Green, D. M. A neural timing theory for response times and the psychophysics of intensity. *Psychological Review*, 1972, *79*, 14–57.

Luce, R. D., & Green, D. M. The response-ratio hypothesis for magnitude estimation. *Journal of Mathematical Psychology*, 1974, *11*, 1–14.

Luce, R. D., Green, D. M., & Weber, D. L. Attention bands in absolute identification. *Perception & Psychophysics*, 1976, *20*, 49–54.

Parducci, A. Direction of shift in the judgment of single stimuli. *Journal of Experimental Psychology*, 1956, *51*, 169–178.

Pollack, I. Neutralization of stimulus bias in auditory rating scales. *Journal of the Acoustical Society of America*, 1964, *36*, 1272–1276.

Ward, L. M. Category judgments of loudness in the absence of an experimenter-induced identification function: Sequential effects and power-function fit. *Journal of Experimental Psychology*, 1972, *94*, 179–184.

Ward, L. M. Repeated magnitude estimates with a variable standard: Sequential effects and other properties. *Perception & Psychophysics*, 1973, *13*, 193–200.

Ward, L. M., & Lockhead, G. R. Sequential effects and memory in category judgments. *Journal of Experimental Psychology*, 1970, *84*, 27–34.

Ward, L. M., & Lockhead, G. R. Response system processes in absolute judgment. *Perception & Psychophysics*, 1971, *9*, 73–78.

(Received February 10, 1976)