Petri Nets

- Formal Definition

\[ N = \{P,T,A,M_0\}, \text{ where} \]
\[ P \text{ is a finite set of places} \]
\[ T \text{ is a finite set of transitions} \]
\[ A \text{ is a finite set of arcs (arrows)} \]
\[ M_0 \text{ is the initial marking of } N \]
Graphical Representation
Graphical Representation
Graphical Representation
Graphical Representation

In 1

Out 1

CR 1

Sem

CR 2

Out 2

In 2

arc

place

transition
Graphical Representation

In_1

arc

place

Out_1

CR_1

Sem

CR_2

In_2
token

Out_2

transition
Graphical Representation

- In
  - In\(_1\)
  - In\(_2\)
- Out
  - Out\(_1\)
  - Out\(_2\)
- arc
- place
- CR\(_1\)
- CR\(_2\)
- Sem
- token
- transition
Petri Nets

- Intuitive Meaning
  - A place holds tokens
  - A transition represents activity
  - An arc connects a place and a transition
  - A marking is an arrangement of tokens in places, representing state
  - An initial marking represents an initial state
Execution Model

- **Input and Output Places**
  - Place $P$ is an *input place* for transition $T$ if there is an arc from $P$ to $T$
  - Place $P$ is an *output place* for transition $T$ if there is an arc from $T$ to $P$

- **Enabled Transition**
  - A transition is *enabled* if there is at least one token at each of its input places
Petri Net Semaphore

In 1

CR 1

Out 1

In 2

CR 2

Out 2

Sem
Enabled Transitions
Execution Model

- Firing a Transition
  - An enabled transition is nondeterministically selected and fired by removing one token from each of its input places and depositing one token at each of its output places.

- Firing Sequence
  - A firing sequence is a sequence $<t_0, t_1, \ldots, t_n>$ such that $t_0$ is enabled and fired in $M_0$, $t_1$ is enabled and fired in $M_1$, etc.
Enabled Transitions
After Firing
Enabled Transition
After Firing

\[ \text{In}_1 \quad \text{Out}_1 \quad \text{CR}_1 \quad \text{CR}_2 \quad \text{Sem} \quad \text{In}_2 \quad \text{Out}_2 \]
Enabled Transition
Breaking the Semaphore

- Lets look at the semaphore example again and see how a change to the initial marking will change the semantics of the Petri Net.
  - In particular, we will break the semantics of the semaphore by adding one token.
Petri Net Semaphore
Enabled Transitions

Lecture 17
After Firing
Enabled Transitions
After Firing
Enabled Transitions
After Firing

\[ \text{In}_1 \rightarrow \text{CR}_1 \rightarrow \text{Out}_1 \]
\[ \text{CR}_2 \rightarrow \text{Sem} \rightarrow \text{CR}_1 \]
\[ \text{In}_2 \rightarrow \text{CR}_2 \rightarrow \text{Out}_2 \]