



Department of Computer Science  
UNIVERSITY OF COLORADO **BOULDER**



## Hidden Markov Models

Natural Language Processing: Jordan  
Boyd-Graber  
University of Colorado Boulder  
LECTURE 20

## Structured Prediction

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- Thus far, we've assumed observations are iid (or at least exchangeable)
- Common tool in computational biology, information extraction, sequence modeling
- I'm going to use linguistic examples (more in 5832), but think about how it might be used in your favorite applications

## Content Questions

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## Administrivia

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- Have everything for last HW
- EC added by special request
- Keep working on projects

## In class ...

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- Finding most likely sequence
- Garden pathing
  - The prime number few
  - The cotton clothing is made of grows in Mississippi

## In class ...

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- Finding most likely sequence
- Garden pathing
  - The prime number few
  - The cotton clothing is made of grows in Mississippi
  - We'll see how Viterbi decoding can be confused by (and solve) part of speech ambiguities

## Model parameters

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$$\pi = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.3 \\ 0.1 \end{bmatrix} \quad \theta = \begin{matrix} & \text{Det} & \text{Adj} & \text{N} & \text{V} \\ \text{Det} & \begin{pmatrix} 0.1 & 0.4 & 0.45 & 0.05 \end{pmatrix} \\ \text{Adj} & \begin{pmatrix} 0.1 & 0.3 & 0.5 & 0.1 \end{pmatrix} \\ \text{N} & \begin{pmatrix} 0.05 & 0.05 & 0.1 & 0.8 \end{pmatrix} \\ \text{V} & \begin{pmatrix} 0.3 & 0.2 & 0.3 & 0.2 \end{pmatrix} \end{matrix} \quad (1)$$

$$\beta = \begin{matrix} & \text{the} & \text{old} & \text{man} & \text{blue} & \text{boat} & \text{a} & \text{an} \\ \text{Det} & \begin{pmatrix} 0.6 & 0.025 & 0.025 & 0.025 & 0.025 & 0.2 & 0.1 \end{pmatrix} \\ \text{Adj} & \begin{pmatrix} 0.033 & 0.3 & 0.1 & 0.3 & 0.1 & 0.033 & 0.033 \end{pmatrix} \\ \text{N} & \begin{pmatrix} 0.033 & 0.1 & 0.4 & 0.1 & 0.3 & 0.033 & 0.033 \end{pmatrix} \\ \text{V} & \begin{pmatrix} 0.033 & 0.1 & 0.4 & 0.2 & 0.2 & 0.033 & 0.033 \end{pmatrix} \end{matrix} \quad (2)$$

## In class . . .

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What is the probability of the sequence “a/Det blue/Adj boat/N”?

(3)

(4)

(5)

(6)

## In class . . .

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What is the probability of the sequence “a/Det blue/Adj boat/N”?

$$\log(\pi_d \beta_{d,a} \theta_{d,a} \beta_{a,blue} \theta_{a,n} \beta_{n,boat}) = \quad (3)$$

(4)

(5)

(6)

## In class . . .

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What is the probability of the sequence “a/Det blue/Adj boat/N”?

$$\log(\pi_d \beta_{d,a} \theta_{d,a} \beta_{a,blue} \theta_{a,n} \beta_{n,boat}) = \quad (3)$$

$$\log(0.3 * 0.6 * 0.4 * 0.3 * 0.5 * 0.1) = \log(0.00108) \quad (4)$$

(5)

(6)

## In class . . .

---

What is the probability of the sequence “a/Det blue/Adj boat/N”?

$$\log(\pi_d \beta_{d,a} \theta_{d,a} \beta_{a,blue} \theta_{a,n} \beta_{n,boat}) = \quad (3)$$

$$\log(0.3 * 0.6 * 0.4 * 0.3 * 0.5 * 0.1) = \log(0.00108) \quad (4)$$

$$-.5 + -.7 + -.4 + -.5 + -0.3 + -0.5 = -3.0 \quad (5)$$

(6)

## Decoding Sentence 2

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- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{array}{c} \\ \\ \\ \end{array} \right) \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\pi_{\text{START}, A} + \beta_{A, \text{the}} = -0.52 + -1.48 = -2.00$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ & & \\ & & \end{matrix} \right) \end{matrix} \quad (7)$$

## Decoding Sentence 2

---

$$\pi_{\text{START}, V} + \beta_{V, \text{the}} = -1.00 + -1.48 = -2.48$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ -2.48 \\ \end{matrix} \right) \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\pi_{\text{START}, D} + \beta_{D, \text{the}} = -0.52 + -0.22 = -0.74$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ -2.48 \\ -0.74 \\ \end{matrix} \right) \end{matrix} \quad (7)$$

## Decoding Sentence 2

---

$$\pi_{\text{START}, N} + \beta_{N, \text{the}} = -0.52 + -1.48 = -2.00$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ -2.48 \\ -0.74 \\ \textcolor{red}{-2.00} \end{matrix} \right) \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\delta_0(D) + \theta_{D, A} + \beta_{A, \text{old}} = -0.74 + -0.40 + -0.52 = \textcolor{red}{-1.67}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ -2.48 \\ -0.74 \\ -2.00 \end{matrix} \right) \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\delta_0(D) + \theta_D, V + \beta_{V, \text{old}} = -0.74 + -1.30 + -1.00 = \textcolor{red}{-3.05}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & \\ V & -2.48 & \textcolor{red}{-3.05} & \\ D & -0.74 & & \\ N & -2.00 & & \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\delta_0(D) + \theta_{D, \text{old}} + \beta_{D, \text{old}} = -0.74 + -1.00 + -1.60 = -3.35$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & \\ V & -2.48 & -3.05 & \\ D & -0.74 & -3.35 & \\ N & -2.00 & & \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\delta_0(D) + \theta_{D, N} + \beta_{N, \text{old}} = -0.74 + -0.35 + -1.00 = \textcolor{red}{-2.09}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & \\ V & -2.48 & -3.05 & \\ D & -0.74 & -3.35 & \\ N & -2.00 & \textcolor{red}{-2.09} & \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\delta_1(A) + \theta_{A, A} + \beta_{A, \text{man}} = -1.67 + -0.52 + -1.00 = \textcolor{red}{-3.19}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ -2.48 \\ -0.74 \\ -2.00 \end{matrix} \middle| \begin{matrix} -1.67 \\ -3.05 \\ -3.35 \\ -2.09 \end{matrix} \right) \end{matrix} \quad (7)$$

## Decoding Sentence 2

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$$\delta_1(N) + \theta_{N, V} + \beta_{V, \text{man}} = -2.09 + -0.10 + -0.40 = -2.59$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & -3.19 \\ V & -2.48 & -3.05 & -2.59 \\ D & -0.74 & -3.35 & \\ N & -2.00 & -2.09 & \end{matrix} \quad (7)$$

## Decoding Sentence 2

---

$$\delta_1(A) + \theta_{A, D} + \beta_{D, \text{man}} = -1.67 + -1.00 + -1.60 = -4.27$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & -3.19 \\ V & -2.48 & -3.05 & -2.59 \\ D & -0.74 & -3.35 & -4.27 \\ N & -2.00 & -2.09 & \end{matrix} \quad (7)$$

## Decoding Sentence 2

---

$$\delta_1(A) + \theta_{A, N} + \beta_{N, \text{man}} = -1.67 + -0.30 + -0.40 = -2.36$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & -3.19 \\ V & -2.48 & -3.05 & -2.59 \\ D & -0.74 & -3.35 & -4.27 \\ N & -2.00 & -2.09 & -2.36 \end{matrix} \quad (7)$$

## Decoding Sentence 2

---

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & -3.19 \\ V & -2.48 & -3.05 & -2.59 \\ D & -0.74 & -3.35 & -4.27 \\ N & -2.00 & -2.09 & -2.36 \end{matrix} \quad (7)$$

- Backpointers

$$\tau = \begin{matrix} & \text{old}_1 & \text{man}_2 \\ A & D & A \\ V & D & N \\ D & D & A \\ N & D & A \end{matrix} \quad (8)$$

## Decoding Sentence 2

---

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & -3.19 \\ V & -2.48 & -3.05 & -2.59 \\ D & -0.74 & -3.35 & -4.27 \\ N & -2.00 & -2.09 & -2.36 \end{matrix} \quad (7)$$

- Backpointers

$$\tau = \begin{matrix} & \text{old}_1 & \text{man}_2 \\ A & \textcolor{red}{D} & A \\ V & D & N \\ D & D & A \\ N & D & \textcolor{red}{A} \end{matrix} \quad (8)$$

## Decoding Sentence 2

---

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 \\ A & -2.00 & -1.67 & -3.19 \\ V & -2.48 & -3.05 & -2.59 \\ D & -0.74 & -3.35 & -4.27 \\ N & -2.00 & -2.09 & -2.36 \end{matrix} \quad (7)$$

- Backpointers

$$\tau = \begin{matrix} & \text{old}_1 & \text{man}_2 \\ A & D & A \\ V & D & N \\ D & D & A \\ N & D & A \end{matrix} \quad (8)$$

- Reconstruction: D A N

## Decoding Sentence 3

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- Scores

$$\delta = \begin{matrix} A \\ V \\ D \\ N \end{matrix} \left( \begin{array}{c} \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \end{array} \right) \quad (9)$$

## Decoding Sentence 3

---

$$\pi_{\text{START}, A} + \beta_{A, \text{the}} = -0.52 + -1.48 = -2.00$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ \end{matrix} \right) \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\pi_{\text{START}, V} + \beta_{V, \text{the}} = -1.00 + -1.48 = -2.48$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ -2.48 \\ \vdots \\ \vdots \end{matrix} \right) \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\pi_{\text{START}, D} + \beta_{D, \text{the}} = -0.52 + -0.22 = \textcolor{red}{-0.74}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 \\ -2.48 \\ \textcolor{red}{-0.74} \end{matrix} \right) \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\pi_{\text{START}}, N + \beta_{N, \text{the}} = -0.52 + -1.48 = \textcolor{red}{-2.00}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 \\ V & -2.48 \\ D & -0.74 \\ N & \textcolor{red}{-2.00} \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_0(D) + \theta_{D, A} + \beta_{A, \text{old}} = -0.74 + -0.40 + -0.52 = \textcolor{red}{-1.67}$$

- Scores

$$\delta = \begin{pmatrix} A & \text{the}_0 \\ V & \text{old}_1 \\ D & \text{man}_2 \\ N & \text{the}_3 \\ & \text{boat}_4 \end{pmatrix} \quad (9)$$

the<sub>0</sub>      old<sub>1</sub>      man<sub>2</sub>      the<sub>3</sub>      boat<sub>4</sub>

- 2.00      **- 1.67**  
- 2.48  
- 0.74  
- 2.00

## Decoding Sentence 3

---

$$\delta_0(D) + \theta_{D, V} + \beta_{V, \text{old}} = -0.74 + -1.30 + -1.00 = \textcolor{red}{-3.05}$$

- Scores

$$\delta = \begin{pmatrix} A & \text{the}_0 \\ V & \text{old}_1 \\ D & \text{man}_2 \\ N & \text{the}_3 \\ & \text{boat}_4 \end{pmatrix} \quad (9)$$

	the <sub>0</sub>	old <sub>1</sub>	man <sub>2</sub>	the <sub>3</sub>	boat <sub>4</sub>
A	-2.00	-1.67			
V	-2.48				
D	-0.74				
N	-2.00				

## Decoding Sentence 3

---

$$\delta_0(D) + \theta_{D, D} + \beta_{D, \text{old}} = -0.74 + -1.00 + -1.60 = \textcolor{red}{-3.35}$$

- Scores

$$\delta = \begin{pmatrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & & & \\ V & -2.48 & -3.05 & & & \\ D & -0.74 & \textcolor{red}{-3.35} & & & \\ N & -2.00 & & & & \end{pmatrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_0(D) + \theta_{D, N} + \beta_{N, \text{old}} = -0.74 + -0.35 + -1.00 = \textcolor{red}{-2.09}$$

- Scores

$$\delta = \begin{pmatrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & & & \\ V & -2.48 & -3.05 & & & \\ D & -0.74 & -3.35 & & & \\ N & -2.00 & \textcolor{red}{-2.09} & & & \end{pmatrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_1(A) + \theta_{A, A} + \beta_{A, \text{man}} = -1.67 + -0.52 + -1.00 = \textcolor{red}{-3.19}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & \textcolor{red}{-3.19} & & \\ V & -2.48 & -3.05 & & & \\ D & -0.74 & -3.35 & & & \\ N & -2.00 & -2.09 & & & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_1(N) + \theta_{N,V} + \beta_{V, \text{ man}} = -2.09 + -0.10 + -0.40 = \textcolor{red}{-2.59}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & & \\ V & -2.48 & -3.05 & \textcolor{red}{-2.59} & & \\ D & -0.74 & -3.35 & & & \\ N & -2.00 & -2.09 & & & \end{matrix} \quad (9)$$

## Decoding Sentence 3

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$$\delta_1(A) + \theta_{A, D} + \beta_{D, \text{man}} = -1.67 + -1.00 + -1.60 = \textcolor{red}{-4.27}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & & \\ V & -2.48 & -3.05 & -2.59 & & \\ D & -0.74 & -3.35 & \textcolor{red}{-4.27} & & \\ N & -2.00 & -2.09 & & & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_1(A) + \theta_{A, N} + \beta_{N, \text{man}} = -1.67 + -0.30 + -0.40 = \textcolor{red}{-2.36}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & & \\ V & -2.48 & -3.05 & -2.59 & & \\ D & -0.74 & -3.35 & -4.27 & & \\ N & -2.00 & -2.09 & \textcolor{red}{-2.36} & & \end{matrix} \quad (9)$$

## Decoding Sentence 3

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$$\delta_2(V) + \theta_{V, A} + \beta_{A, \text{the}} = -2.59 + -0.70 + -1.48 = \color{red}{-4.77}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ \begin{matrix} A \\ V \\ D \\ N \end{matrix} & \left( \begin{matrix} -2.00 & -1.67 & -3.19 & \color{red}{-4.77} \\ -2.48 & -3.05 & -2.59 & \\ -0.74 & -3.35 & -4.27 & \\ -2.00 & -2.09 & -2.36 & \end{matrix} \right) \end{matrix} \quad (9)$$

## Decoding Sentence 3

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$$\delta_2(N) + \theta_{N, V} + \beta_{V, \text{the}} = -2.36 + -0.10 + -1.48 = \textcolor{red}{-3.94}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & \\ V & -2.48 & -3.05 & -2.59 & \textcolor{red}{-3.94} & \\ D & -0.74 & -3.35 & -4.27 & & \\ N & -2.00 & -2.09 & -2.36 & & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_2(V) + \theta_{V, D} + \beta_{D, \text{the}} = -2.59 + -0.52 + -0.22 = \textcolor{red}{-3.33}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & \\ V & -2.48 & -3.05 & -2.59 & -3.94 & \\ D & -0.74 & -3.35 & -4.27 & \textcolor{red}{-3.33} & \\ N & -2.00 & -2.09 & -2.36 & & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_2(V) + \theta_{V, N} + \beta_{N, \text{the}} = -2.59 + -0.52 + -1.48 = \textcolor{red}{-4.59}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & \\ V & -2.48 & -3.05 & -2.59 & -3.94 & \\ D & -0.74 & -3.35 & -4.27 & -3.33 & \\ N & -2.00 & -2.09 & -2.36 & \textcolor{red}{-4.59} & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_3(D) + \theta_{D, A} + \beta_{A, \text{boat}} = -3.33 + -0.40 + -1.00 = \textcolor{red}{-4.73}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & \textcolor{red}{-4.73} \\ V & -2.48 & -3.05 & -2.59 & -3.94 & \\ D & -0.74 & -3.35 & -4.27 & -3.33 & \\ N & -2.00 & -2.09 & -2.36 & -4.59 & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_3(D) + \theta_{D, V} + \beta_{V, \text{boat}} = -3.33 + -1.30 + -0.70 = \textcolor{red}{-5.33}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & -4.73 \\ V & -2.48 & -3.05 & -2.59 & -3.94 & \textcolor{red}{-5.33} \\ D & -0.74 & -3.35 & -4.27 & -3.33 & \\ N & -2.00 & -2.09 & -2.36 & -4.59 & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_3(D) + \theta_{D, D} + \beta_{D, \text{boat}} = -3.33 + -1.00 + -1.60 = \textcolor{red}{-5.93}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & -4.73 \\ V & -2.48 & -3.05 & -2.59 & -3.94 & -5.33 \\ D & -0.74 & -3.35 & -4.27 & -3.33 & \textcolor{red}{-5.93} \\ N & -2.00 & -2.09 & -2.36 & -4.59 & \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

$$\delta_3(D) + \theta_{D, N} + \beta_{N, \text{boat}} = -3.33 + -0.35 + -0.52 = \textcolor{red}{-4.20}$$

- Scores

$$\delta = \begin{matrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & -4.73 \\ V & -2.48 & -3.05 & -2.59 & -3.94 & -5.33 \\ D & -0.74 & -3.35 & -4.27 & -3.33 & -5.93 \\ N & -2.00 & -2.09 & -2.36 & -4.59 & \textcolor{red}{-4.20} \end{matrix} \quad (9)$$

## Decoding Sentence 3

---

- Scores

$$\delta = \begin{pmatrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & -4.73 \\ V & -2.48 & -3.05 & -2.59 & -3.94 & -5.33 \\ D & -0.74 & -3.35 & -4.27 & -3.33 & -5.93 \\ N & -2.00 & -2.09 & -2.36 & -4.59 & -4.20 \end{pmatrix} \quad (9)$$

- Backpointers

$$\tau = \begin{pmatrix} & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & D & A & V & D \\ V & D & N & N & D \\ D & D & A & V & D \\ N & D & A & V & D \end{pmatrix} \quad (10)$$

## Decoding Sentence 3

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- Scores

$$\delta = \begin{pmatrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & -4.73 \\ V & -2.48 & -3.05 & -2.59 & -3.94 & -5.33 \\ D & -0.74 & -3.35 & -4.27 & -3.33 & -5.93 \\ N & -2.00 & -2.09 & -2.36 & -4.59 & -4.20 \end{pmatrix} \quad (9)$$

- Backpointers

$$\tau = \begin{pmatrix} & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & D & A & V & D \\ V & D & \textcolor{red}{N} & N & D \\ D & D & A & \textcolor{red}{V} & D \\ N & \textcolor{red}{D} & A & V & \textcolor{red}{D} \end{pmatrix} \quad (10)$$

## Decoding Sentence 3

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- Scores

$$\delta = \begin{pmatrix} & \text{the}_0 & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & -2.00 & -1.67 & -3.19 & -4.77 & -4.73 \\ V & -2.48 & -3.05 & -2.59 & -3.94 & -5.33 \\ D & -0.74 & -3.35 & -4.27 & -3.33 & -5.93 \\ N & -2.00 & -2.09 & -2.36 & -4.59 & -4.20 \end{pmatrix} \quad (9)$$

- Backpointers

$$\tau = \begin{pmatrix} & \text{old}_1 & \text{man}_2 & \text{the}_3 & \text{boat}_4 \\ A & D & A & V & D \\ V & D & N & N & D \\ D & D & A & V & D \\ N & D & A & V & D \end{pmatrix} \quad (10)$$

- Reconstruction: D N V D N