



# Constituency Parsing

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INTRO / CHART PARSING

## A More Grounded Syntax Theory

- A central question in linguistics is **how do we know when a sentence is grammatical?**
- Chomsky's generative grammars attempted to mathematically formalize this question
- Linguistic phrases contained a universal, hierarchical structure formalized as parse trees

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- A central question in linguistics is **how do we know when a sentence is grammatical?**
- Chomsky's generative grammars attempted to mathematically formalize this question
- Linguistic phrases contained a universal, hierarchical structure formalized as parse trees
- Today
  - A formalization
  - Foundation of all computational syntax
  - Learnable from data

## Context Free Grammars

### Definition

- $N$ : finite set of **non-terminal** symbols
- $\Sigma$ : finite set of terminal symbols
- $R$ : productions of the form  $X \rightarrow Y_1 \dots Y_n$ , where  $X \in N$ ,  $Y \in (N \cup \Sigma)$
- $S$ : a start symbol within  $N$

Examples of non-terminals:

- np for “noun phrase”
- vp for “verb phrase”
- Often correspond to multiword syntactic abstractions

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Examples of terminals:

- *“dog”*
- *“play”*
- *“the”*

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Examples of productions:

- $n \rightarrow \text{"dog"}$
- $np \rightarrow n$
- $np \rightarrow \text{adj } n$

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- $S$ : a **start symbol** within  $N$

In NLP applications, by convention we use  $S$  as the start symbol

## Flexibility of CFG Productions

- Unary rules:  $nn \rightarrow \text{"man"}$
- Mixing terminals and nonterminals on RHS:
  - $np \rightarrow \text{"Congress"} \setminus \text{t } \text{"the"} \text{"pooch"}$
  - $np \rightarrow \text{"the"} nn$
- Empty terminals
  - $np \rightarrow \epsilon$
  - $adj \rightarrow \epsilon$



## Derivations

- A derivation is a sequence of strings  $s_1 \dots s_T$  where
- $s_1 \equiv S$ , the start symbol
- $s_T \in \Sigma^*$ : i.e., the final string is only terminals
- $s_i, \forall i > 1$ , is derived from  $s_{i-1}$  by replacing some non-terminal  $X$  in  $s_{i-1}$  and replacing it by some  $\beta$ , where  $x \rightarrow \beta \in R$ .

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- Example: parse tree

## Example Derivation

### Productions

$s \rightarrow np \ vp$	$np \rightarrow Det \ nn$	$vp \rightarrow vz$
$vp \rightarrow AdvP \ vz$	$np \rightarrow AdjP \ nn$	$np \rightarrow pro$
$Det \rightarrow "the"$	$Det \rightarrow "a"$	$Det \rightarrow "an"$
$nn \rightarrow "dot"$	$nn \rightarrow "cat"$	$nn \rightarrow "mouse"$
$vz \rightarrow "barked"$	$vz \rightarrow "ran"$	$vz \rightarrow "sat"$
$\vdots$	$\vdots$	$\vdots$

$s_1 =$

S

## Example Derivation

### Productions

$s \rightarrow np \ vp$

$vp \rightarrow AdvP \ vz$

$Det \rightarrow \text{"the"}$

$nn \rightarrow \text{"dot"}$

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$\vdots$

$np \rightarrow Det \ nn$

$np \rightarrow AdjP \ nn$

$Det \rightarrow \text{"a"}$

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$vp \rightarrow vz$

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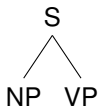
$Det \rightarrow \text{"an"}$

$nn \rightarrow \text{"mouse"}$

$vz \rightarrow \text{"sat"}$

$\vdots$

$S_2 =$



## Productions

s  $\rightarrow$  np vpvp  $\rightarrow$  AdvP vzDet  $\rightarrow$  "the"nn  $\rightarrow$  "dot"vz  $\rightarrow$  "barked"

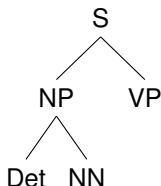
⋮

np  $\rightarrow$  Det nnnp  $\rightarrow$  AdjP nnDet  $\rightarrow$  "a"nn  $\rightarrow$  "cat"vz  $\rightarrow$  "ran"

⋮

vp  $\rightarrow$  vznp  $\rightarrow$  proDet  $\rightarrow$  "an"nn  $\rightarrow$  "mouse"vz  $\rightarrow$  "sat"

⋮

s<sub>3</sub> =

## Productions

s → np vp

vp → AdvP vz

Det → "the"

nn → "dot"

vz → "barked"

⋮

np → Det nn

np → AdjP nn

Det → "a"

nn → "cat"

vz → "ran"

⋮

vp → vz

np → pro

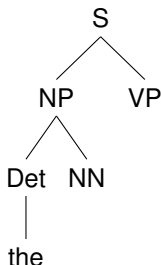
Det → "an"

nn → "mouse"

vz → "sat"

⋮

$S_4 =$



## Productions

s → np vp

vp → AdvP vz

Det → "the"

nn → "dot"

vz → "barked"

⋮

np → Det nn

np → AdjP nn

Det → "a"

nn → "cat"

vz → "ran"

⋮

vp → vz

np → pro

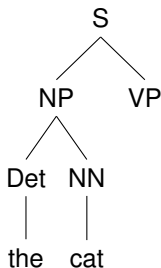
Det → "an"

nn → "mouse"

vz → "sat"

⋮

S<sub>5</sub> =



## Productions

s → np vp

vp → AdvP vz

Det → "the"

nn → "dot"

vz → "barked"

⋮

np → Det nn

np → AdjP nn

Det → "a"

nn → "cat"

vz → "ran"

⋮

vp → vz

np → pro

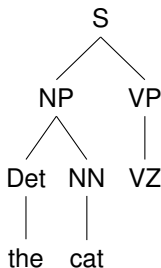
Det → "an"

nn → "mouse"

vz → "sat"

⋮

$S_6 =$





## Productions

s → np vp

vp → AdvP vz

Det → "the"

nn → "dot"

vz → "barked"

⋮

np → Det nn

np → AdjP nn

Det → "a"

nn → "cat"

vz → "ran"

⋮

vp → vz

np → pro

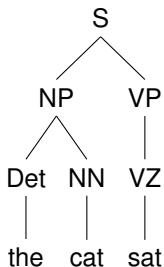
Det → "an"

nn → "mouse"

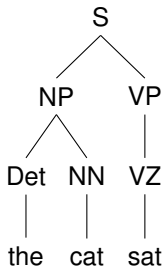
vz → "sat"

⋮

$S_7 =$



## Example Derivation



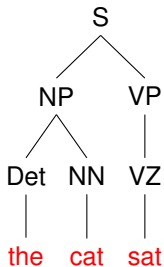
## Ambiguous Yields

The **yield** of a parse tree is the collection of terminals produced by the parse tree. Given a yield  $s$ .

## Parsing / Decoding

Given, a yield  $s$  and a grammar  $G$ , determine the set of parse trees that could have produced that sequence of terminals:  $T_G(s)$ .

## Example Derivation



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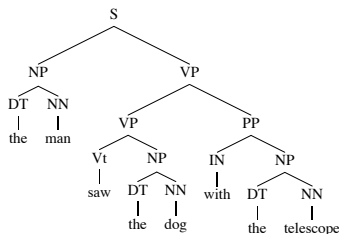
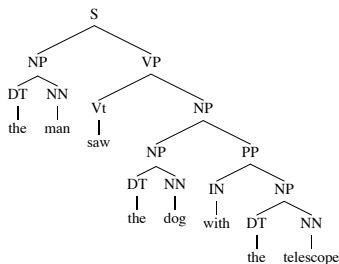
## Parsing / Decoding

Given, a yield  $s$  and a grammar  $G$ , determine the set of parse trees that could have produced that sequence of terminals:  $T_G(s)$ .

## Ambiguity

Example sentence: “The man saw the dog with the telescope”

- Grammatical:  $T_G(s) > 0$
- Ambiguous:  $T_G(s) > 1$

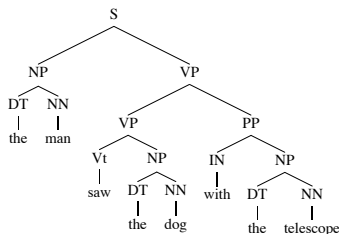
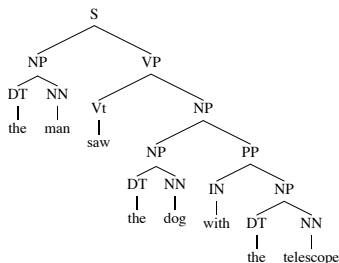


- Which should we prefer?

## Ambiguity

Example sentence: “The man saw the dog with the telescope”

- Grammatical:  $T_G(s) > 0$
- Ambiguous:  $T_G(s) > 1$



- Which should we prefer?
- One is more probable than the other
- Add probabilities!

## Goals

- What we want is a probability distribution over possible parse trees

$$t \in T_G(s)$$

$$\forall t, p(t) \geq 0 \quad \sum_{t \in T_G(s)} p(t) = 1 \quad (1)$$

- Rest of this lecture:
  - How do we define the function  $p(t)$  (parameterization)
  - How do we learn  $p(t)$  from data (estimation)
  - Given a sentence, how do we find the possible parse trees (parsing / decoding)

## Parameterization

- For every production  $\alpha \rightarrow \beta$ , we assume we have a function  $q(\alpha \rightarrow \beta)$
- We consider it a **conditional probability** of  $\beta$  (LHS) being derived from  $\alpha$  (RHS)

$$\sum_{\alpha \rightarrow \beta \in R: \alpha = X} q(\alpha \rightarrow \beta) = 1 \quad (2)$$

- The total probability of a tree  $t \equiv \{\alpha_1 \rightarrow \beta_1 \dots \alpha_n \rightarrow \beta_n\}$  is

$$p(t) = \prod_{i=1}^n q(\alpha_i \rightarrow \beta_i) \quad (3)$$

## Estimation



- Get a bunch of grad students to make parse trees for a million sentences
- Mitch Marcus: Penn Treebank (Wall Street Journal)
- To compute the conditional probability of a rule,

$$q(\text{np} \rightarrow \text{Det adj nn}) \approx \frac{\text{Count}(\text{np} \rightarrow \text{Det adj nn})}{\text{Count}(\text{np})}$$

- Where “Count” is the number of times that derivation appears in the sentences



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- Where “Count” is the number of times that derivation appears in the sentences
- Why no smoothing?

## Dynamic Programming

- Like for dependency parsing, we build a chart to consider all possible subtrees
- First, however, we'll just consider whether a sentence is grammatical or not
- Build up a chart with all possible derivations of spans
- Then see entry with start symbol over the entire sentence: those are all grammatical parses

## CYK Algorithm (deterministic)

### Assumptions

Assumes binary grammar (not too difficult to extend) and no recursive rules

Given sentence  $\vec{w}$  of length  $N$ , grammar  $(N, \Sigma, R, S)$

Initialize array  $C[s, t, n]$  as array of booleans, all false ( $\perp$ )

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**for**  $i = 0 \dots N$  **do**

**for** For each production  $r_j \equiv N_a \rightarrow w_i$  **do**

        set  $C[i, i, a] \leftarrow \top$

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**for**  $l = 2 \dots n$  (length of span) **do**

**for**  $s = 1 \dots N - l + 1$  (start of span) **do**

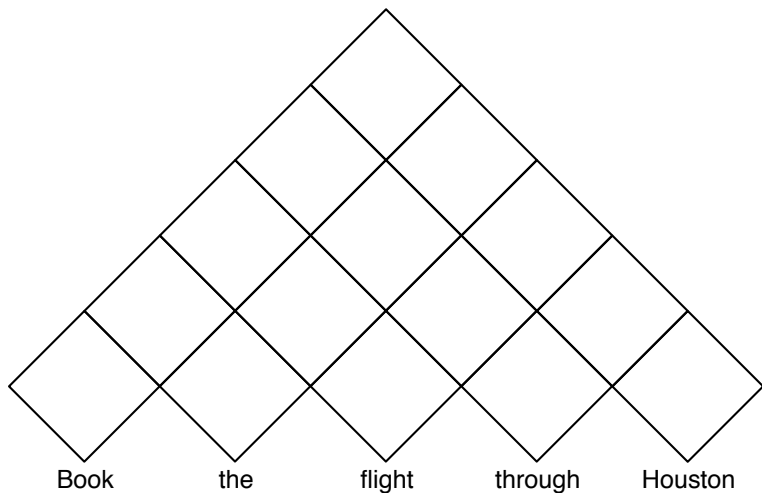
**for**  $k = 1 \dots l - 1$  (pivot within span) **do**

**for** each production  $r \equiv \alpha \rightarrow \beta\gamma$  **do**

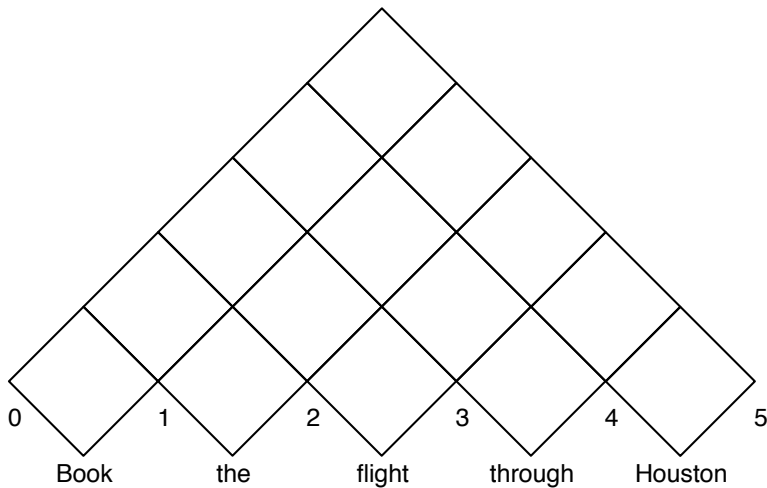
**if**  $\neg C[s, s + l, \alpha]$  **then**

$C[s, s + l, \alpha] \leftarrow C[s, s + k - 1, \beta] \wedge C[s + k, s + l, \gamma]$

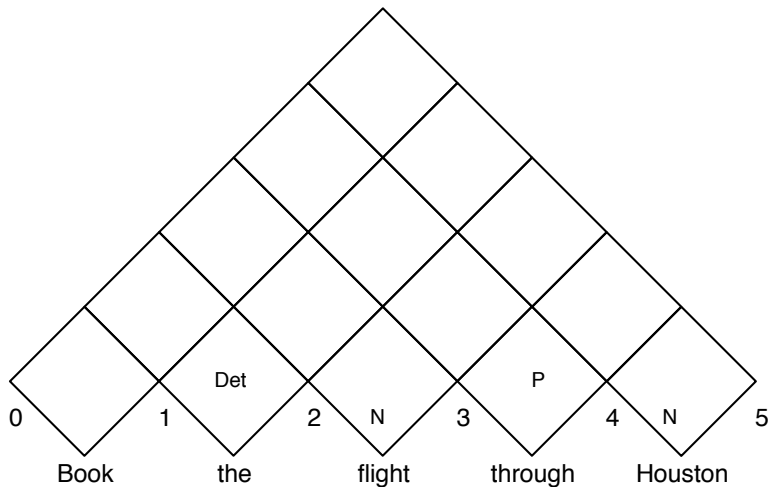
## Chart Parsing



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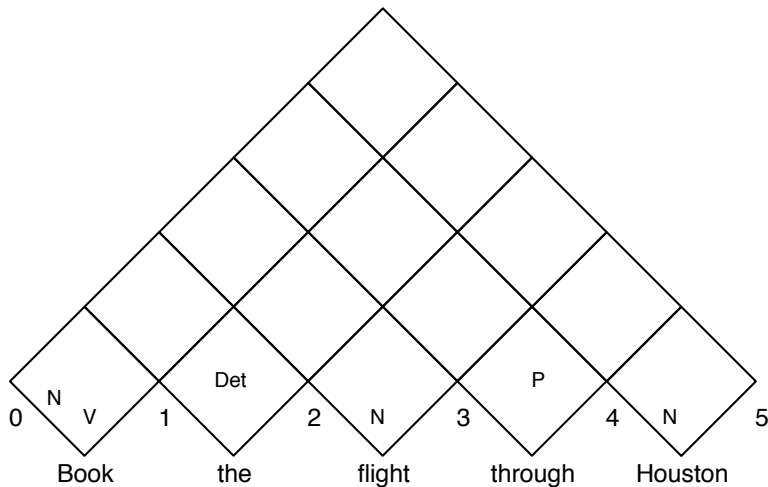


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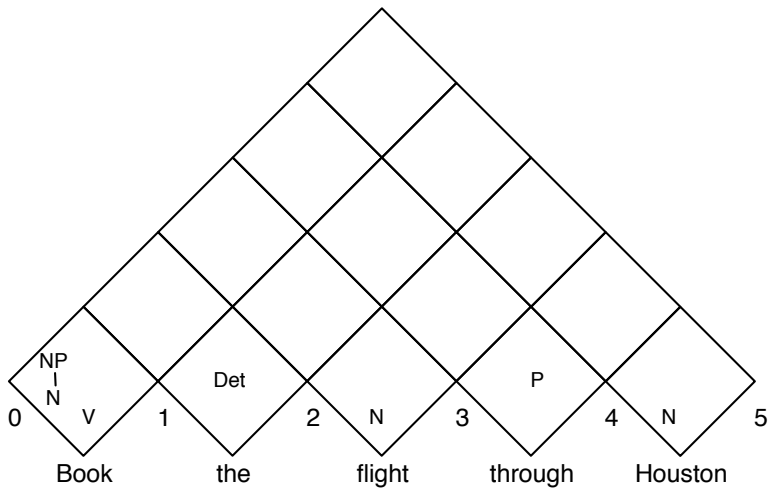




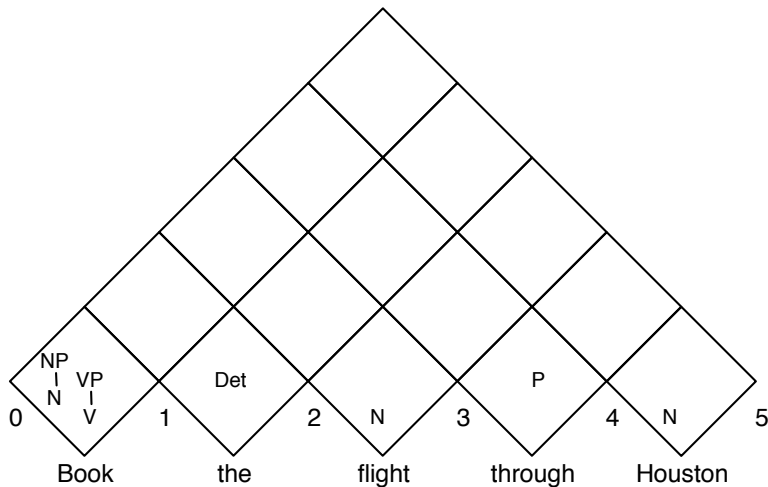
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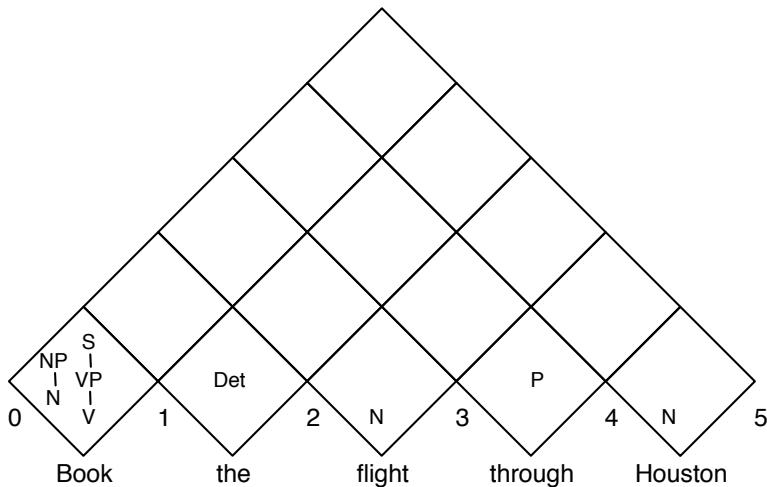
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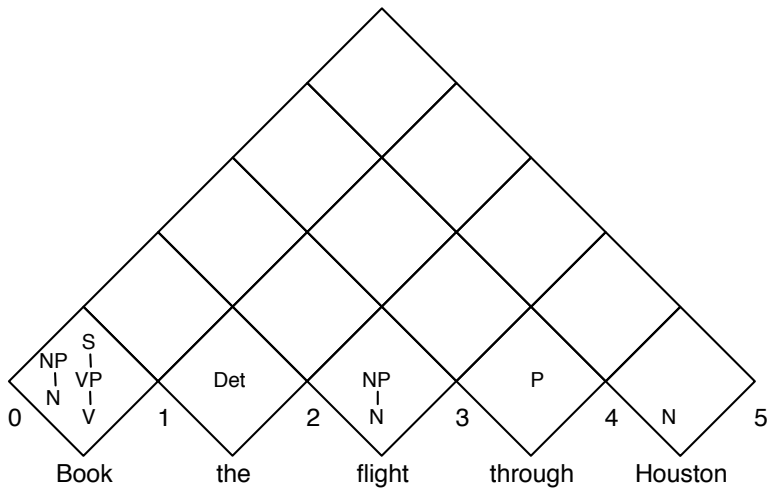
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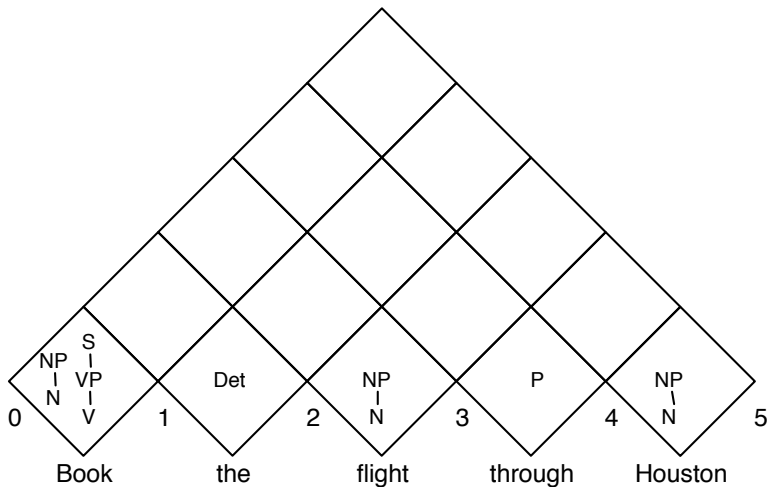
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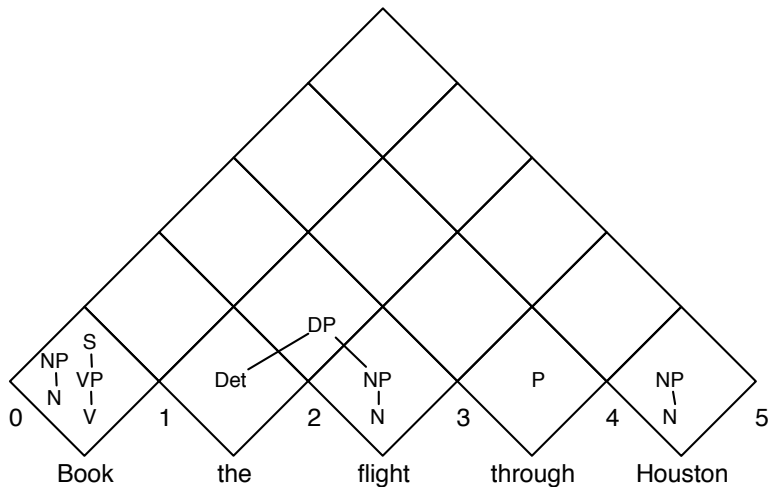
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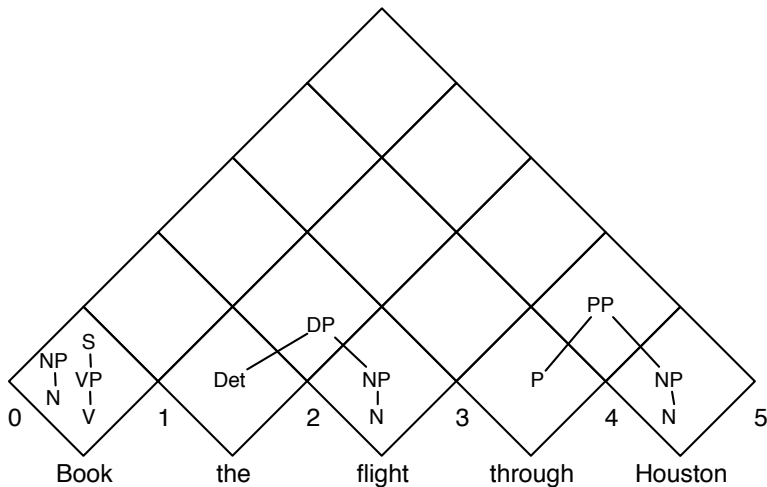
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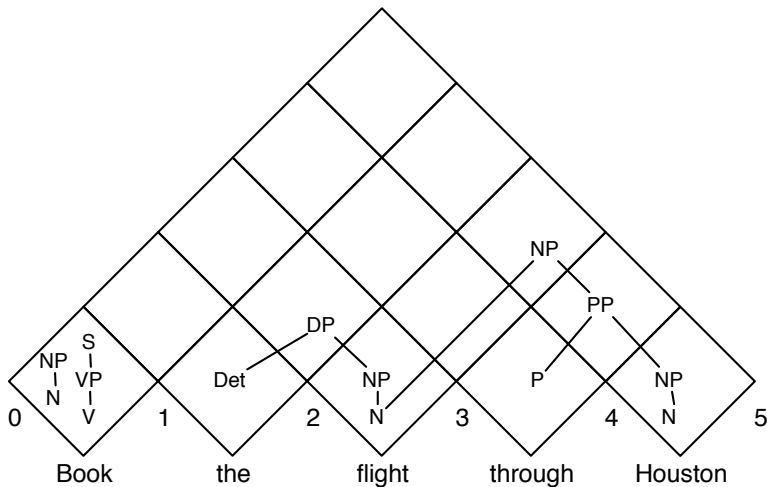


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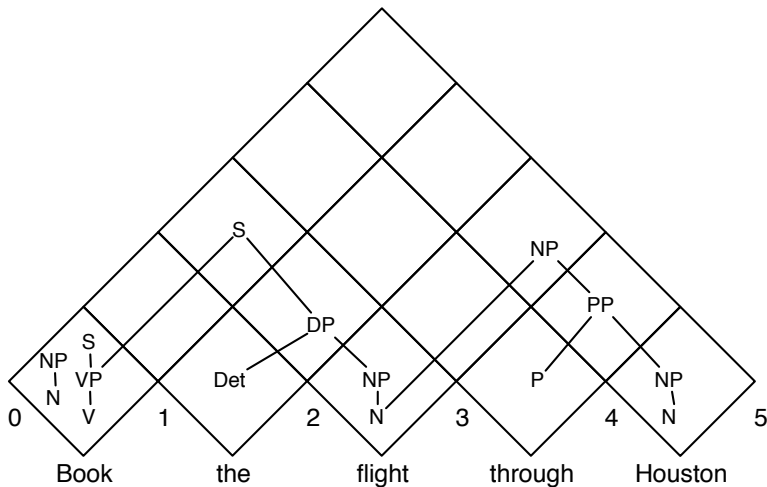




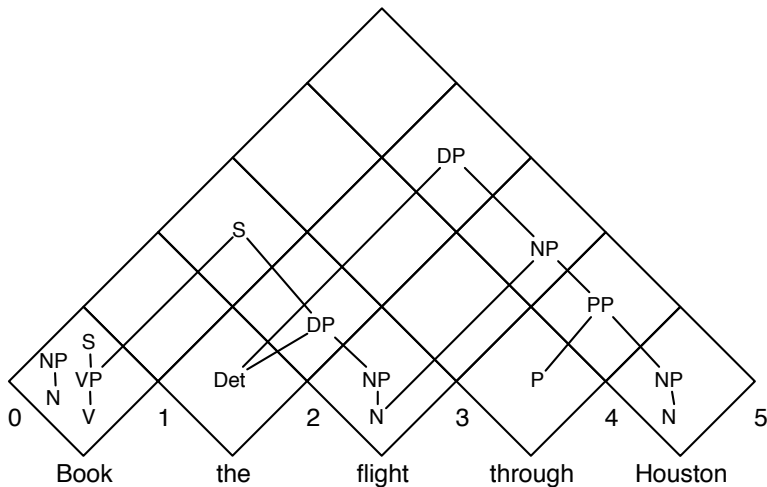
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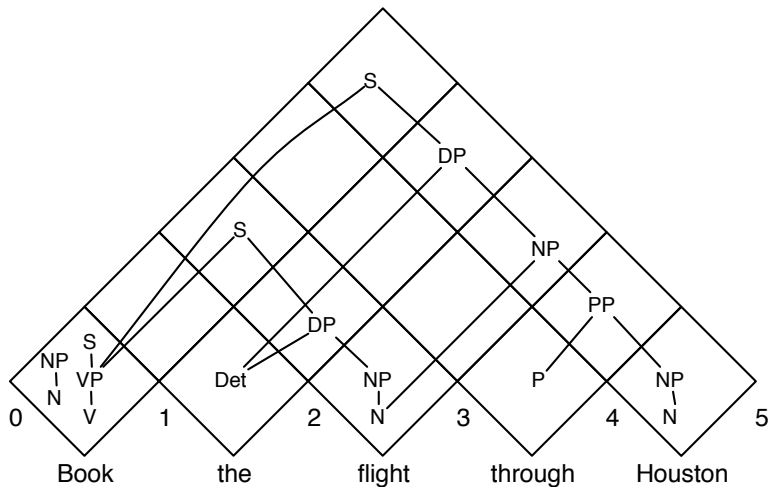
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## Complexity?

- Chart has  $n^2$  cells

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- Each cell has  $n$  options
- Times the number of productions  $|G|$
- Thus,  $O(n^3|G|)$



## How to deal with PCFG ambiguity

- In addition to keeping track of non-terminals in cell, also include **max** probability of forming non-terminal from sub-trees

$$C[s, s+k, \alpha] \leftarrow \max(C[s, s+k, \alpha], C[s, s+l-1, \beta] \cdot C[s+l, s+k, \gamma])$$

- The score associated with S in the top of the chart is the best overall parse-tree (**given the yield**)

## Recap

- Hierarchical syntax model: context free grammar
- Probabilistic interpretation: learn from data to solve ambiguity
- In class (next time):
  - Work through example to resolve ambiguity
  - Scoring a sentence

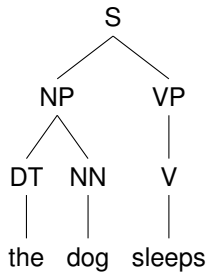
## A pcfg

Assume the following grammar

s	→	np	vp	1.0	v	→	sleeps	0.4
vp	→	v	np	0.7	v	→	saw	0.6
vp	→	vp	pp	0.2	nn	→	man	0.1
vp	→	v		0.1	nn	→	woman	0.1
np	→	dt	nn	0.2	nn	→	telescope	0.3
np	→	np	pp	0.8	nn	→	dog	0.5
pp	→	p	np	1.0	dt	→	the	1.0
					p	→	with	0.6
					p	→	in	0.4

## Evaluating the probability of a sentence

What is the probability of the parse



## Evaluating the probability of a sentence

$$\underbrace{1.0}_{\text{det} \rightarrow \text{the}} \cdot \underbrace{0.5}_{\text{n} \rightarrow \text{dog}} \cdot \underbrace{1.0}_{\text{v} \rightarrow \text{sleeps}} \cdot \underbrace{0.1}_{\text{vp} \rightarrow \text{v}} \cdot \underbrace{0.2}_{\text{np} \rightarrow \text{dt n}} \cdot \underbrace{1.0}_{\text{s} \rightarrow \text{np vp}} = 0.002$$

## Span 0

1.  $C[8, 8, nn] = \ln(0.3) = -1.2$
2.  $C[7, 7, dt] = \ln(1.0) = 0.0$
3.  $C[6, 6, p] = \ln(0.6) = -0.51$
4.  $C[5, 5, nn] = \ln(0.5) = -0.69$
5.  $C[4, 4, dt] = \ln(1.0) = 0.0$
6.  $C[3, 3, v] = \ln(0.6) = -0.51$
7.  $C[3, 3, vp] = \ln(0.6) + \ln(0.1) = -2.8$
8.  $C[2, 2, nn] = \ln(0.1) = -2.3$
9.  $C[1, 1, dt] = \ln(1.0) = 0.0$

## Span 1

$$1. C[1,2,np] = \underbrace{0.0}_{C[1,1,DT]} + \ln(\underbrace{-2.3}_{C[2,2,NN]}) + \ln(\underbrace{0.2}_{np \rightarrow dt \ n}) = -2.3 + -1.6 = -3.9$$

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$$2. C[4,5,np] = \underbrace{0.0}_{C[4,4,DT]} + \underbrace{-0.69}_{C[5,5,NN]} + \ln(\underbrace{0.2}_{np \rightarrow dt n}) = -0.69 + -1.6 = -2.3$$



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$$1. C[1,2,np] = \underbrace{0.0}_{C[1,1,DT]} + \ln\left(\underbrace{-2.3}_{C[2,2,NN]}\right) + \ln\left(\underbrace{0.2}_{np \rightarrow dt n}\right) = -2.3 + -1.6 = -3.9$$

$$2. C[4,5,np] = \underbrace{0.0}_{C[4,4,DT]} + \underbrace{-0.69}_{C[5,5,NN]} + \ln\left(\underbrace{0.2}_{np \rightarrow dt n}\right) = -0.69 + -1.6 = -2.3$$

$$3. C[7,8,np] = \underbrace{0.0}_{C[7,7,DT]} + \underbrace{-1.2}_{C[8,8,NN]} + \ln\left(\underbrace{0.2}_{np \rightarrow dt n}\right) = -1.2 + -1.6 = -2.8$$

## Span 2

$$1. C[1,3,s] = \underbrace{-3.9}_{C[1,2,NP]} + \underbrace{-2.8}_{C[3,3,VP]} + \ln(\underbrace{1.0}_{s \rightarrow np \text{ vp}}) = -6.7$$

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$$2. C[3, 5, vp] = \underbrace{-0.5}_{C[3,3, V]} + \underbrace{-2.3}_{C[4,5, NP]} + \ln\left(\underbrace{0.7}_{vp \rightarrow v \text{ np}}\right) = -2.8 - 0.36 = -3.2$$

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$$1. C[1, 3, s] = \underbrace{-3.9}_{C[1,2, NP]} + \underbrace{-2.8}_{C[3,3, VP]} + \ln\left(\underbrace{1.0}_{s \rightarrow np \text{ vp}}\right) = -6.7$$

$$2. C[3, 5, vp] = \underbrace{-0.5}_{C[3,3, V]} + \underbrace{-2.3}_{C[4,5, NP]} + \ln\left(\underbrace{0.7}_{vp \rightarrow v \text{ np}}\right) = -2.8 - 0.36 = -3.2$$

$$3. C[6, 8, pp] = \underbrace{-0.51}_{C[6,6, P]} + \underbrace{-2.8}_{C[7,8, NP]} + \ln\left(\underbrace{1.0}_{pp \rightarrow p \text{ np}}\right) = -3.3 + -1.6 = -3.3$$

## Span 4

$$1. C[1,5,s] = \underbrace{-3.9}_{C[1,2,NP]} + \underbrace{-3.2}_{C[3,5,VP]} + \ln(\underbrace{1.0}_{s \rightarrow np vp}) = -7.1$$

## Span 4

$$1. C[1,5,s] = \underbrace{-3.9}_{C[1,2,NP]} + \underbrace{-3.2}_{C[3,5,VP]} + \ln\left(\underbrace{1.0}_{s \rightarrow np\ vp}\right) = -7.1$$

$$2. C[4,8,np] = \underbrace{-2.3}_{C[4,5,NP]} + \underbrace{-3.3}_{C[6,8,PP]} + \ln\left(\underbrace{0.8}_{np \rightarrow np\ pp}\right) = -5.6 + -0.2 = -5.8$$

## Span 5

$$C[3, 8, vp] = \max( \tag{4}$$

$$\underbrace{-3.2}_{C[3,5,VP]} + \underbrace{-3.3}_{C[6,8,PP]} + \underbrace{-1.6}_{vp \rightarrow vp\ pp} , \tag{5}$$

$$\underbrace{-0.5}_{C[3,3,V]} + \underbrace{-5.8}_{C[4,8,NP]} + \underbrace{-0.36}_{vp \rightarrow v\ np} ) \tag{6}$$

$$= \max(-8.1, -6.7) = -6.7 \tag{7}$$

## Span 7

$$1. C[1,8,s] = \underbrace{-3.9}_{C[1,2,NP]} + \underbrace{-6.7}_{C[3,8,VP]} = -10.6$$