

# CSCI 5582 Artificial Intelligence

Lecture 21  
Jim Martin

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## Today 11/14

- Review
- Hypothesis Learning
  - Version Spaces
- Break
- Relational Learning
  - ILP

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## Review

- Supervised machine learning
  - Naïve Bayes
  - Decision trees
  - Decision lists
  - Ensembles

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## Classifiers

- These all provide a way to separate objects into classes based on intrinsic features of the object (encoded as sets of feature/value pairs).
- They don't necessarily provide a definition for the concept learned
- They can't deal with relational data.

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## Classifiers

- Uncle?

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## Concept Learning

- In **concept learning** we'd like to learn something akin to a definition: necessary and sufficient conditions for membership in a category
  - Rules out all non-members
  - Includes all members
- And we'd like to be able to deal with **relational data**
- Assume we're given positive and negative examples of the concept to be learned.

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## Data Mining

- The field of data mining is concerned with the extraction of possibly useful rules or patterns from large amounts of data.

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## Concept Learning

- And most importantly the concept to be learned is expressed in terms of predicates/propositions that are already known (that is we have a domain theory of some kind).

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## Basics

- In the context of concept learning, a **hypothesis** is just a theory of the concept that
  - Includes all members of the category
  - Excludes all non-members
- A **false negative** is...
- A **false positive** is...

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## Concept Learning: Search

- Again its just search. We're searching through the space of possible hypotheses to find one (all?) that do exactly what we want: cover all and only the concepts we're trying to learn.

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## Current Best Hypothesis

Maintain a single hypothesis at a time.

Perform surgery on it on demand.

- If I give it a positive example and it covers it...
- If I give it a negative example and it rejects it...

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## Current Best Hypothesis

- If I give a positive example and it rejects it...
  - False negative.
    - Adjust the theory so that
      1. It...
      2. And it...

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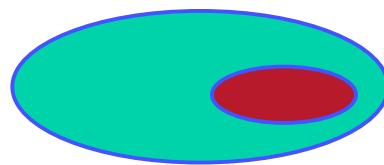
## Current Best Hypothesis

- If I give it a negative example and it accepts it
  - False positive
    - Adjust the theory so that it
      1. ?
      2. ?

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## CBH

- How?
  - Depends on the language being used. But the critical notion to exploit is generalization/specialization



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## CBH

- If you need to cover a falsely rejected positive example...
  - You need to generalize your hypothesis
- If you need to reject a false accepted negative example...
  - You need to specialize your hypothesis

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## How...

- Depends on the language... typically
  - Dropping/adding conditions for membership
  - Adding/removing disjuncts from a definition

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## So...

- Search is just specializing/generalizing a single hypothesis in response to **each** successive training example
- Until you cover all and only.
- But....

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## But

- The backtracking inherent in CBH is pretty horrible.
- It turns out it isn't really required
- CBH is making commitments early on that it really doesn't have to make
  - Exploit the hierarchy inherent in the logical structure of the hypotheses

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## Version Space Learning

- You can represent the space of hypothesis by representing certain boundaries (without representing the hypotheses) themselves.
- In response to false positives and false negatives you simply adjust the boundaries.
- At the end the space of hypotheses within the boundaries are all consistent with the training data.

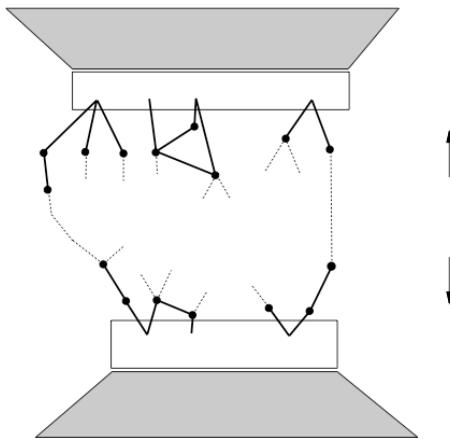
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## Version Space Learning

- I give you a single positive training example...
  - What's the most general theory you can come up with?
  - What's the most specific theory you can come up with?

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VS



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## Version Space Learning

- Termination....
  - When I run out of training examples
  - When the VS collapses
    - There are no theories left in the space

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## Break

- Look at
  - holmes.txt and tarzan.txt in
  - [www.cs.colorado.edu/~martin/Csci5582](http://www.cs.colorado.edu/~martin/Csci5582)

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## Break

- I'll go over the quiz topics Thursday

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## Relational Learning and Inductive Logic Programming

- Fixed feature vectors are a very limited representation of objects.
- Examples or target concept may require relational representation that includes multiple entities with relationships among them.
- First-order predicate logic is a more powerful representation for handling such relational descriptions.

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## ILP Example

- Learn definitions of family relationships given data for primitive types and relations.  
 $\text{brother}(A,C), \text{parent}(C,B) \rightarrow \text{uncle}(A,B)$   
 $\text{husband}(A,C), \text{sister}(C,D), \text{parent}(D,B) \rightarrow \text{uncle}(A,B)$
- Given the relevant predicates and a database populated with positive and negative examples
- By database I mean sets of tuples for each of the relevant relations

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## FOIL

### First-Order Inductive Logic

- Top-down sequential covering algorithm to learn first order theories.
- Background knowledge provided extensionally (ie. A model)
- Start with the most general rule possible. ( $T \rightarrow P(x)$ )
- Specialize it on demand...
- Specializations of a clause include adding all possible literals one at a time to the antecedent...
  - $A \rightarrow P$
  - $B \rightarrow P$
  - $C \rightarrow P\dots$

Where  $A$ ,  $B$  and  $C$  are predicates already in the domain theory.

We're working top-down from the most general hypothesis so what's driving things?

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## FOIL

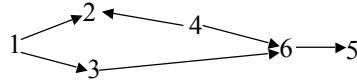
- At a high level.
  - Start with the most general  $H$
  - Repeatedly constructs clauses that cover a subset of the positive examples and none of the negative examples.
  - Then remove the covered positive examples
  - Constructs another clause
  - Repeat until all the positive examples are covered.

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## FOIL Training Data

- Background knowledge consists of complete set of tuples for each background predicate for this universe.
- Example: Consider learning a definition for the target predicate path for finding a path in a directed acyclic graph.

```
path(X,Y) :- edge(X,Y).  
path(X,Y) :- edge(X,Z), path(Z,Y).
```

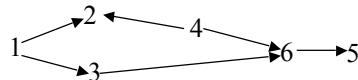


```
edge: {<1,2>,<1,3>,<3,6>,<4,2>,<4,6>,<6,5>}  
path: {<1,2>,<1,3>,<1,6>,<1,5>,<3,6>,<3,5>,  
<4,2>,<4,6>,<4,5>,<6,5>}
```

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## FOIL Negative Training Data

- Negative examples of target predicate can be provided directly, or generated indirectly by making a *closed world assumption*.
  - Every pair of constants  $\langle X, Y \rangle$  not in positive tuples for path predicate.

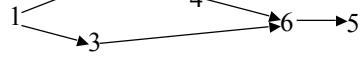


Negative path tuples:

```
{<1,1>,<1,4>,<2,1>,<2,2>,<2,3>,<2,4>,<2,5>,<2,6>,  
<3,1>,<3,2>,<3,3>,<3,4>,<4,1>,<4,3>,<4,4>,<5,1>,  
<5,2>,<5,3>,<5,4>,<5,5>,<5,6>,<6,1>,<6,2>,<6,3>,  
<6,4>,<6,6>}
```

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## Sample FOIL Induction



Pos: { $\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 6 \rangle, \langle 1, 5 \rangle, \langle 3, 6 \rangle, \langle 3, 5 \rangle,$   
 $\langle 4, 2 \rangle, \langle 4, 6 \rangle, \langle 4, 5 \rangle, \langle 6, 5 \rangle$ }

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 2, 5 \rangle, \langle 2, 6 \rangle,$   
 $\langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle, \langle 5, 1 \rangle,$   
 $\langle 5, 2 \rangle, \langle 5, 3 \rangle, \langle 5, 4 \rangle, \langle 5, 5 \rangle, \langle 5, 6 \rangle, \langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle,$   
 $\langle 6, 4 \rangle, \langle 6, 6 \rangle$ }

Start with clause:

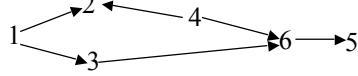
path(X, Y) :- .

Possible literals to add:

edge(X, X), edge(Y, Y), edge(X, Y), edge(Y, X), edge(X, Z),  
edge(Y, Z), edge(Z, X), edge(Z, Y), path(X, X), path(Y, Y),  
path(X, Y), path(Y, X), path(X, Z), path(Y, Z), path(Z, X),  
path(Z, Y), X=Y,

plus negations of all of these. CSCI 5582 Fall 2006

## Sample FOIL Induction



Pos: { $\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 6 \rangle, \langle 1, 5 \rangle, \langle 3, 6 \rangle, \langle 3, 5 \rangle,$   
 $\langle 4, 2 \rangle, \langle 4, 6 \rangle, \langle 4, 5 \rangle, \langle 6, 5 \rangle$ }

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 2, 5 \rangle, \langle 2, 6 \rangle,$   
 $\langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle, \langle 5, 1 \rangle,$   
 $\langle 5, 2 \rangle, \langle 5, 3 \rangle, \langle 5, 4 \rangle, \langle 5, 5 \rangle, \langle 5, 6 \rangle, \langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle,$   
 $\langle 6, 4 \rangle, \langle 6, 6 \rangle$ }

Test:

path(X, Y) :- edge(X, X) .

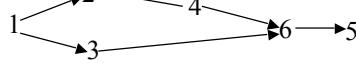
Covers 0 positive examples

Covers 6 negative examples

Not a good literal to try.

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## Sample FOIL Induction



Pos: { $\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 6 \rangle, \langle 1, 5 \rangle, \langle 3, 6 \rangle, \langle 3, 5 \rangle,$   
 $\langle 4, 2 \rangle, \langle 4, 6 \rangle, \langle 4, 5 \rangle, \langle 6, 5 \rangle$ }

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 2, 5 \rangle, \langle 2, 6 \rangle,$   
 $\langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle, \langle 5, 1 \rangle,$   
 $\langle 5, 2 \rangle, \langle 5, 3 \rangle, \langle 5, 4 \rangle, \langle 5, 5 \rangle, \langle 5, 6 \rangle, \langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle,$   
 $\langle 6, 4 \rangle, \langle 6, 6 \rangle$ }

Test:

`path(X, Y) :- edge(X, Y).`

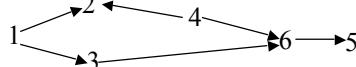
Covers 6 positive examples

Covers 0 negative examples

Chosen as best literal. Result is base clause.

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## Sample FOIL Induction



Pos: { $\langle 1, 6 \rangle, \langle 1, 5 \rangle, \langle 3, 5 \rangle,$   
 $\langle 4, 5 \rangle$ }

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 2, 5 \rangle, \langle 2, 6 \rangle,$   
 $\langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle, \langle 5, 1 \rangle,$   
 $\langle 5, 2 \rangle, \langle 5, 3 \rangle, \langle 5, 4 \rangle, \langle 5, 5 \rangle, \langle 5, 6 \rangle, \langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle,$   
 $\langle 6, 4 \rangle, \langle 6, 6 \rangle$ }

Test:

`path(X, Y) :- edge(X, Y).`

Covers 6 positive examples

Covers 0 negative examples

Chosen as best literal. Result is base clause.

Remove covered positive tuples

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## Sample FOIL Induction



Pos: { $<1, 6>$ ,  $<1, 5>$ ,  $<3, 5>$ ,  
 $<4, 5>$ }

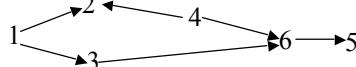
Neg: { $<1, 1>$ ,  $<1, 4>$ ,  $<2, 1>$ ,  $<2, 2>$ ,  $<2, 3>$ ,  $<2, 4>$ ,  $<2, 5>$ ,  $<2, 6>$ ,  
 $<3, 1>$ ,  $<3, 2>$ ,  $<3, 3>$ ,  $<3, 4>$ ,  $<4, 1>$ ,  $<4, 3>$ ,  $<4, 4>$ ,  $<5, 1>$ ,  
 $<5, 2>$ ,  $<5, 3>$ ,  $<5, 4>$ ,  $<5, 5>$ ,  $<5, 6>$ ,  $<6, 1>$ ,  $<6, 2>$ ,  $<6, 3>$ ,  
 $<6, 4>$ ,  $<6, 6>$ }

Start new clause

path(X, Y) :- .

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## Sample FOIL Induction



Pos: { $<1, 6>$ ,  $<1, 5>$ ,  $<3, 5>$ ,  
 $<4, 5>$ }

Neg: { $<1, 1>$ ,  $<1, 4>$ ,  $<2, 1>$ ,  $<2, 2>$ ,  $<2, 3>$ ,  $<2, 4>$ ,  $<2, 5>$ ,  $<2, 6>$ ,  
 $<3, 1>$ ,  $<3, 2>$ ,  $<3, 3>$ ,  $<3, 4>$ ,  $<4, 1>$ ,  $<4, 3>$ ,  $<4, 4>$ ,  $<5, 1>$ ,  
 $<5, 2>$ ,  $<5, 3>$ ,  $<5, 4>$ ,  $<5, 5>$ ,  $<5, 6>$ ,  $<6, 1>$ ,  $<6, 2>$ ,  $<6, 3>$ ,  
 $<6, 4>$ ,  $<6, 6>$ }

Test:

path(X, Y) :- edge(X, Y) .

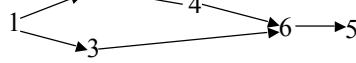
Covers 0 positive examples

Covers 0 negative examples

Not a good literal.

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## Sample FOIL Induction



Pos: { $\langle 1, 6 \rangle, \langle 1, 5 \rangle, \langle 3, 5 \rangle,$   
 $\langle 4, 5 \rangle \}$

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 2, 5 \rangle, \langle 2, 6 \rangle,$   
 $\langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle, \langle 5, 1 \rangle,$   
 $\langle 5, 2 \rangle, \langle 5, 3 \rangle, \langle 5, 4 \rangle, \langle 5, 5 \rangle, \langle 5, 6 \rangle, \langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle,$   
 $\langle 6, 4 \rangle, \langle 6, 6 \rangle \}$

Test:

`path(X, Y) :- edge(X, Z).`

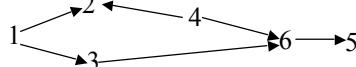
Covers all 4 positive examples

Covers 14 of 26 negative examples

Eventually chosen as best possible literal

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## Sample FOIL Induction



Pos: { $\langle 1, 6 \rangle, \langle 1, 5 \rangle, \langle 3, 5 \rangle,$   
 $\langle 4, 5 \rangle \}$

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle,$   
 $\langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle,$   
 $\langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle,$   
 $\langle 6, 4 \rangle, \langle 6, 6 \rangle \}$

Test:

`path(X, Y) :- edge(X, Z).`

Covers all 4 positive examples

Covers 15 of 26 negative examples

Eventually chosen as best possible literal

Negatives still covered, remove uncovered examples.

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## Sample FOIL Induction



Pos: { $\langle 1, 6, 2 \rangle, \langle 1, 6, 3 \rangle, \langle 1, 5, 2 \rangle, \langle 1, 5, 3 \rangle, \langle 3, 5, 2 \rangle, \langle 3, 5, 3 \rangle, \langle 4, 5, 2 \rangle, \langle 4, 5, 3 \rangle$ }

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 1, 6 \rangle, \langle 2, 1 \rangle, \langle 2, 4 \rangle, \langle 2, 6 \rangle, \langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 3, 6 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle, \langle 4, 6 \rangle, \langle 5, 1 \rangle, \langle 5, 2 \rangle, \langle 5, 3 \rangle, \langle 5, 6 \rangle, \langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle, \langle 6, 4 \rangle, \langle 6, 6 \rangle$ }

Test:

`path(X, Y) :- edge(X, Z).`

Covers all 4 positive examples

Covers 15 of 26 negative examples

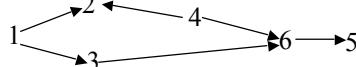
Eventually chosen as best possible literal

Negatives still covered, remove uncovered examples.

Expand tuples to account for possible  $Z$  values.

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## Sample FOIL Induction



Pos: { $\langle 1, 6, 2 \rangle, \langle 1, 6, 3 \rangle, \langle 1, 5, 2 \rangle, \langle 1, 5, 3 \rangle, \langle 3, 5, 2 \rangle, \langle 3, 5, 3 \rangle, \langle 4, 5, 2 \rangle, \langle 4, 5, 3 \rangle, \langle 4, 5, 6 \rangle$ }

Neg: { $\langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 1, 6 \rangle, \langle 2, 1 \rangle, \langle 2, 4 \rangle, \langle 2, 6 \rangle, \langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle, \langle 3, 6 \rangle, \langle 4, 1 \rangle, \langle 4, 3 \rangle, \langle 4, 4 \rangle, \langle 4, 6 \rangle, \langle 5, 1 \rangle, \langle 5, 2 \rangle, \langle 5, 3 \rangle, \langle 5, 6 \rangle, \langle 6, 1 \rangle, \langle 6, 2 \rangle, \langle 6, 3 \rangle, \langle 6, 4 \rangle, \langle 6, 6 \rangle$ }

Test:

`path(X, Y) :- edge(X, Z).`

Covers all 4 positive examples

Covers 15 of 26 negative examples

Eventually chosen as best possible literal

Negatives still covered, remove uncovered examples.

Expand tuples to account for possible  $Z$  values.

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## Sample FOIL Induction



Pos: { $<1, 6, 2>, <1, 6, 3>, <1, 5, 2>, <1, 5, 3>, <3, 5, 6>, <4, 5>$ }

Neg: { $<1, 1>, <1, 4>, <3, 1>, <3, 2>, <3, 3>, <3, 4>, <4, 1>, <4, 3>, <4, 4>, <6, 1>, <6, 2>, <6, 3>, <6, 4>, <6, 6>$ }

Test:

`path(X, Y) :- edge(X, Z).`

Covers all 4 positive examples

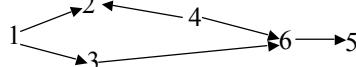
Covers 15 of 26 negative examples

Eventually chosen as best possible literal

Negatives still covered, remove uncovered examples.

Expand tuples to account for possible  $Z$  values.

## Sample FOIL Induction



Pos: { $<1, 6, 2>, <1, 6, 3>, <1, 5, 2>, <1, 5, 3>, <3, 5, 6>, <4, 5, 6>$ }

Neg: { $<1, 1>, <1, 4>, <3, 1>, <3, 2>, <3, 3>, <3, 4>, <4, 1>, <4, 3>, <4, 4>, <6, 1>, <6, 2>, <6, 3>, <6, 4>, <6, 6>$ }

Test:

`path(X, Y) :- edge(X, Z).`

Covers all 4 positive examples

Covers 15 of 26 negative examples

Eventually chosen as best possible literal

Negatives still covered, remove uncovered examples.

Expand tuples to account for possible  $Z$  values.

## Sample FOIL Induction



Pos: { $<1, 6, 2>, <1, 6, 3>, <1, 5, 2>, <1, 5, 3>, <3, 5, 6>, <4, 5, 6>$ }

Neg: { $<1, 1, 2>, <1, 1, 3>, <1, 4, 2>, <1, 4, 3>, <3, 1, 6>, <3, 2, 6>, <3, 3, 6>, <3, 4, 6>, <4, 1, 2>, <4, 1, 6>, <4, 3, 2>, <4, 3, 6>, <4, 4, 2>, <4, 4, 6>, <6, 1, 5>, <6, 2, 5>, <6, 3, 5>, <6, 4, 5>, <6, 6, 5>$ }

Test:

`path(X, Y) :- edge(X, Z).`

Covers all 4 positive examples

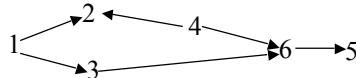
Covers 15 of 26 negative examples

Eventually chosen as best possible literal

Negatives still covered, remove uncovered examples.

Expand tuples to account for possible  $Z$  values.

## Sample FOIL Induction



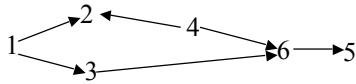
Pos: { $<1, 6, 2>, <1, 6, 3>, <1, 5, 2>, <1, 5, 3>, <3, 5, 6>, <4, 5, 6>$ }

Neg: { $<1, 1, 2>, <1, 1, 3>, <1, 4, 2>, <1, 4, 3>, <3, 1, 6>, <3, 2, 6>, <3, 3, 6>, <3, 4, 6>, <4, 1, 2>, <4, 1, 6>, <4, 3, 2>, <4, 3, 6>, <4, 4, 2>, <4, 4, 6>, <6, 1, 5>, <6, 2, 5>, <6, 3, 5>, <6, 4, 5>, <6, 6, 5>$ }

Continue specializing clause:

`path(X, Y) :- edge(X, Z).`

## Sample FOIL Induction



Pos: { $\langle 1, 6, 2 \rangle$ ,  $\langle 1, 6, 3 \rangle$ ,  $\langle 1, 5, 2 \rangle$ ,  $\langle 1, 5, 3 \rangle$ ,  $\langle 3, 5, 6 \rangle$ ,  
 $\langle 4, 5, 6 \rangle$ }

Neg: { $\langle 1, 1, 2 \rangle$ ,  $\langle 1, 1, 3 \rangle$ ,  $\langle 1, 4, 2 \rangle$ ,  $\langle 1, 4, 3 \rangle$ ,  $\langle 3, 1, 6 \rangle$ ,  $\langle 3, 2, 6 \rangle$ ,  
 $\langle 3, 3, 6 \rangle$ ,  $\langle 3, 4, 6 \rangle$ ,  $\langle 4, 1, 2 \rangle$ ,  $\langle 4, 1, 6 \rangle$ ,  $\langle 4, 3, 2 \rangle$ ,  $\langle 4, 3, 6 \rangle$ ,  
 $\langle 4, 4, 2 \rangle$ ,  $\langle 4, 4, 6 \rangle$ ,  $\langle 6, 1, 5 \rangle$ ,  $\langle 6, 2, 5 \rangle$ ,  $\langle 6, 3, 5 \rangle$ ,  
 $\langle 6, 4, 5 \rangle$ ,  $\langle 6, 6, 5 \rangle$ }

Test:

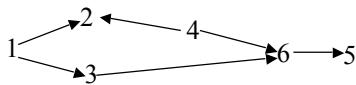
`path(X, Y) :- edge(X, Z), edge(Z, Y).`

Covers 3 positive examples

Covers 0 negative examples

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## Sample FOIL Induction



Pos: { $\langle 1, 6, 2 \rangle$ ,  $\langle 1, 6, 3 \rangle$ ,  $\langle 1, 5, 2 \rangle$ ,  $\langle 1, 5, 3 \rangle$ ,  $\langle 3, 5, 6 \rangle$ ,  
 $\langle 4, 5, 6 \rangle$ }

Neg: { $\langle 1, 1, 2 \rangle$ ,  $\langle 1, 1, 3 \rangle$ ,  $\langle 1, 4, 2 \rangle$ ,  $\langle 1, 4, 3 \rangle$ ,  $\langle 3, 1, 6 \rangle$ ,  $\langle 3, 2, 6 \rangle$ ,  
 $\langle 3, 3, 6 \rangle$ ,  $\langle 3, 4, 6 \rangle$ ,  $\langle 4, 1, 2 \rangle$ ,  $\langle 4, 1, 6 \rangle$ ,  $\langle 4, 3, 2 \rangle$ ,  $\langle 4, 3, 6 \rangle$ ,  
 $\langle 4, 4, 2 \rangle$ ,  $\langle 4, 4, 6 \rangle$ ,  $\langle 6, 1, 5 \rangle$ ,  $\langle 6, 2, 5 \rangle$ ,  $\langle 6, 3, 5 \rangle$ ,  
 $\langle 6, 4, 5 \rangle$ ,  $\langle 6, 6, 5 \rangle$ }

Test:

`path(X, Y) :- edge(X, Z), path(Z, Y).`

Covers 4 positive examples    Covers 0 negative examples

Eventually chosen as best literal; completes clause.

Definition complete, since all original  $\langle X, Y \rangle$  tuples are covered  
(by way of covering some  $\langle X, Y, Z \rangle$  tuple)

## More Realistic Applications

- Classifying chemical compounds as mutagenic (cancer causing) based on their graphical molecular structure and chemical background knowledge.
- Classifying web documents based on both the content of the page and its links to and from other pages with particular content.
  - A web page is a university faculty home page if:
    - It contains the words "Professor" and "University", and
    - It is pointed to by a page with the word "faculty", and
    - It points to a page with the words "course" and "exam"

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## Rule Learning and ILP Summary

- There are effective methods for learning symbolic rules from data using greedy sequential covering and top-down or bottom-up search.
- These methods have been extended to first-order logic to learn relational rules and recursive Prolog programs.
- Knowledge represented by rules is generally more interpretable by people, allowing human insight into what is learned and possible human approval and correction of learned knowledge.

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