Maximum Likelihood Estimation

Introduction to Data Science Algorithms

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Getting Started: Poisson

- Recall the density function

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{1}$$

- Taking the log makes math easier, doesn't change answer (monotonic)
- If we observe $x_1 \dots x_N$, then log likelihood is

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Poisson MLE Parameter

$$\ell = (\log \lambda)\left(\sum_i x_i\right) - N\lambda - \sum_i \log x_i! \tag{3}$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{\sum_i x_i}{\lambda} + -N + 0 \tag{4}$$

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Solve for λ:

$$0 = \frac{\sum_i x_i}{\lambda} - N \tag{5}$$

$$N = \frac{\sum_i x_i}{\lambda} \tag{6}$$

$$\lambda = \frac{\sum_i x_i}{N} \tag{7}$$

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Which makes sense!