University of Colorado
Department of Computer Science

Chaotic Dynamics – CSCI 4446/5446
Spring 2023

Problem Set 12

Issued:	11 April 2023
Due:	*25 April 2023*

Note: only undergraduates are required to turn in this problem set, but *all* students should do the reading listed below.

Project presentations: are scheduled during lecture on 25 and 27 April. I'll send around a scheduling doodle next week. Please make sure to look over the presentation hints that are posted on the class webpage before you write your talk.

Undergrads: you'll be writing short reports on each talk, so bring your computer on those days.

Reading: *Classical Mechanics Notes; Berreby paper (see the PS12 bullet on the course webpage for a link to this).*

Online assignment: Tuesday 4/11 and Thursday 4/13: unit 7.6 video. Tuesday 4/18 and Thursday 4/20: unit 10.1 and 10.3 videos. Friday 4/21: quizzes 10.1 and 10.3

Bibliography:

- H. Abelson, "The Bifurcation Interpreter: A step towards the automatic analysis of dynamical systems," *International Journal of Computers and Mathematics with Applications* **20**:13 (1990).
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- J.D. Farmer and J.J. Sidorowich, "Exploiting chaos to predict the future and reduce noise," in *Evolution, Learning and Cognition*, World Scientific, 1988.
- M. Hénon and C. Heiles, "The applicability of the third integral of motion: Some numerical experiments," *Astronomical Journal* **69**:73 (1964)
- M. Hénon, "Numerical exploration of Hamiltonian systems," in *Chaotic Behaviour of Deterministic Systems*, North-Holland, 1983.
- I. Percival, "Chaos in Hamiltonian systems," *Proceedings of the Royal Society, London* **413**:131-144 (1987), published in *Dynamical Chaos*, M. Berry, I. Percival, and N. Weiss, eds., University Press, Cambridge UK, 1987.

- I. Peterson, *Newton's Clock: Chaos in the Solar System*, Freeman, 1993.
- G. Sussman and J. Wisdom, "Numerical evidence that the motion of Pluto is chaotic," *Science* **241**:433-437, 1988.
- A. Weigend and N. Gershenfeld, eds., *Time Series Prediction: Forecasting the Future and Understanding the Past*, Santa Fe Institute Studies in the Sciences of Complexity, 1993.
- J. Wisdom, "Chaotic behavior in the solar system," *Nuclear Physics B* **2**:391-414, 1987.
- J. Wisdom, "Is the solar system stable? and Can we use chaos to make measurements?," in *Chaos/XAOC : Soviet-American perspectives on nonlinear science*, D. K. Campbell, editor, American Institute of Physics, New York, 1990.
- K. Yip, *KAM: A System for Intelligently Guiding Numerical Experimentation by Computer*, MIT Press, 1991.
- K. Yip, "Understanding complex dynamics by visual and symbolic reasoning," *Artificial Intelligence* **51**, Special Volume on Qualitative Reasoning About Physical Systems II, 1991.

Problems:

Adapt your RK4 algorithm from PS4 to the two-body equations — the system of equations labeled (4) in the *Classical Mechanics Notes*. To do this, you'll have to pull these equations apart into twelve first-order ODEs by rewriting them out in terms of their coordinates, as I show in the *Notes*.

Plug your answers to problem 4(d) on PS11 into these ODEs and integrate them for half a dozen orbits, using at least 1000 timesteps per orbit period. Plot your results in physical space: one point for each star, looking down from above on the plane of the orbit. Turn in a plot and draw a few arrows to show which way each star is going.

Does your binary orbit properly? Does the center of mass appear to move? Which way?