

# Adding Object-Oriented Capabilities to Mathematica

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# Roadmap

## About Mathematica

- ▶ Environment
- ▶ Language features
- ▶ Programming paradigms

## What might object-oriented Mathematica be like?

- ▶ Ongoing efforts to add capabilities
- ▶ The *Objectica* add-on

## Discussion of object orientation in Mathematica

- ▶ Emergence
- ▶ Best uses

## Resources

- ▶ 2



# Mathematica

## What is it?

- ▶ Software for making computations and visualizing results
  - ▶ Interactive exploration is a key feature
- ▶ Originally developed and released by Stephen Wolfram in 1988; now sold by Wolfram Research

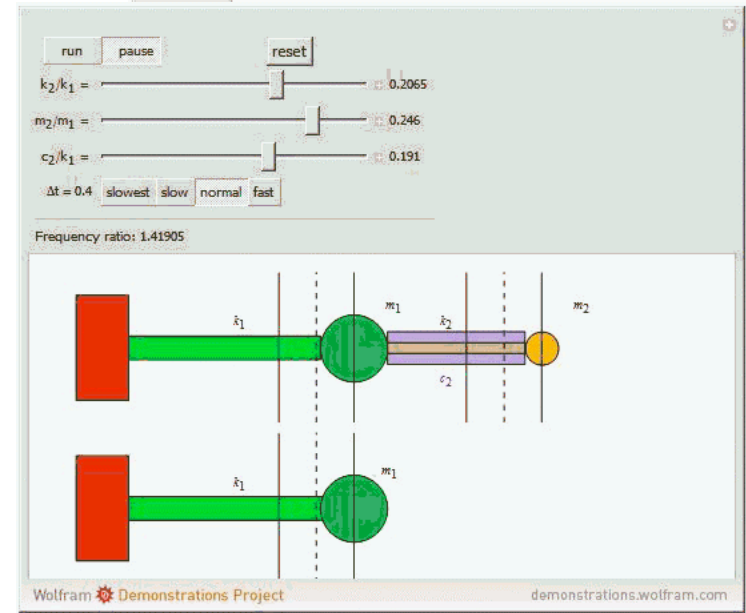
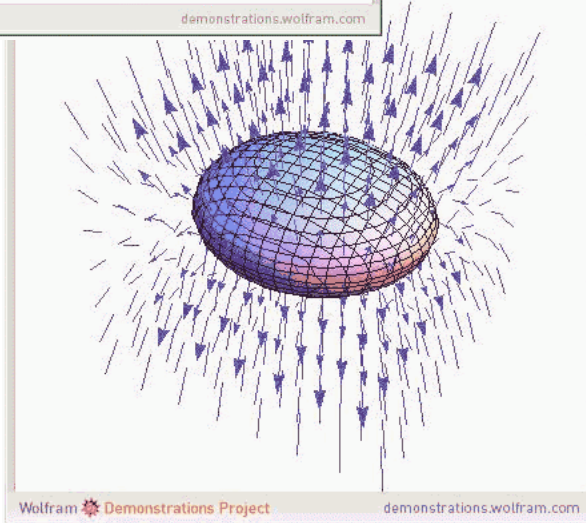
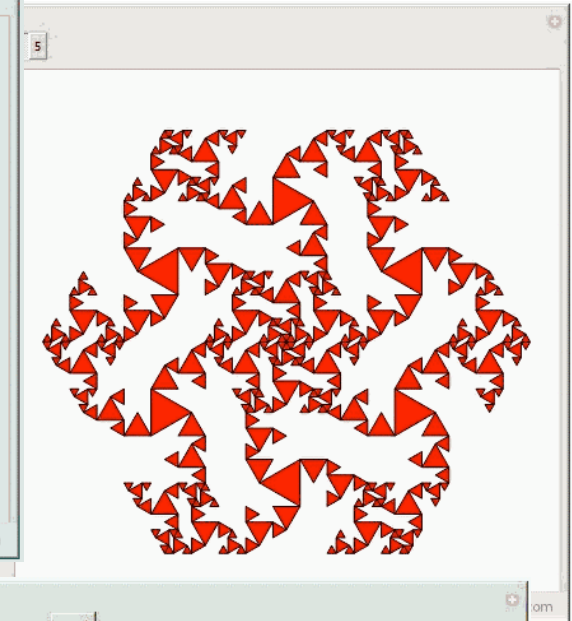
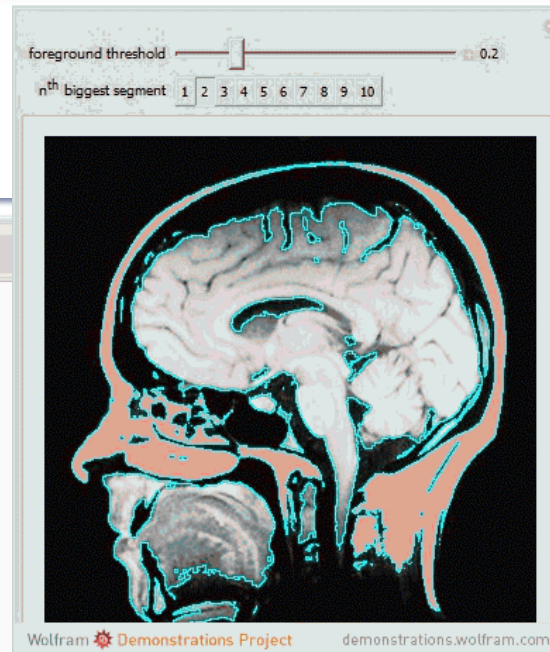
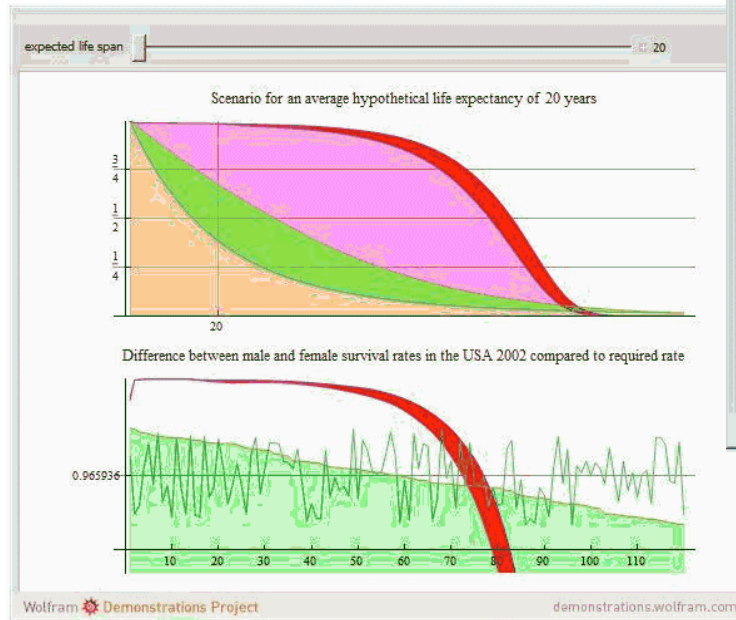
## Who uses it?

- ▶ Millions of users
  - ▶ STEM / Medicine
  - ▶ Business
  - ▶ Social sciences
  - ▶ Education
  - ▶ Arts

“Mathematica has become a standard in a great many organizations, and it is used today in all of the Fortune 50 companies, all of the 15 major departments of the U.S. government, and all of the world’s 50 largest universities.”



# Mathematica



# Mathematica: Environment

## Interactive user interfaces

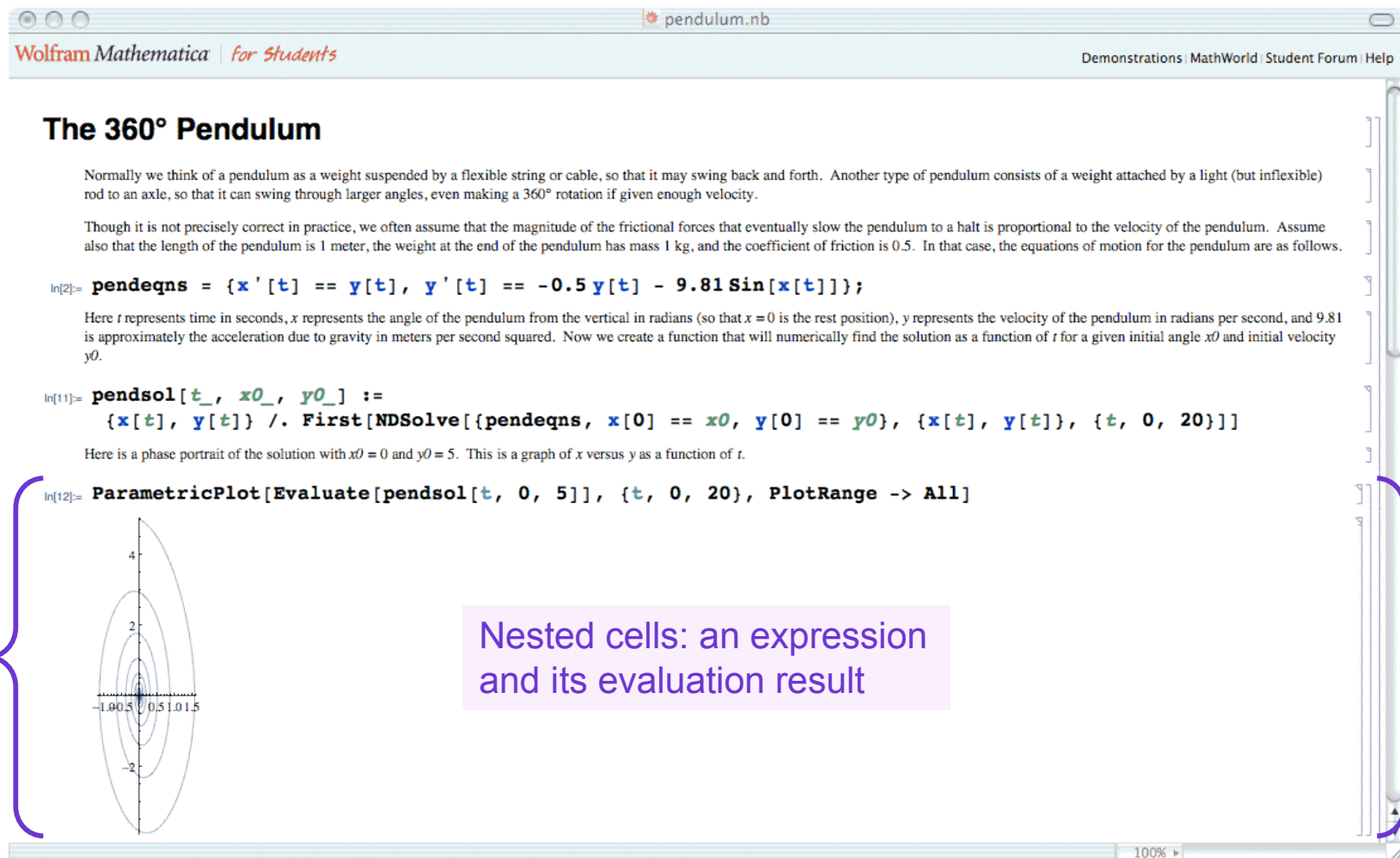
- ▶ Notebooks
  - ▶ Evaluate expression in any *cell*, see results immediately
  - ▶ May include explanatory text
  - ▶ Mathematica help files are also notebooks
- ▶ Workbench (Eclipse IDE)
- ▶ Web-based player for local and remote content
  - ▶ Replaced desktop-based player

## Computation engine

- ▶ Kernel accessible from above interfaces and as a service to other programs



# Mathematica: Environment



**The 360° Pendulum**

Normally we think of a pendulum as a weight suspended by a flexible string or cable, so that it may swing back and forth. Another type of pendulum consists of a weight attached by a light (but inflexible) rod to an axle, so that it can swing through larger angles, even making a 360° rotation if given enough velocity.

Though it is not precisely correct in practice, we often assume that the magnitude of the frictional forces that eventually slow the pendulum to a halt is proportional to the velocity of the pendulum. Assume also that the length of the pendulum is 1 meter, the weight at the end of the pendulum has mass 1 kg, and the coefficient of friction is 0.5. In that case, the equations of motion for the pendulum are as follows.

```
In[2]:= pendeqns = {x'[t] == y[t], y'[t] == -0.5 y[t] - 9.81 Sin[x[t]]};
```

Here  $t$  represents time in seconds,  $x$  represents the angle of the pendulum from the vertical in radians (so that  $x=0$  is the rest position),  $y$  represents the velocity of the pendulum in radians per second, and 9.81 is approximately the acceleration due to gravity in meters per second squared. Now we create a function that will numerically find the solution as a function of  $t$  for a given initial angle  $x0$  and initial velocity  $y0$ .

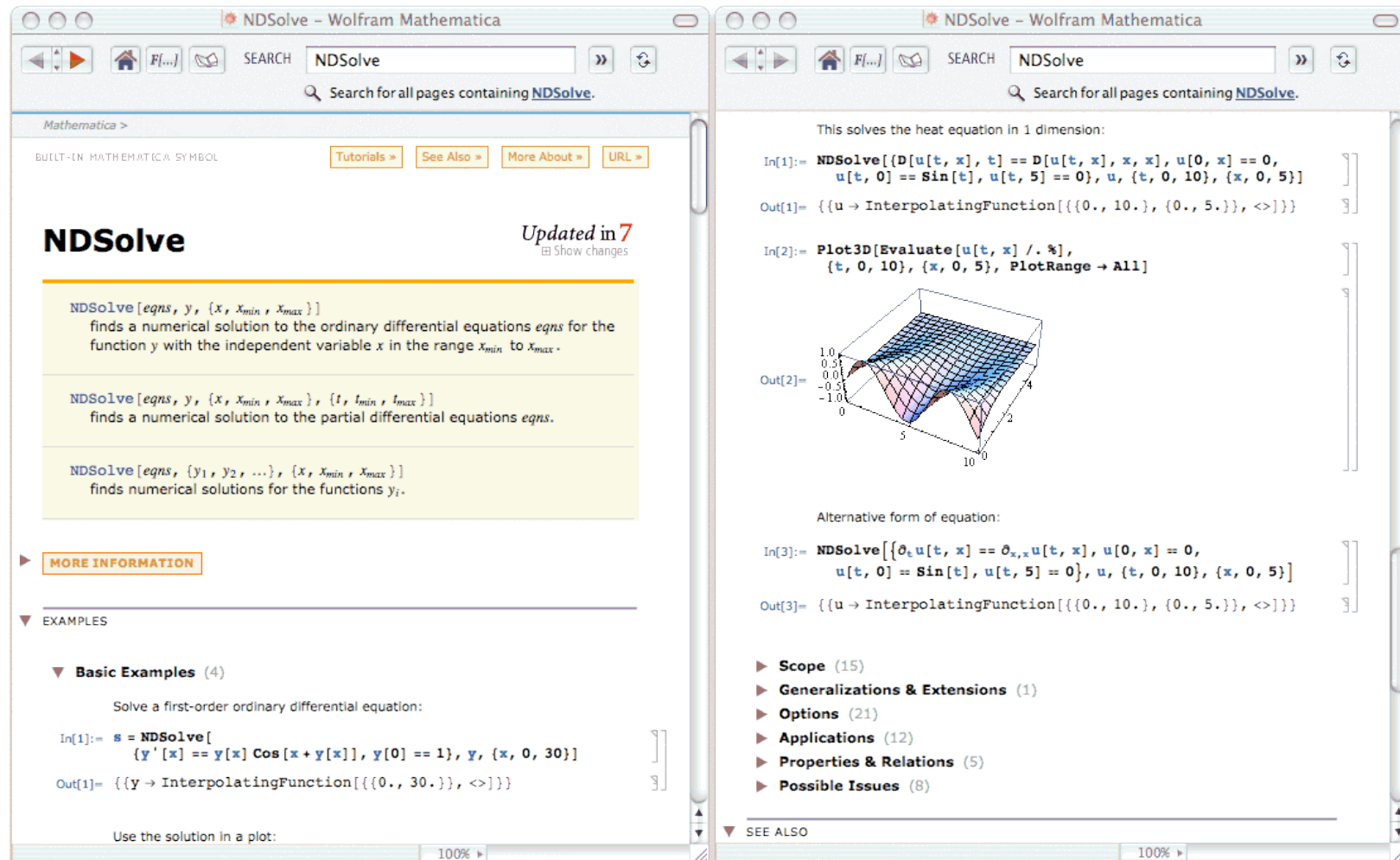
```
In[11]:= pendsol[t_, x0_, y0_] :=  
  {x[t], y[t]} /. First[NDSolve[{pendeqns, x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, 0, 20}]]
```

Here is a phase portrait of the solution with  $x0=0$  and  $y0=5$ . This is a graph of  $x$  versus  $y$  as a function of  $t$ .

```
In[12]:= ParametricPlot[Evaluate[pendsol[t, 0, 5]], {t, 0, 20}, PlotRange -> All]
```

Nested cells: an expression and its evaluation result

# Mathematica: Environment



**NDSolve** Updated in 7

**NDSolve**  
finds a numerical solution to the ordinary differential equations *eqns* for the function *y* with the independent variable *x* in the range  $x_{min}$  to  $x_{max}$ .

**NDSolve**  
finds a numerical solution to the partial differential equations *eqns*.

**NDSolve**  
finds numerical solutions for the functions  $y_i$ .

**EXAMPLES**

**Basic Examples (4)**

Solve a first-order ordinary differential equation:

```
In[1]:= s = NDSolve[
  {y'[x] == y[x] Cos[x + y[x]], y[0] == 1}, y, {x, 0, 30}]
Out[1]:= {{y -> InterpolatingFunction[{{0., 30.}}, <>]}}
```

Use the solution in a plot:

This solves the heat equation in 1 dimension:

```
In[1]:= NDSolve[{D[u[t, x], t] == D[u[t, x], x, x], u[0, x] == 0,
  u[t, 0] == Sin[t], u[t, 5] == 0}, u, {t, 0, 10}, {x, 0, 5}]
Out[1]:= {{u -> InterpolatingFunction[{{0., 10.}}, {0., 5.}], <>}}
```

```
In[2]:= Plot3D[Evaluate[u[t, x] /. %],
  {t, 0, 10}, {x, 0, 5}, PlotRange -> All]
Out[2]=
```

Alternative form of equation:

```
In[3]:= NDSolve[{D[u[t, x], t] == D[u[t, x], x, x], u[0, x] == 0,
  u[t, 0] == Sin[t], u[t, 5] == 0}, u, {t, 0, 10}, {x, 0, 5}]
Out[3]:= {{u -> InterpolatingFunction[{{0., 10.}}, {0., 5.}], <>}}
```

**Scope (15)**  
**Generalizations & Extensions (1)**  
**Options (21)**  
**Applications (12)**  
**Properties & Relations (5)**  
**Possible Issues (8)**

**SEE ALSO**



# Mathematica: Environment

## Interactive user interfaces

- ▶ Notebooks
  - ▶ Evaluate expressions in *cells*, see results immediately
  - ▶ May include explanatory text
  - ▶ Mathematica help files are also notebooks
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# Mathematica: Language Features

Numerous built-in functions and libraries

- ▶ Chooses best algorithm

Scoping

- ▶ Modules  
(lexical scoping)
- ▶ Blocks  
(dynamic scoping, less commonly used)

Exception handling

String / list manipulation

Rules and pattern matching

Symbolic computation

- ▶ Here is a typical numerical computation.

```
In[1]:= 3 + 62 - 1
```

```
Out[1]= 64
```

This is a symbolic computation.

```
In[2]:= 3 x - x + 2
```

```
Out[2]= 2 + 2 x
```



# Mathematica: Programming Paradigms

## Major paradigms

- ▶ Procedural
- ▶ Functional
- ▶ **Object-oriented** (*or so they say...*)

## Additional paradigms

- ▶ List-based
- ▶ Rule-based
- ▶ String-based



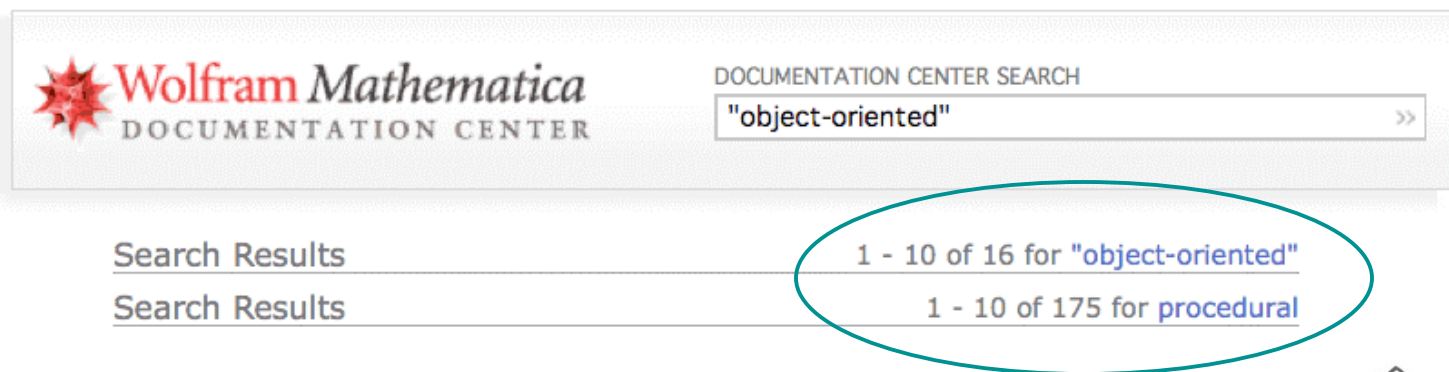
# Object-Oriented Mathematica

Wolfram's early claims of object-oriented capabilities have faded over time...

- ▶ Then “It is very easy to do object-oriented programming in Mathematica. The basic idea is to associate Mathematica transformation rules with the objects they act on rather than with the functions they perform.”

— *The Mathematica Book, First Edition*

- ▶ Now



The screenshot shows the Wolfram Mathematica Documentation Center search interface. The search bar contains the text "object-oriented". Below the search bar, the results are displayed in two columns. The left column shows "Search Results" and "Search Results". The right column shows "1 - 10 of 16 for 'object-oriented'" and "1 - 10 of 175 for procedural". A red circle highlights the results for "object-oriented".

# Object-Oriented Mathematica: Built-In Capabilities

## Object-Oriented Definition

**UpSet** ( $\wedge=$ ) — associate a definition with an inner construct

**TagSet** ( $/: \dots =$ ) — associate a definition with any construct

*Still not giving up...*

## Functional Programming

Long viewed as an important theoretical idea, functional programming finally became truly convenient and practical with the introduction of *Mathematica*'s symbolic language. Treating expressions like  $f[x]$  as both symbolic data and the application of a function  $f$  provides a uniquely powerful way to integrate structure and function—and an efficient, elegant representation of many common computations.

**Function** ( $\&$ ) — specify a pure function (e.g.  $(\# + 1) \&$ )

$\#, \#\#$  — slots for variables in a pure function

### Applying Functions to Lists »

**Map** ( $/@$ ) — map across a list:  $f /@ \{x, y, z\} \rightarrow \{f[x], f[y], f[z]\}$

**Apply** ( $@@, @@@$ ) — apply to a list:  $f @@ \{x, y, z\} \rightarrow f[x, y, z]$

**MapIndexed** — map with index information:  $\{f[x, \{1\}], f[y, \{2\}], f[z, \{3\}]\}$

**MapThread** • **MapAt** • **MapAll** • **Scan** • ...

### Iteratively Applying Functions »

**Nest**, **NestList** — nest a function:  $f[f[f[x]]]$  etc.

**Fold**, **FoldList** — fold in a list of values:  $f[f[f[x, 1], 2], 3]$  etc.

**FixedPoint**, **FixedPointList** — repeatedly nest until a fixed point

**NestWhile** • **NestWhileList** • **TakeWhile** • **LengthWhile** • ...

### List-Oriented Functions

**Select** — select from a list according to a function

**Array** — create an array from a function

**Sort**, **Split** — sort, split according to a function

### Functional Composition Operations

**Identity** • **Composition** • **Operate** • **Through** • **Distribute**

## Procedural Programming

*Mathematica* stands out from traditional computer languages in supporting many programming paradigms. Procedural programming is the only paradigm available in languages like C and Java, as well as most scripting languages. *Mathematica* supports all standard procedural programming constructs, but often extends them through integration into its more general symbolic programming environment.

$x = \text{value}$  (**Set**) — set the value for a variable

$\text{expr}_1 \text{expr}_2 \text{expr}_3$  (**CompoundExpression**) — execute expressions in sequence

### Assignments »

$=$  •  $+=$  •  $++$  •  $*$  •  $AppendTo$  • ...

### Loops »

**Do** • **While** • **For** • **Table** • **Nest** • ...

### Conditionals »

**If** • **Which** • **Switch** • **And**( $\&\&$ ) • **Equal**( $==$ ) • **Less**( $<$ ) • ...

### Flow Control »

**Return** • **Throw** • **Catch** • **TimeConstrained** • ...

### Scoping Constructs »

**Module** • **With** • **Block** • ...

### Input, Output, Etc. »

**Print** • **Input** • **Pause** • **Import** • **OpenRead** • ...



# Object-Oriented Mathematica: Built-In Capabilities

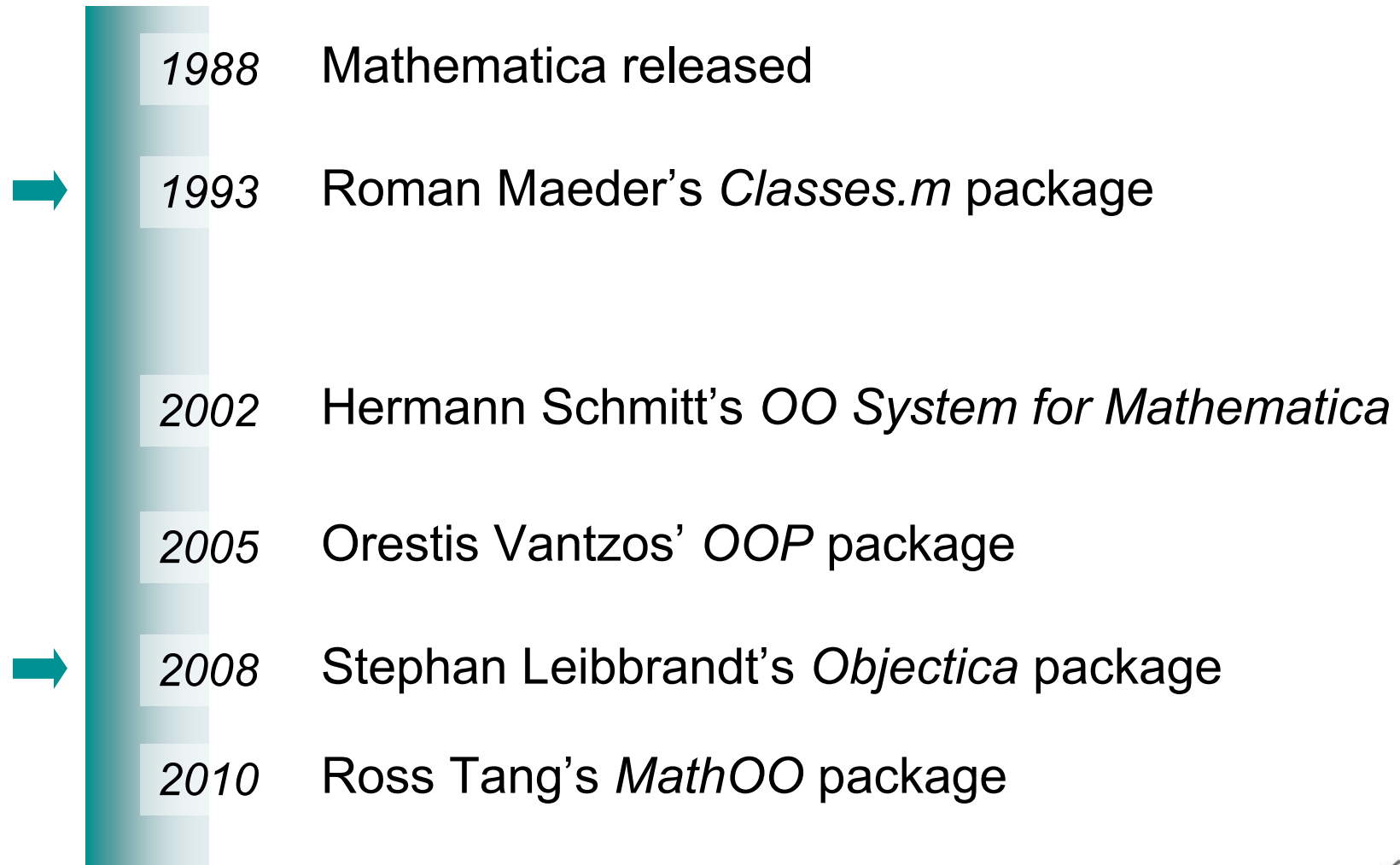
## Stack example using TagSet (/: ... =)

- ▶ `stackobj /: push[stackobj[stack_, item_]] := Append[stack, item];`  
`stackobj /: pop[stackobj[stack_]] := Most[stack];`
- ▶ `mystack = {1, 2, 3}`  
`myitem = 4`
- ▶ `mystack = push[stackobj[mystack, myitem]]`
  - ▶ `{1, 2, 3, 4}`
- ▶ `mystack = pop[stackobj[mystack]]`
  - ▶ `{1, 2, 3}`
- ▶ `mystack = pop[stackobj[mystack]]`
  - ▶ `{1, 2}`

*In web forums, highly experienced Mathematica programmers suggest that inheritance, etc. is possible; no examples found*



# Object-Oriented Mathematica: Ongoing Enhancement Efforts



# Mathematica: Ongoing Enhancement Efforts

Maeder (1993), Leibbrandt (2008) packages  
most significant

- ▶ Sanctioned in some way by Wolfram
- ▶ Available as add-ons

Other packages offered by Mathematica enthusiasts

- ▶ Some Q&A in user community
- ▶ Varying levels of capability, documentation

Syntactic differences (as would be expected)

- ▶ Maeder `translateBy[sphere1, {1, 0, 0}]`
- ▶ Vantzios `sphere1::translateBy[{1, 0, 0}]`
- ▶ Leibbrandt `sphere1.translateBy[{1, 0, 0}]`





# Mathematica: Maeder's Classes.m

## First serious effort at an add-on

- ▶ Originally promising, but...
  - ▶ Weakly documented
  - ▶ Support later withdrawn

“Mathematica will surely become the prototyping tool par excellence for object oriented programming.”

— *Mastering Mathematica*

## Sample code

- ▶ `Class[ Account, Object,  
    {bal, own},  
    {  
        {new, (new[super]; bal = #1; own = #2)&},  
        {balance, bal&},  
        {deposit, Function[bal += #1]},  
        {withdraw, Function[bal -= #1]},  
        {owner, own&}  
    }  
]`



# Object-Oriented Mathematica: The Objectica Add-On

## What is it?

- ▶ Package for adding object-oriented features to Mathematica
  - ▶ Sales literature emphasizes “abstract data types, inheritance, encapsulation, and polymorphism”
- ▶ Developed by Stephan Leibbrandt in 2008; sold by Symbols and Numbers (Germany)

## Who uses it?

- ▶ Size of user population unclear

### “Typical Users

- Software engineers of other object oriented languages for building prototypes
- Developers of big Mathematica projects in order to structure the problem
- Engineers to image real objects”



# Object-Oriented Mathematica: The Objectica Add-On

## Abstract data types

- ▶ Confusing terminology here; should really say **abstract base classes**, which are appropriately implemented

## Inheritance and polymorphism

- ▶ Clear syntax, handled well

## Encapsulation

- ▶ Some difficulties here with respect to class / subclass relationships (see *Virtual* later on)
- ▶ Can hide data and methods with *Private* option



# Object-Oriented Mathematica: The Objectica Add-On

## More features

- ▶ Interfaces
  - ▶ Very much like abstract classes
  - ▶ Classes can use multiple interfaces, but can have only one parent class
- ▶ Anonymous classes
  - ▶ Available, but poorly documented

## Overall assessment

- ▶ Implements object orientation well, aside from encapsulation (open to programmer error)
- ▶ Well-documented, for the most part



# Object-Oriented Mathematica: The Objectica Add-On

Comparison with other languages				
	<i>Ruby</i>	<i>Java</i>	<i>Python</i>	<i>Objectica</i>
<i>Typing</i>	Dynamic	Static	Dynamic	Dynamic
<i>Inheritance</i>	Single with mixins	Single with interfaces	Multiple	Single with interfaces
<i>Method overloading</i>	No	Yes	No	Yes
<i>Class vars &amp; methods</i>	Yes	Yes	No	Yes



# Object-Oriented Mathematica: Objectica Usage Basics

## Package loading options

- ▶ `Needs["Class`Class`"]`
- ▶ `Get["Class`Class`"]`
- ▶ `Get["<path to Class.m>"]`
- ▶ Note use of Class continues (name originated by Roman Maeder)



# Mathematica: Objectica Usage Basics

## Class definition

- ▶ `Class[base] := {  
    Virtual.st = 0,  
    f[x_?Positive] := x^2,  
    f[x_?Negative] := x^3 + st  
}`
- ▶ Default constructor created
- ▶ Use *Virtual* to ensure that child's instance of `st` is accessed when `f[x_?Negative]` is called
  - ▶ Good practice in case children come later

## Subclass definition

- ▶ `Class[child, base] := {  
    st = 20,  
    f[x_?Positive] := x,  
    g[y_] := Sin[y]  
}`
- ▶ More explicit version using *Override*  
`Class[child, base] := {  
    Override.Virtual.st = 20,  
    Override.f[x_?Positive] := x,  
    g[y_] := Sin[y]  
}`





# Object-Oriented Mathematica: Objectica Usage Basics

## Creating a new object

- ▶ `baseObj = New.base[]`
  - ▶ Note use of dot notation
- ▶ Alternatively, `New[baseObj].base[]`

## Redefining class to include constructor

- ▶ `Class[base] := {  
 Virtual.st = 0,  
 base[st_] := (This.st = st),  
 f[x_?Positive] := x^2,  
 f[x_?Negative] := x^3 + st  
}`
- ▶ `baseObj1 = New.base[10]`



# Object-Oriented Mathematica: Objectica Usage Basics

## Setting an instance variable

- ▶ `baseObj.st = 100;`

## Calling a method

- ▶ `baseObj.f[-5]`



# Mathematica: Objectica Usage Basics

## Abstract class with polymorphism

- ▶ Class definition (note use of *Abstract*)
  - ▶ `Class[Room] := { ... }`
  - ▶ `SetAttributes[Class[Room], Abstract]`
- ▶ Alternatively, define in one step
  - ▶ `Abstract.Class[Room] := { ... }`
- ▶ Subclass definitions (note use of *Super*)
  - ▶ `Class[Single, Room] := { Single[person_String] := Super[person] }`  
`Class[Double, Room] := { ... }`
- ▶ Calling `Price` method polymorphically
  - ▶ `New.Single["Mr. Smith"].Price[]`  
`New.Double["Mr. Smith", "Mrs. Smith"].Price[]`



# Object-Oriented Mathematica: Objectica Usage Basics

## Class with interfaces

- ▶ `Interface[one] := {f[x_] := 0};`  
`Interface[two] := {g[x_] := 0};`  
`Class[base2] := {h[x_] := x^2};`

```
Class[child, base2, one, two] := {  
  Override.f[x_] := x^3,  
  Override.g[x_] := x^4  
};
```

## An anonymous class

- ▶ `New.base[].{ ... }`



# Object-Oriented Mathematica: Objectica Usage Basics

## Public, protected, and private data

- ▶ `Class[base3] := {`  
    `e[x_] := x^2,`  
    `Public.f[x_] := x^3,`      ← *any caller*  
    `Protected.g[x_] := x^4,`      ← *child classes*  
    `Private.h[x_] := x^5`      ← *this class*  
};
- ▶ Alternatively, `SetAttributes[Room.Persons, Private]`

## Class method definition and call

- ▶ `Class[base4] := {`  
    `Static.z[x_] := x^2`  
}
- ▶ `base4.z[5]`



# Object-Oriented Mathematica: Objectica Usage Example

## Stack example, revisited

- ▶ `Class[stackobj2] := {  
 Virtual.stack = {},  
 stackobj2[stack_] := (This.stack = stack),  
 push[item_] := stack = Append[stack, item],  
 pop[] := stack = Most[stack]  
}`
- ▶ `mystack2 = New.stackobj2[{1, 2, 3}]`
- ▶ `mystack2.push[myitem]`
  - ▶ `{1, 2, 3, 4}`
- ▶ `mystack2.pop[]`
  - ▶ `{1, 2, 3}`



# Object-Oriented Mathematica: Discussion

## Why so slow to emerge?

- ▶ Existing programming paradigms are powerful
- ▶ Mathematica programmers know workarounds
  - ▶ Rules, patterns
  - ▶ Interfaces to OO languages such as Java, C++
- ▶ Big shift in thinking required (and not desired)
  - ▶ E.g., pushback on new OO graph features in Mathematica 8

## Best uses

- ▶ Large applications with significant complexity
- ▶ Real-world applications with hierarchical structure
- ▶ User interface programming
- ▶ Situations where encapsulation would be helpful





# Resources for Further Exploration

## Mathematica's built-in object-oriented abilities

- ▶ *The Mathematica Book, First Edition*  
section on “object-oriented” programming  
<http://reference.wolfram.com/legacy/v1/contents/4.1.6.pdf>
- ▶ Online documentation of “object-oriented” functionality
  - ▶ UpSet  
<http://reference.wolfram.com/mathematica/ref/UpSet.html>
  - ▶ TagSet  
<http://reference.wolfram.com/mathematica/ref/TagSet.html>

## Mathematica linking to object-oriented languages

- ▶ <http://reference.wolfram.com/mathematica/guide/SystemsInterfacesAndDeploymentOverview.html>
- ▶ <http://reference.wolfram.com/mathematica/tutorial/MathLinkAndExternalProgramCommunicationOverview.html>



# Resources for Further Exploration

## Objectica product pages

- ▶ Wolfram  
<http://www.wolfram.com/products/applications/objectica/>
- ▶ Symbols and Numbers  
<http://www.objectica.net/>

## Objectica presentations

- ▶ *From Symbols to Objects*  
2010 Wolfram Technology Conference  
<http://library.wolfram.com/infocenter/Conferences/7871/>
- ▶ *Object-Oriented Modeling with Objectica*  
2007 Wolfram Technology Conference  
<http://library.wolfram.com/infocenter/Conferences/6923/>



# Resources for Further Exploration

## Other efforts

- ▶ Roman Maeder's Classes.m package
  - ▶ *The Mathematica Journal* 3:1, pp. 23-31 (1993)  
<http://library.wolfram.com/infocenter/Articles/3243/>
  - ▶ Gray, J., *Mastering Mathematica*, chapter 9
  - ▶ Maeder, R. *The Mathematica Programmer*, chapter 4  
<http://www.mathconsult.ch/showroom/pubs/MathProg/htmls/1-04.htm>
- ▶ Hermann Schmitt's OO System for Mathematica
  - ▶ An OO System for Mathematica, Version 3  
[http://www.schmittther.de/oosys\\_en/introduction.html](http://www.schmittther.de/oosys_en/introduction.html)



# Resources for Further Exploration

## Other efforts

- ▶ Orestis Vantzios' OOP package
  - ▶ *From Symbols to Objects*  
2005 Wolfram Technology Conference  
<http://library.wolfram.com/infocenter/Conferences/5773/>
- ▶ Ross Tang's MathOO package
  - ▶ Code repository  
<http://code.google.com/p/mathoo-packages/>
  - ▶ Additional documentation  
<http://www.voofie.com/concept/MathOO/>
    - ▶ Read in date order, not display order



# Credits

## Slide 4 pictures

- ▶ <http://demonstrations.wolfram.com/NegligibleSenescenceScenario/>
- ▶ <http://demonstrations.wolfram.com/SegmentingAMedicalImage/>
- ▶ <http://demonstrations.wolfram.com/RecursiveExercisesIIIFirePatterns/>
- ▶ <http://demonstrations.wolfram.com/TunedMassDamper/>
- ▶ <http://demonstrations.wolfram.com/SurfacesAndGradients/>

## Slide 6 notebook

- ▶ <http://www.math.umd.edu/undergraduate/schol/primer/Notebooks/pendulum.nb>

## Slide 7 pictures

- ▶ Mathematica documentation



# Credits

## Slide 9 picture

- ▶ <http://reference.wolfram.com/mathematica/tutorial/SymbolicComputation.html>

## Slide 12 pictures

- ▶ Mathematica documentation

## Slide 16 code

- ▶ <http://library.wolfram.com/infocenter/Articles/3243/>

## Slide 20 table

- ▶ [http://www.schmitther.de/oosys\\_en/comp\\_tab.html](http://www.schmitther.de/oosys_en/comp_tab.html)

## Slides 20–27 code

- ▶ Objectica documentation

