Ranking

Jordan Boyd-Graber
University of Colorado Boulder
LECTURE 16

Roadmap

- Combining rankings: taking advantage of multiple weak rankers
- Maximum margin ranking: support vector machines
- Reduction to classification: optimizing

Ranking

- Web search (Google uses $>$ 200 features)
- Movie rankings
- Dating

Plan

An Efficient Boosting Algorithm for Combining Preferences

Freund, Iyer, Schapire, Singer. JMLR, 2003.

- Feedback function: $\Phi : \mathcal{X} \times \mathcal{X} \mapsto \mathbb{R}$
  - $\phi(x_0, x_1) > 0$: x_1 is preferred to x_0
  - $\phi(x_0, x_1) < 0$: x_0 is preferred to x_1
  - $\phi(x_0, x_1) == 0$: no preference
- Want to learn distribution $D(x_0, x_1) \equiv c \cdot \max\{0, \Phi(x_0, x_1)\}$ s.t.

$$\sum_{x,x'} D(x, x') = 1 \tag{1}$$

What's the goal?

- Minimize the number of misranked pairs under final ranking

$$\sum_{x,x'} D(x,x') \cdot \mathbb{1}\left[H(x') \leq H(x)\right] = \Pr_{(x,y)\sim D}[H(y) \leq H(x)] \quad (2)$$

- Choose entries with high weight in D to be *important* (can't get them wrong)

What's the input

- Weak rankings of the form $h_t : \mathcal{X} \mapsto \mathbb{R}$
- Could be different systems / users / feature sets
- Will combine them into a final ranking of the same form

What's a weak ranking?

- A function of the form

$$h(x) = \begin{cases} 1 & \text{if } f_i(x) > \theta \\ 0 & \text{if } f_i(x) \leq \theta \\ q_{\text{def}} & \text{if } f_i(x) == \perp \end{cases} \tag{3}$$

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- How to find q_{def} and θ?
- Binary search over how much it improves ranking implied by D (i.e., gets high D weights right)

Algorithm

- Initialize D_1
- For $t = 1 \dots T$:
  - Get weak ranking $h_t : \mathcal{X} \mapsto \mathbb{R}$
  - Choose α_t
  - Update distribution

$$D_{t+1}(x, y) \propto D_t(x, y) \cdot \exp\left\{\alpha_t \left[h_t(x) - h_t(y)\right]\right\} \tag{4}$$

  - Final ranking is

$$H(x) = \sum_{1}^{T} \alpha_t h_t(x) \tag{5}$$

Learning rate

- α_t encodes importance of individual weak learner
- In general decreases over iterations
- Find weighted discrepancy

$$r = \sum_{x,y} D(x,y)\left[h(y) - h(x)\right] \tag{6}$$

- Use $\alpha = \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right)$

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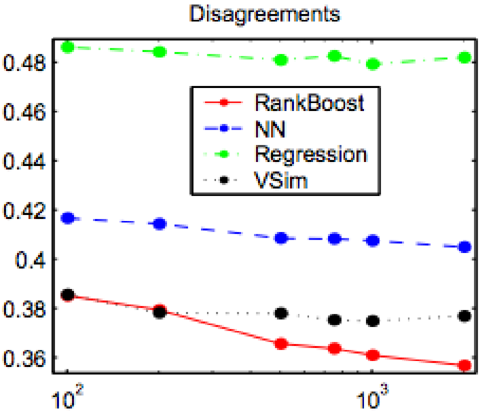
- As r gets smaller, weak learner t will have lower weight

Performance

- Works better than individual features or nearest neighbor



Plan

Examples as feature vectors

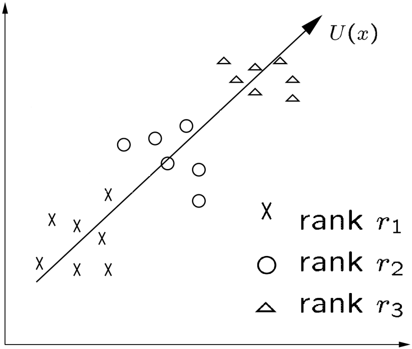
Every example has a feature vector $f(x)$

example	docID	query	cosine score	ω	judgment
Φ_1	37	linux operating system	0.032	3	*relevant*
Φ_2	37	penguin logo	0.02	4	*nonrelevant*
Φ_3	238	operating system	0.043	2	*relevant*
Φ_4	238	runtime environment	0.004	2	*nonrelevant*
Φ_5	1741	kernel layer	0.022	3	*relevant*
Φ_6	2094	device driver	0.03	2	*relevant*
Φ_7	3191	device driver	0.027	5	*nonrelevant*

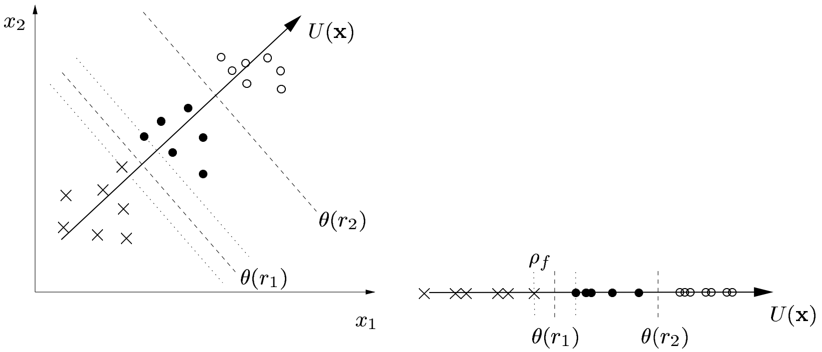
Turning features to rank

- Have a series of "levels" or ranks $y = 1 \ldots$
- We want to find a function to separate examples

$$f(x) \equiv \langle w \cdot \phi(x) \rangle \quad (7)$$



Maximizing the margin



Using SVM-light

- Each example has a rank
- and a query id
- and lots of features

Using SVM-light

```
# query 1
3 qid:1 1:1 2:1 3:0 4:0.2 5:0
2 qid:1 1:0 2:0 3:1 4:0.1 5:1
1 qid:1 1:0 2:1 3:0 4:0.4 5:0
1 qid:1 1:0 2:0 3:1 4:0.3 5:0
# query 2
1 qid:2 1:0 2:0 3:1 4:0.2 5:0
2 qid:2 1:1 2:0 3:1 4:0.4 5:0
1 qid:2 1:0 2:0 3:1 4:0.1 5:0
1 qid:2 1:0 2:0 3:1 4:0.2 5:0
# query 3
2 qid:3 1:0 2:0 3:1 4:0.1 5:1
3 qid:3 1:1 2:1 3:0 4:0.3 5:0
4 qid:3 1:1 2:0 3:0 4:0.4 5:1
1 qid:3 1:0 2:1 3:1 4:0.5 5:0
```

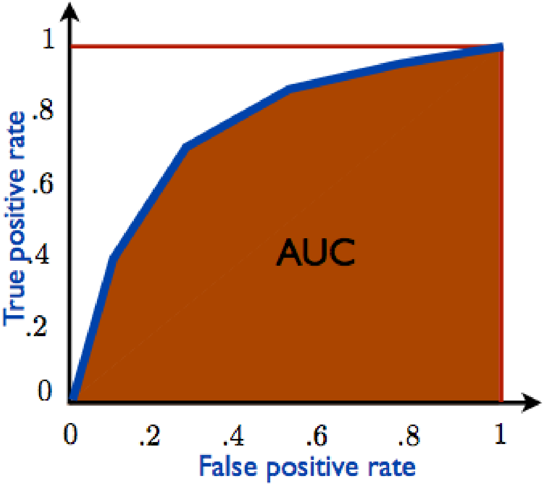
Plan

Are all pairs important?

- Often we care about the *top* of the result list
- Regression (as in previous section) not robust when there's one right answer and many wrong ones
- Measured by the AUC: area under the curve
  - Imagine two classes: winners and losers
  - We want there to be a consecutive run of winners before losers in the results (extends to greater number of classes)
  - Want to minimize probability of losers before winners in an ordering π on a set of examples $S = (x_1, y_1) \dots$

$$l(\pi, S) = \frac{\sum_{i \neq j} \mathbb{1}\left[y_i > y_j\right] \pi(x_i, x_j)}{\sum_{i < j} \mathbb{1}\left[y_i \neq y_j\right]} \tag{8}$$

roc curve



Reduction to Classification

Robust Reductions from Ranking to Classification

Maria-Florina Blcan, Nikhil Bansal, Alina Beygelzimer, Don Coppersmith, John Langford, Gregory B. Sorkin. JMLR, 2008.

- Produces a ranking using a classifier
- If regret of classifier is r, loss of classifier is at most $2r$
- Thus, if binary error rate is 20% due to inherent noise and 5% due to errors made by the classifier
- Then AUC regret is at most 10%

Algorithm

- Learn a classifier
  - Given a random pair of examples, learn a classifier c to predict whether it should prefer x_1 to x_2
  - Return the classifier c
- Get a ranking from the resulting classifier tournament
  - For an example x, define the degree

$$\deg(x) = |\{x' : c(x, x') == 1, x' \in U| \tag{9}$$

  - Sort by the degree of the node (number of matches it won)

Efficiency

- For ranking a large list, complexity $O(n^2)$ is unacceptable
- Possible to use variant of QuickSort $O(n \log n)$
- Has the same regret performance, but is randomized

Training Preference Classifier

- How do you balance positive and negative classes?
- Requires cross-validation: try many options on held out data
- Weighting positive classes is important:
  - Some frameworks allow you to weight examples
  - In other cases, you can just duplicate positive

Recap

- Ranking is an important problem
- Multiple approaches
  - Combining weak rankers
  - Max-margin
  - Tournament classification