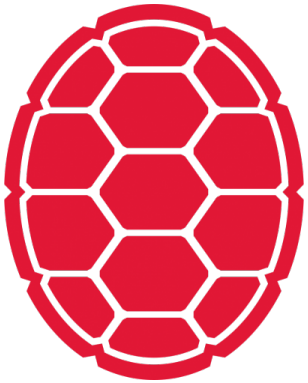

# Long Short Term Memory Networks

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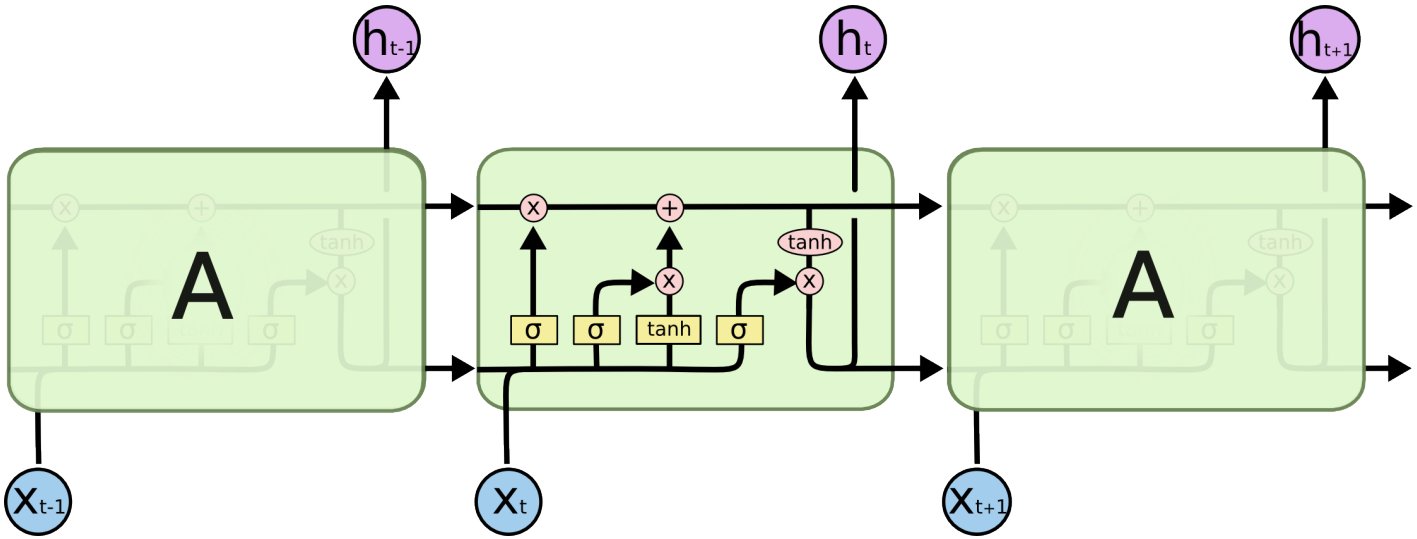
LSTM EXAMPLE

## Recap of LSTM




Three gates: input ($i_t$), forget ($f_t$), out ($o_t$)

$$i_t = \sigma(W_{ii}x_t + b_{ii} + W_{hi}h_{t-1} + b_{hi})$$
$$f_t = \sigma(W_{if}x_t + b_{if} + W_{hf}h_{t-1} + b_{hf})$$
$$o_t = \sigma(W_{io}x_t + b_{io} + W_{ho}h_{t-1} + b_{ho})$$

New memory input: $\tilde{c}_t$

$$\tilde{c}_t = \tanh(W_{ic}x_t + b_{ic} + W_{hc}h_{t-1} + b_{hc})$$

Memorize and forget:

$$c_t = f_t * c_{t-1} + i_t * \tilde{c}_t$$
$$h_t = o_t * \tanh(c_t)$$

## Figuring out this LSTM

| A | B |
|---|---|
| 1.0 0.0 | 0.0 1.0 |

- input sequence: A, A, B, B, A, B, A

$$x_1 = [1.0, 0.0] \quad x_2 = [1.0, 0.0] \quad x_3 = [0.0, 1.0] \quad \ldots$$

- prediction output:

$$y_t = \text{softmax}(h_t) \quad [\text{number of hidden nodes} = 2]$$

## Parameters that take $x_t$ as input

### Input Gate

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}$$

$$b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

### Memory Cell

$$W_{ic} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix}$$

$$b_{ic} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

### Forget Gate

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}$$

$$b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

### Output Gate

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}$$

$$b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix}$$

## Parameters that take $h_{t-1}$ as input

### Input Gate

$$W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix}$$

$$b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

### Memory Cell

$$W_{hc} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}$$

$$b_{hc} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

### Forget Gate

$$W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix}$$

$$b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

### Output Gate

$$W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}$$

$$b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

## Inputs

- Initial hidden states:

$$h_0 = [0.0, 0.0]^\top$$

- Initial memory input:

$$c_0 = [0.0, 0.0]^\top$$

- Input sequences in time: A, A, B, B, A, B, A

$$x_1 = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} \quad x_3 = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} \quad \dots$$

## Input Gate at $t = 1$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top \qquad h^{(0)} = [0.00, 0.00]^\top$$

## Input Gate at $t = 1$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top \qquad h^{(0)} = [0.00, 0.00]^\top$$

$$\begin{aligned} i^{(1)} &= \sigma(W_{ii}x^{(1)} + b_{ii} + W_{hi}h^{(0)} + b_{hi}) && (1) \\ &= \sigma([30.00, -30.00]^\top) && (2) \\ &= [1.00, 0.00]^\top && (3) \end{aligned}$$

## Forget Gate at $t = 1$: $f^{(1)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \qquad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top \qquad h^{(0)} = [0.00, 0.00]^\top$$

## Forget Gate at $t = 1$: $f^{(1)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top \qquad h^{(0)} = [0.00, 0.00]^\top$$

$$f^{(1)} = \sigma(W_{if} x^{(1)} + b_{if} + W_{hf} h^{(0)} + b_{hf}) \tag{4}$$

$$= \sigma([-30.00, 0.00]^\top) \tag{5}$$

$$= [0.00, 0.50]^\top \tag{6}$$

## Output Gate at $t = 1$: $o^{(1)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top \qquad h^{(0)} = [0.00, 0.00]^\top$$

## Output Gate at $t = 1$: $o^{(1)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top \qquad h^{(0)} = [0.00, 0.00]^\top$$

$$o^{(1)} = \sigma(W_{io}x^{(1)} + b_{io} + W_{ho}h^{(0)} + b_{ho}) \tag{7}$$

$$= \sigma([30.00, 30.00]^\top) \tag{8}$$

$$= [1.00, 1.00]^\top \tag{9}$$

## Memory Contribution at $t = 1$: $\tilde{c}^{(1)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top$$

$$h^{(0)} = [0.00, 0.00]^\top$$

## Memory Contribution at $t = 1$: $\tilde{c}^{(1)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(1)} = [1.00, 0.00]^\top \qquad h^{(0)} = [0.00, 0.00]^\top$$

$$\begin{aligned} \tilde{c}^{(1)} &= \tanh(W_{i\tilde{c}} x^{(1)} + b_{i\tilde{c}} + W_{h\tilde{c}} h^{(0)} + b_{h\tilde{c}}) && (10) \\ &= \tanh([30.00, 0.00]^\top) && (11) \\ &= [1.00, 0.00]^\top && (12) \end{aligned}$$

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

$$c_1 = f_1 \circ c_0 + i_1 \circ \tilde{c}_1 \tag{13}$$

(14)

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

$$c_1 = f_1 \circ c_0 + i_1 \circ \tilde{c}_1 \tag{13}$$

$$= [0.00, 0.50]^\top \circ [0.00, 0.00]^\top + [1.00, 0.00]^\top \circ [1.00, 0.00]^\top \tag{14}$$

$$\tag{15}$$

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

$$c_1 = f_1 \circ c_0 + i_1 \circ \tilde{c}_1 \tag{13}$$

$$= [0.00, 0.50]^\top \circ [0.00, 0.00]^\top + [1.00, 0.00]^\top \circ [1.00, 0.00]^\top \tag{14}$$

$$= [1.00, 0.00]^\top \tag{15}$$

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

$$c_1 = [1.00, 0.00]^\top \tag{13}$$

- New hidden ($h_1$)

$$h_1 \tag{14}$$

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

$$c_1 = [1.00, 0.00]^\top \tag{13}$$

- New hidden ($h_1$)

$$h_1 = o_1 \circ \tanh(c_1) \tag{14}$$

$$\tag{15}$$

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

$$c_1 = [1.00, 0.00]^\top \tag{13}$$

- New hidden ($h_1$)

$$h_1 = o_1 \circ \tanh(c_1) \tag{14}$$

$$= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) \tag{15}$$

$$\tag{16}$$

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

$$c_1 = [1.00, 0.00]^\top \tag{13}$$

- New hidden ($h_1$)

$$\begin{aligned} h_1 &= o_1 \circ \tanh(c_1) && (14)\\ &= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) && (15)\\ &= [0.76, 0.00]^\top && (16) \end{aligned}$$

## Forward message at time step 1

| $f_1$ | $c_0$ | $i_1$ | $\tilde{c}_1$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_1$)

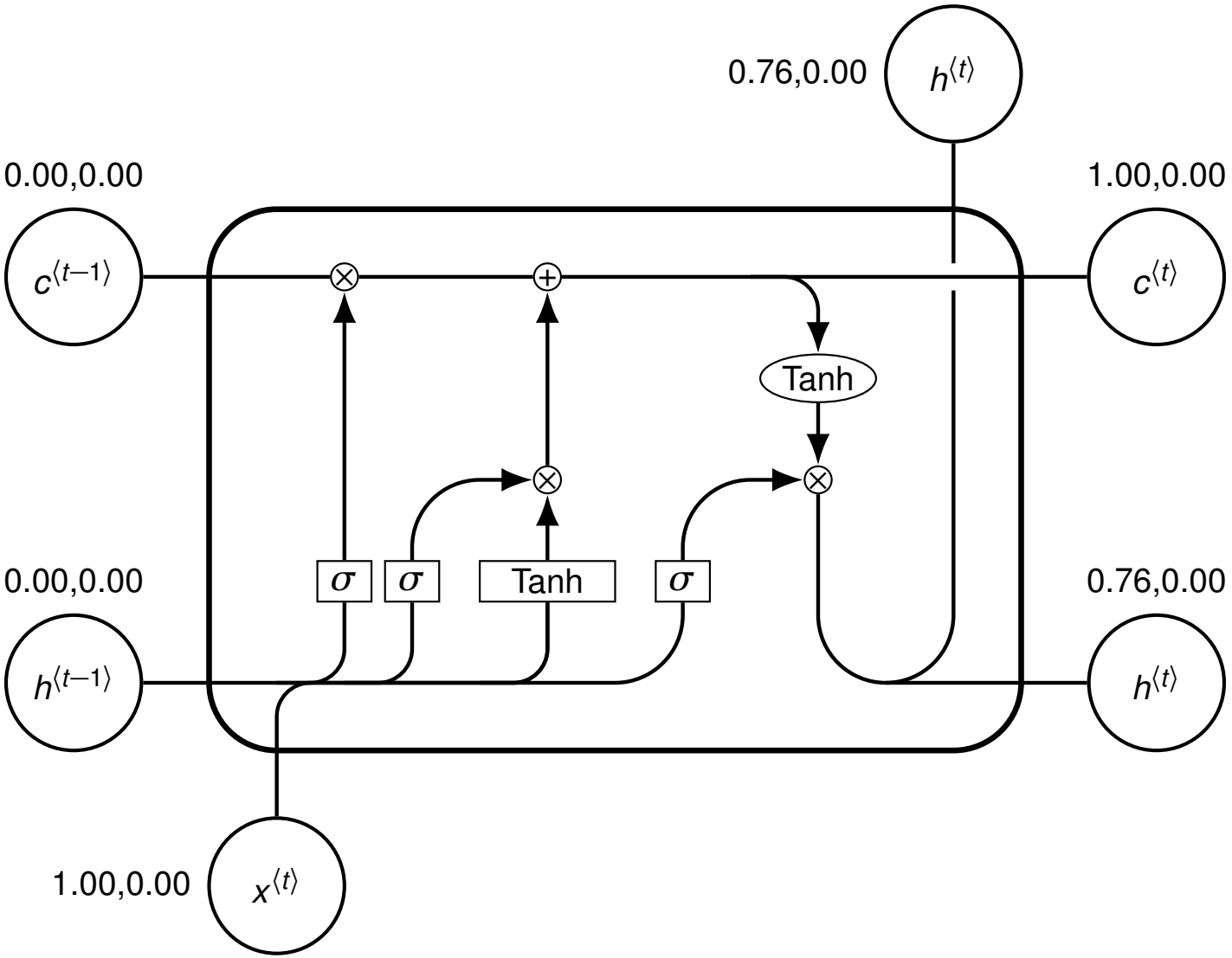
$$c_1 = [1.00, 0.00]^\top \tag{13}$$

- New hidden ($h_1$)

$$h_1 = [0.76, 0.00]^\top \tag{14}$$

- Prediction $y_1 = \text{softmax}(h_1) = 0$

## Summary at $t = 1$



## Input Gate at $t = 2$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top \qquad h^{(1)} = [0.76, 0.00]^\top$$

## Input Gate at $t = 2$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top \qquad h^{(1)} = [0.76, 0.00]^\top$$

$$\begin{aligned} i^{(2)} &= \sigma(W_{ii}x^{(2)} + b_{ii} + W_{hi}h^{(1)} + b_{hi}) && (15) \\ &= \sigma([30.00, 15.70]^\top) && (16) \\ &= [1.00, 1.00]^\top && (17) \end{aligned}$$

## Forget Gate at $t = 2$: $f^{(2)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top \qquad h^{(1)} = [0.76, 0.00]^\top$$

## Forget Gate at $t = 2$: $f^{(2)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top \qquad h^{(1)} = [0.76, 0.00]^\top$$

$$f^{(2)} = \sigma\left(W_{if} x^{(2)} + b_{if} + W_{hf} h^{(1)} + b_{hf}\right) \tag{18}$$

$$= \sigma\left([-30.00, 0.00]^\top\right) \tag{19}$$

$$= [0.00, 0.50]^\top \tag{20}$$

## Output Gate at $t = 2$: $o^{(2)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top \qquad h^{(1)} = [0.76, 0.00]^\top$$

## Output Gate at $t = 2$: $o^{(2)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top \qquad h^{(1)} = [0.76, 0.00]^\top$$

$$o^{(2)} = \sigma(W_{io}x^{(2)} + b_{io} + W_{ho}h^{(1)} + b_{ho}) \tag{21}$$

$$= \sigma([30.00, 30.00]^\top) \tag{22}$$

$$= [1.00, 1.00]^\top \tag{23}$$

## Memory Contribution at $t = 2$: $\tilde{c}^{(2)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top$$

$$h^{(1)} = [0.76, 0.00]^\top$$

## Memory Contribution at $t = 2$: $\tilde{c}^{(2)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(2)} = [1.00, 0.00]^\top \qquad h^{(1)} = [0.76, 0.00]^\top$$

$$\begin{aligned} \tilde{c}^{(2)} &= \tanh(W_{i\tilde{c}} x^{(2)} + b_{i\tilde{c}} + W_{h\tilde{c}} h^{(1)} + b_{h\tilde{c}}) && (24) \\ &= \tanh([30.00, 0.00]^\top) && (25) \\ &= [1.00, 0.00]^\top && (26) \end{aligned}$$

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

$$c_2 = f_2 \circ c_1 + i_2 \circ \tilde{c}_2 \tag{27}$$

(28)

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

$$c_2 = f_2 \circ c_1 + i_2 \circ \tilde{c}_2 \tag{27}$$

$$= [0.00, 0.50]^\top \circ [1.00, 0.00]^\top + [1.00, 1.00]^\top \circ [1.00, 0.00]^\top \tag{28}$$

$$\tag{29}$$

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

$$c_2 = f_2 \circ c_1 + i_2 \circ \tilde{c}_2 \tag{27}$$

$$= [0.00, 0.50]^\top \circ [1.00, 0.00]^\top + [1.00, 1.00]^\top \circ [1.00, 0.00]^\top \tag{28}$$

$$= [1.00, 0.00]^\top \tag{29}$$

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

$$c_2 = [1.00, 0.00]^\top \tag{27}$$

- New hidden ($h_2$)

$$h_2 \tag{28}$$

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

$$c_2 = [1.00, 0.00]^\top \tag{27}$$

- New hidden ($h_2$)

$$h_2 = o_2 \circ \tanh(c_2) \tag{28}$$

$$\tag{29}$$

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

$$c_2 = [1.00, 0.00]^\top \tag{27}$$

- New hidden ($h_2$)

$$h_2 = o_2 \circ \tanh(c_2) \tag{28}$$

$$= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) \tag{29}$$

$$\tag{30}$$

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

$$c_2 = [1.00, 0.00]^\top \tag{27}$$

- New hidden ($h_2$)

$$\begin{aligned} h_2 &= o_2 \circ \tanh(c_2) && (28)\\ &= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) && (29)\\ &= [0.76, 0.00]^\top && (30) \end{aligned}$$

## Forward message at time step 2

| $f_2$ | $c_1$ | $i_2$ | $\tilde{c}_2$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_2$)

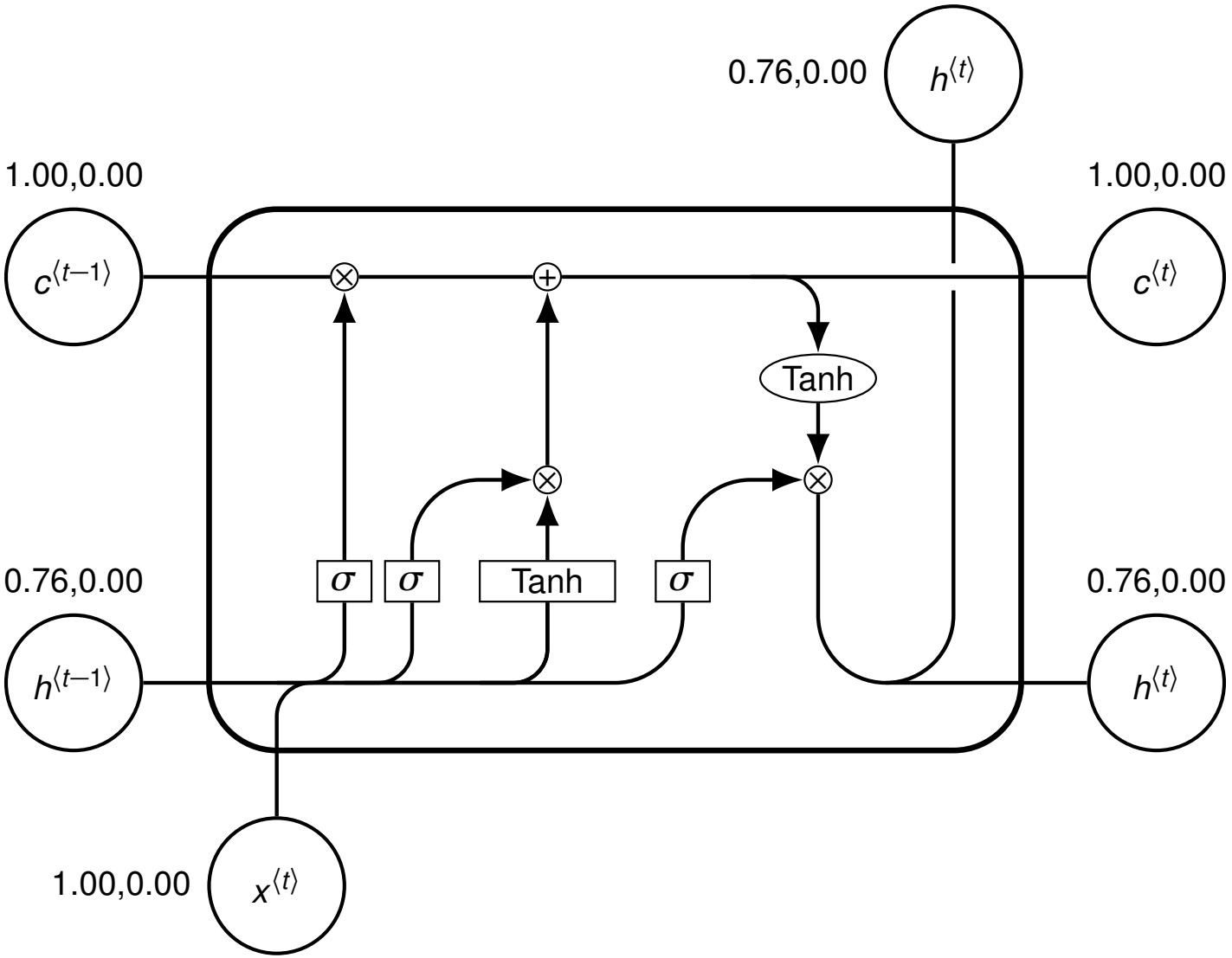
$$c_2 = [1.00, 0.00]^\top \tag{27}$$

- New hidden ($h_2$)

$$h_2 = [0.76, 0.00]^\top \tag{28}$$

- Prediction $y_2 = \text{softmax}(h_2) = 0$

## Summary at $t = 2$



## Input Gate at $t = 3$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top \qquad h^{(2)} = [0.76, 0.00]^\top$$

## Input Gate at $t = 3$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top \qquad h^{(2)} = [0.76, 0.00]^\top$$

$$i^{(3)} = \sigma(W_{ii}x^{(3)} + b_{ii} + W_{hi}h^{(2)} + b_{hi}) \tag{29}$$

$$= \sigma([0.00, 15.70]^\top) \tag{30}$$

$$= [0.50, 1.00]^\top \tag{31}$$

## Forget Gate at $t = 3$: $f^{(3)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top \qquad h^{(2)} = [0.76, 0.00]^\top$$

## Forget Gate at $t = 3$: $f^{(3)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top \qquad h^{(2)} = [0.76, 0.00]^\top$$

$$f^{(3)} = \sigma(W_{if} x^{(3)} + b_{if} + W_{hf} h^{(2)} + b_{hf}) \tag{32}$$

$$= \sigma([-30.00, 0.00]^\top) \tag{33}$$

$$= [0.00, 0.50]^\top \tag{34}$$

## Output Gate at $t = 3$: $o^{(3)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top \qquad h^{(2)} = [0.76, 0.00]^\top$$

## Output Gate at $t = 3$: $o^{(3)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top \qquad h^{(2)} = [0.76, 0.00]^\top$$

$$o^{(3)} = \sigma(W_{io}x^{(3)} + b_{io} + W_{ho}h^{(2)} + b_{ho}) \tag{35}$$

$$= \sigma([30.00, 30.00]^\top) \tag{36}$$

$$= [1.00, 1.00]^\top \tag{37}$$

## Memory Contribution at $t = 3$: $\tilde{c}^{(3)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top$$

$$h^{(2)} = [0.76, 0.00]^\top$$

## Memory Contribution at $t = 3$: $\tilde{c}^{(3)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(3)} = [0.00, 1.00]^\top \qquad h^{(2)} = [0.76, 0.00]^\top$$

$$\begin{aligned} \tilde{c}^{(3)} &= \tanh(W_{i\tilde{c}} x^{(3)} + b_{i\tilde{c}} + W_{h\tilde{c}} h^{(2)} + b_{h\tilde{c}}) && (38) \\ &= \tanh([0.00, 30.00]^\top) && (39) \\ &= [0.00, 1.00]^\top && (40) \end{aligned}$$

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

$$c_3 = f_3 \circ c_2 + i_3 \circ \tilde{c}_3 \tag{41}$$

(42)

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

$$c_3 = f_3 \circ c_2 + i_3 \circ \tilde{c}_3 \tag{41}$$

$$= [0.00, 0.50]^\top \circ [1.00, 0.00]^\top + [0.50, 1.00]^\top \circ [0.00, 1.00]^\top \tag{42}$$

$$\tag{43}$$

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

$$c_3 = f_3 \circ c_2 + i_3 \circ \tilde{c}_3 \tag{41}$$

$$= [0.00, 0.50]^\top \circ [1.00, 0.00]^\top + [0.50, 1.00]^\top \circ [0.00, 1.00]^\top \tag{42}$$

$$= [0.00, 1.00]^\top \tag{43}$$

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

$$c_3 = [0.00, 1.00]^\top \tag{41}$$

- New hidden ($h_3$)

$$h_3 \tag{42}$$

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

$$c_3 = [0.00, 1.00]^\top \tag{41}$$

- New hidden ($h_3$)

$$h_3 = o_3 \circ \tanh(c_3) \tag{42}$$

$$\tag{43}$$

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

$$c_3 = [0.00, 1.00]^\top \tag{41}$$

- New hidden ($h_3$)

$$h_3 = o_3 \circ \tanh(c_3) \tag{42}$$

$$= [1.00, 1.00]^\top \circ \tanh([0.00, 1.00]^\top) \tag{43}$$

$$\tag{44}$$

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

$$c_3 = [0.00, 1.00]^\top \tag{41}$$

- New hidden ($h_3$)

$$\begin{aligned} h_3 &= o_3 \circ \tanh(c_3) && (42)\\ &= [1.00, 1.00]^\top \circ \tanh([0.00, 1.00]^\top) && (43)\\ &= [0.00, 0.76]^\top && (44) \end{aligned}$$

## Forward message at time step 3

| $f_3$ | $c_2$ | $i_3$ | $\tilde{c}_3$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_3$)

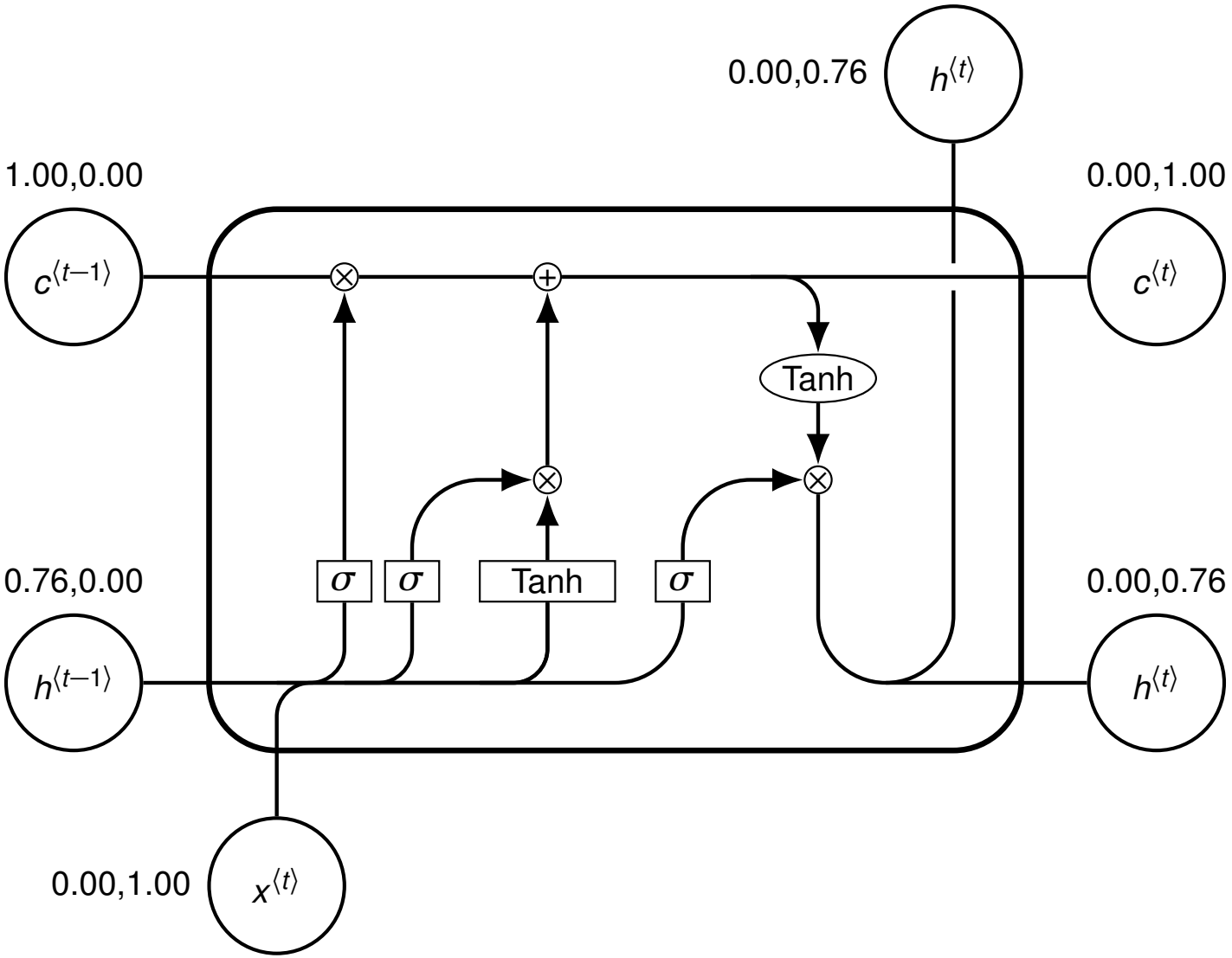
$$c_3 = [0.00, 1.00]^\top \tag{41}$$

- New hidden ($h_3$)

$$h_3 = [0.00, 0.76]^\top \tag{42}$$

- Prediction $y_3 = \text{softmax}(h_3) = 1$

## Summary at $t = 3$



## Input Gate at $t = 4$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top \qquad h^{(3)} = [0.00, 0.76]^\top$$

## Input Gate at $t = 4$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top \qquad h^{(3)} = [0.00, 0.76]^\top$$

$$\begin{aligned} i^{(4)} &= \sigma(W_{ii}x^{(4)} + b_{ii} + W_{hi}h^{(3)} + b_{hi}) && (43) \\ &= \sigma([0.00, -30.00]^\top) && (44) \\ &= [0.50, 0.00]^\top && (45) \end{aligned}$$

## Forget Gate at $t = 4$: $f^{(4)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top \qquad h^{(3)} = [0.00, 0.76]^\top$$

## Forget Gate at $t = 4$: $f^{(4)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top \qquad h^{(3)} = [0.00, 0.76]^\top$$

$$f^{(4)} = \sigma(W_{if} x^{(4)} + b_{if} + W_{hf} h^{(3)} + b_{hf}) \tag{46}$$

$$= \sigma([-30.00, -22.85]^\top) \tag{47}$$

$$= [0.00, 0.00]^\top \tag{48}$$

## Output Gate at $t = 4$: $o^{(4)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top \qquad h^{(3)} = [0.00, 0.76]^\top$$

## Output Gate at $t = 4$: $o^{(4)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top \qquad h^{(3)} = [0.00, 0.76]^\top$$

$$o^{(4)} = \sigma(W_{io}x^{(4)} + b_{io} + W_{ho}h^{(3)} + b_{ho}) \tag{49}$$

$$= \sigma([30.00, 30.00]^\top) \tag{50}$$

$$= [1.00, 1.00]^\top \tag{51}$$

## Memory Contribution at $t = 4$: $\tilde{c}^{(4)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top$$

$$h^{(3)} = [0.00, 0.76]^\top$$

## Memory Contribution at $t = 4$: $\tilde{c}^{(4)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(4)} = [0.00, 1.00]^\top \qquad h^{(3)} = [0.00, 0.76]^\top$$

$$\begin{aligned} \tilde{c}^{(4)} &= \tanh(W_{i\tilde{c}} x^{(4)} + b_{i\tilde{c}} + W_{h\tilde{c}} h^{(3)} + b_{h\tilde{c}}) && (52) \\ &= \tanh([0.00, 30.00]^\top) && (53) \\ &= [0.00, 1.00]^\top && (54) \end{aligned}$$

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

$$c_4 = f_4 \circ c_3 + i_4 \circ \tilde{c}_4 \tag{55}$$

(56)

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

$$c_4 = f_4 \circ c_3 + i_4 \circ \tilde{c}_4 \tag{55}$$

$$= [0.00, 0.00]^\top \circ [0.00, 1.00]^\top + [0.50, 0.00]^\top \circ [0.00, 1.00]^\top \tag{56}$$

$$\tag{57}$$

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

$$c_4 = f_4 \circ c_3 + i_4 \circ \tilde{c}_4 \tag{55}$$

$$= [0.00, 0.00]^\top \circ [0.00, 1.00]^\top + [0.50, 0.00]^\top \circ [0.00, 1.00]^\top \tag{56}$$

$$= [0.00, 0.00]^\top \tag{57}$$

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

$$c_4 = [0.00, 0.00]^\top \tag{55}$$

- New hidden ($h_4$)

$$h_4 \tag{56}$$

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

$$c_4 = [0.00, 0.00]^\top \tag{55}$$

- New hidden ($h_4$)

$$h_4 = o_4 \circ \tanh(c_4) \tag{56}$$

$$\tag{57}$$

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

$$c_4 = [0.00, 0.00]^\top \tag{55}$$

- New hidden ($h_4$)

$$h_4 = o_4 \circ \tanh(c_4) \tag{56}$$
$$= [1.00, 1.00]^\top \circ \tanh([0.00, 0.00]^\top) \tag{57}$$
$$\tag{58}$$

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

$$c_4 = [0.00, 0.00]^\top \tag{55}$$

- New hidden ($h_4$)

$$\begin{aligned} h_4 &= o_4 \circ \tanh(c_4) && (56)\\ &= [1.00, 1.00]^\top \circ \tanh([0.00, 0.00]^\top) && (57)\\ &= [0.00, 0.00]^\top && (58) \end{aligned}$$

## Forward message at time step 4

| $f_4$ | $c_3$ | $i_4$ | $\tilde{c}_4$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[0.50, 0.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_4$)

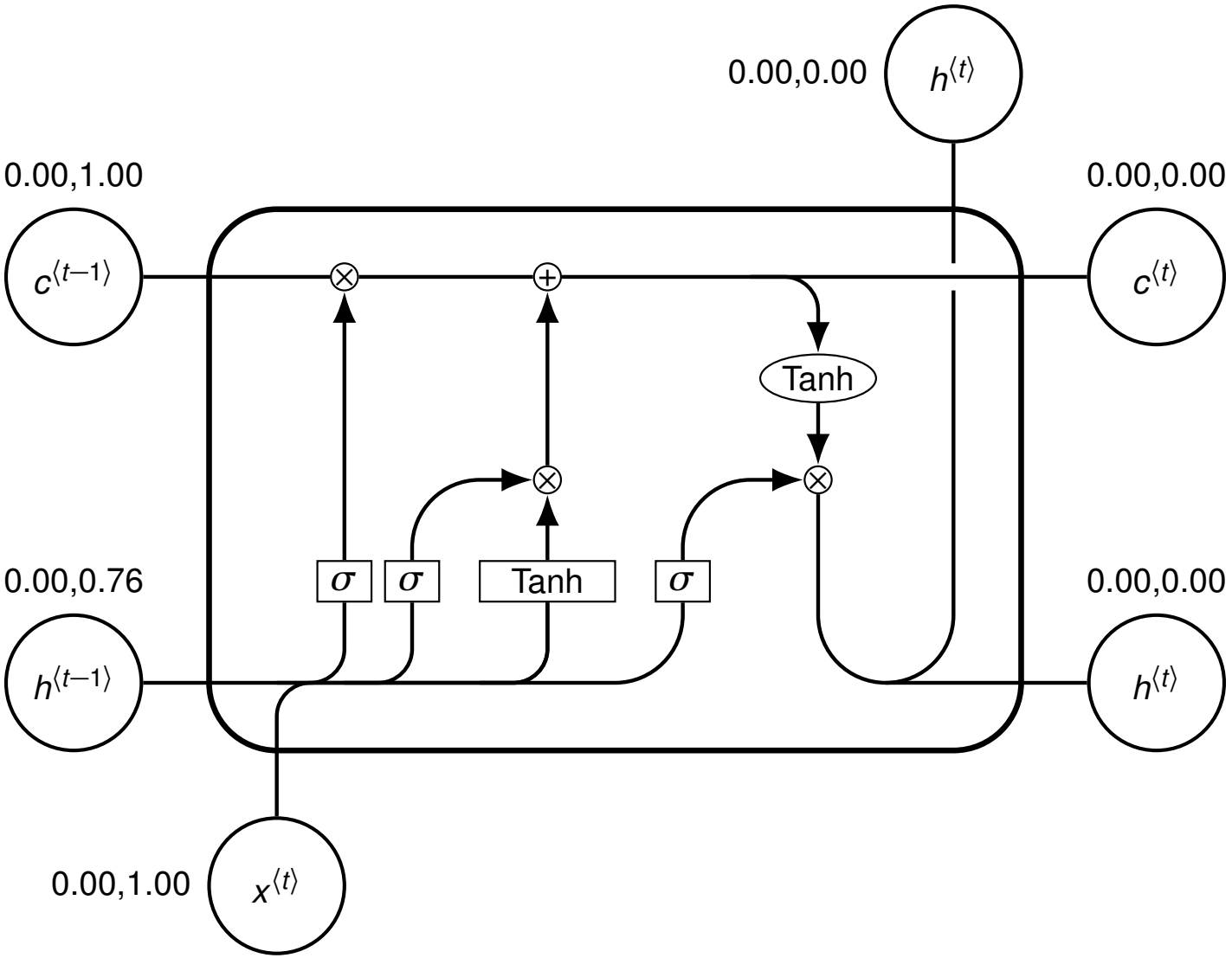
$$c_4 = [0.00, 0.00]^\top \tag{55}$$

- New hidden ($h_4$)

$$h_4 = [0.00, 0.00]^\top \tag{56}$$

- Prediction $y_4 = \text{softmax}(h_4) = 1$

## Summary at $t = 4$



## Input Gate at $t = 5$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top \qquad h^{(4)} = [0.00, 0.00]^\top$$

## Input Gate at $t = 5$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top \qquad h^{(4)} = [0.00, 0.00]^\top$$

$$\begin{aligned} i^{(5)} &= \sigma(W_{ii}x^{(5)} + b_{ii} + W_{hi}h^{(4)} + b_{hi}) && (57) \\ &= \sigma([30.00, -30.00]^\top) && (58) \\ &= [1.00, 0.00]^\top && (59) \end{aligned}$$

## Forget Gate at $t = 5$: $f^{(5)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top \qquad h^{(4)} = [0.00, 0.00]^\top$$

## Forget Gate at $t = 5$: $f^{(5)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top \qquad h^{(4)} = [0.00, 0.00]^\top$$

$$f^{(5)} = \sigma(W_{if} x^{(5)} + b_{if} + W_{hf} h^{(4)} + b_{hf}) \tag{60}$$

$$= \sigma([-30.00, -0.00]^\top) \tag{61}$$

$$= [0.00, 0.50]^\top \tag{62}$$

## Output Gate at $t = 5$: $o^{(5)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top \qquad h^{(4)} = [0.00, 0.00]^\top$$

## Output Gate at $t = 5$: $o^{(5)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top \qquad h^{(4)} = [0.00, 0.00]^\top$$

$$o^{(5)} = \sigma(W_{io}x^{(5)} + b_{io} + W_{ho}h^{(4)} + b_{ho}) \tag{63}$$

$$= \sigma([30.00, 30.00]^\top) \tag{64}$$

$$= [1.00, 1.00]^\top \tag{65}$$

## Memory Contribution at $t = 5$: $\tilde{c}^{(5)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top$$

$$h^{(4)} = [0.00, 0.00]^\top$$

## Memory Contribution at $t = 5$: $\tilde{c}^{(5)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(5)} = [1.00, 0.00]^\top \qquad h^{(4)} = [0.00, 0.00]^\top$$

$$\begin{aligned} \tilde{c}^{(5)} &= \tanh(W_{i\tilde{c}} x^{(5)} + b_{i\tilde{c}} + W_{h\tilde{c}} h^{(4)} + b_{h\tilde{c}}) && (66) \\ &= \tanh([30.00, 0.00]^\top) && (67) \\ &= [1.00, 0.00]^\top && (68) \end{aligned}$$

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

$$c_5 = f_5 \circ c_4 + i_5 \circ \tilde{c}_5 \tag{69}$$

(70)

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

$$c_5 = f_5 \circ c_4 + i_5 \circ \tilde{c}_5 \tag{69}$$

$$= [0.00, 0.50]^\top \circ [0.00, 0.00]^\top + [1.00, 0.00]^\top \circ [1.00, 0.00]^\top \tag{70}$$

$$\tag{71}$$

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

$$c_5 = f_5 \circ c_4 + i_5 \circ \tilde{c}_5 \tag{69}$$

$$= [0.00, 0.50]^\top \circ [0.00, 0.00]^\top + [1.00, 0.00]^\top \circ [1.00, 0.00]^\top \tag{70}$$

$$= [1.00, 0.00]^\top \tag{71}$$

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

$$c_5 = [1.00, 0.00]^\top \tag{69}$$

- New hidden ($h_5$)

$$h_5 \tag{70}$$

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

$$c_5 = [1.00, 0.00]^\top \tag{69}$$

- New hidden ($h_5$)

$$h_5 = o_5 \circ \tanh(c_5) \tag{70}$$

$$\tag{71}$$

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

$$c_5 = [1.00, 0.00]^\top \tag{69}$$

- New hidden ($h_5$)

$$h_5 = o_5 \circ \tanh(c_5) \tag{70}$$

$$= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) \tag{71}$$

$$\tag{72}$$

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

$$c_5 = [1.00, 0.00]^\top \tag{69}$$

- New hidden ($h_5$)

$$\begin{aligned} h_5 &= o_5 \circ \tanh(c_5) && (70)\\ &= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) && (71)\\ &= [0.76, 0.00]^\top && (72) \end{aligned}$$

## Forward message at time step 5

| $f_5$ | $c_4$ | $i_5$ | $\tilde{c}_5$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[0.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_5$)

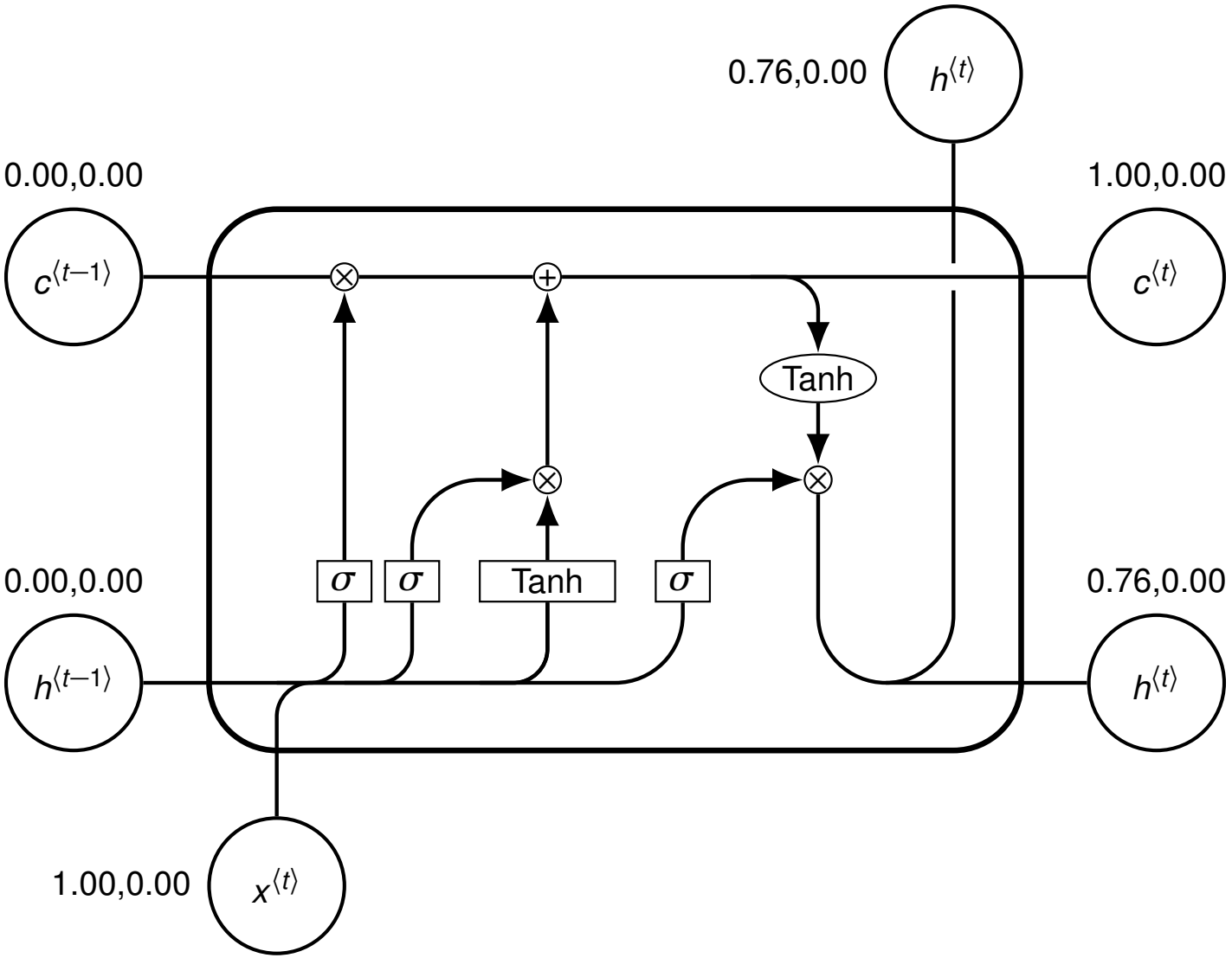
$$c_5 = [1.00, 0.00]^\top \tag{69}$$

- New hidden ($h_5$)

$$h_5 = [0.76, 0.00]^\top \tag{70}$$

- Prediction $y_5 = \text{softmax}(h_5) = 0$

## Summary at $t = 5$



## Input Gate at $t = 6$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top \qquad h^{(5)} = [0.76, 0.00]^\top$$

## Input Gate at $t = 6$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top \qquad h^{(5)} = [0.76, 0.00]^\top$$

$$\begin{aligned} i^{(6)} &= \sigma(W_{ii}x^{(6)} + b_{ii} + W_{hi}h^{(5)} + b_{hi}) && (71) \\ &= \sigma([0.00, 15.70]^\top) && (72) \\ &= [0.50, 1.00]^\top && (73) \end{aligned}$$

## Forget Gate at $t = 6$: $f^{(6)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top \qquad h^{(5)} = [0.76, 0.00]^\top$$

## Forget Gate at $t = 6$: $f^{(6)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top \qquad h^{(5)} = [0.76, 0.00]^\top$$

$$f^{(6)} = \sigma(W_{if} x^{(6)} + b_{if} + W_{hf} h^{(5)} + b_{hf}) \tag{74}$$

$$= \sigma([-30.00, -0.00]^\top) \tag{75}$$

$$= [0.00, 0.50]^\top \tag{76}$$

## Output Gate at $t = 6$: $o^{(6)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top \qquad h^{(5)} = [0.76, 0.00]^\top$$

## Output Gate at $t = 6$: $o^{(6)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top \qquad h^{(5)} = [0.76, 0.00]^\top$$

$$o^{(6)} = \sigma(W_{io}x^{(6)} + b_{io} + W_{ho}h^{(5)} + b_{ho}) \tag{77}$$

$$= \sigma([30.00, 30.00]^\top) \tag{78}$$

$$= [1.00, 1.00]^\top \tag{79}$$

## Memory Contribution at $t = 6$: $\tilde{c}^{(6)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top$$

$$h^{(5)} = [0.76, 0.00]^\top$$

## Memory Contribution at $t = 6$: $\tilde{c}^{(6)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(6)} = [0.00, 1.00]^\top \qquad h^{(5)} = [0.76, 0.00]^\top$$

$$\begin{aligned} \tilde{c}^{(6)} &= \tanh(W_{i\tilde{c}} x^{(6)} + b_{i\tilde{c}} + W_{h\tilde{c}} h^{(5)} + b_{h\tilde{c}}) && (80) \\ &= \tanh([0.00, 30.00]^\top) && (81) \\ &= [0.00, 1.00]^\top && (82) \end{aligned}$$

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

$$c_6 = f_6 \circ c_5 + i_6 \circ \tilde{c}_6 \tag{83}$$

(84)

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

$$c_6 = f_6 \circ c_5 + i_6 \circ \tilde{c}_6 \tag{83}$$

$$= [0.00, 0.50]^\top \circ [1.00, 0.00]^\top + [0.50, 1.00]^\top \circ [0.00, 1.00]^\top \tag{84}$$

$$\tag{85}$$

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

$$
\begin{aligned}
c_6 &= f_6 \circ c_5 + i_6 \circ \tilde{c}_6 && (83)\\
&= [0.00, 0.50]^\top \circ [1.00, 0.00]^\top + [0.50, 1.00]^\top \circ [0.00, 1.00]^\top && (84)\\
&= [0.00, 1.00]^\top && (85)
\end{aligned}
$$

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

$$c_6 = [0.00, 1.00]^\top \tag{83}$$

- New hidden ($h_6$)

$$h_6 \tag{84}$$

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

$$c_6 = [0.00, 1.00]^\top \tag{83}$$

- New hidden ($h_6$)

$$h_6 = o_6 \circ \tanh(c_6) \tag{84}$$

$$\tag{85}$$

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

$$c_6 = [0.00, 1.00]^\top \tag{83}$$

- New hidden ($h_6$)

$$h_6 = o_6 \circ \tanh(c_6) \tag{84}$$

$$= [1.00, 1.00]^\top \circ \tanh([0.00, 1.00]^\top) \tag{85}$$

$$\tag{86}$$

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

$$c_6 = [0.00, 1.00]^\top \tag{83}$$

- New hidden ($h_6$)

$$\begin{aligned} h_6 &= o_6 \circ \tanh(c_6) && (84)\\ &= [1.00, 1.00]^\top \circ \tanh([0.00, 1.00]^\top) && (85)\\ &= [0.00, 0.76]^\top && (86) \end{aligned}$$

## Forward message at time step 6

| $f_6$ | $c_5$ | $i_6$ | $\tilde{c}_6$ |
|---|---|---|---|
| $[0.00, 0.50]^\top$ | $[1.00, 0.00]^\top$ | $[0.50, 1.00]^\top$ | $[0.00, 1.00]^\top$ |

- Message forward ($c_6$)

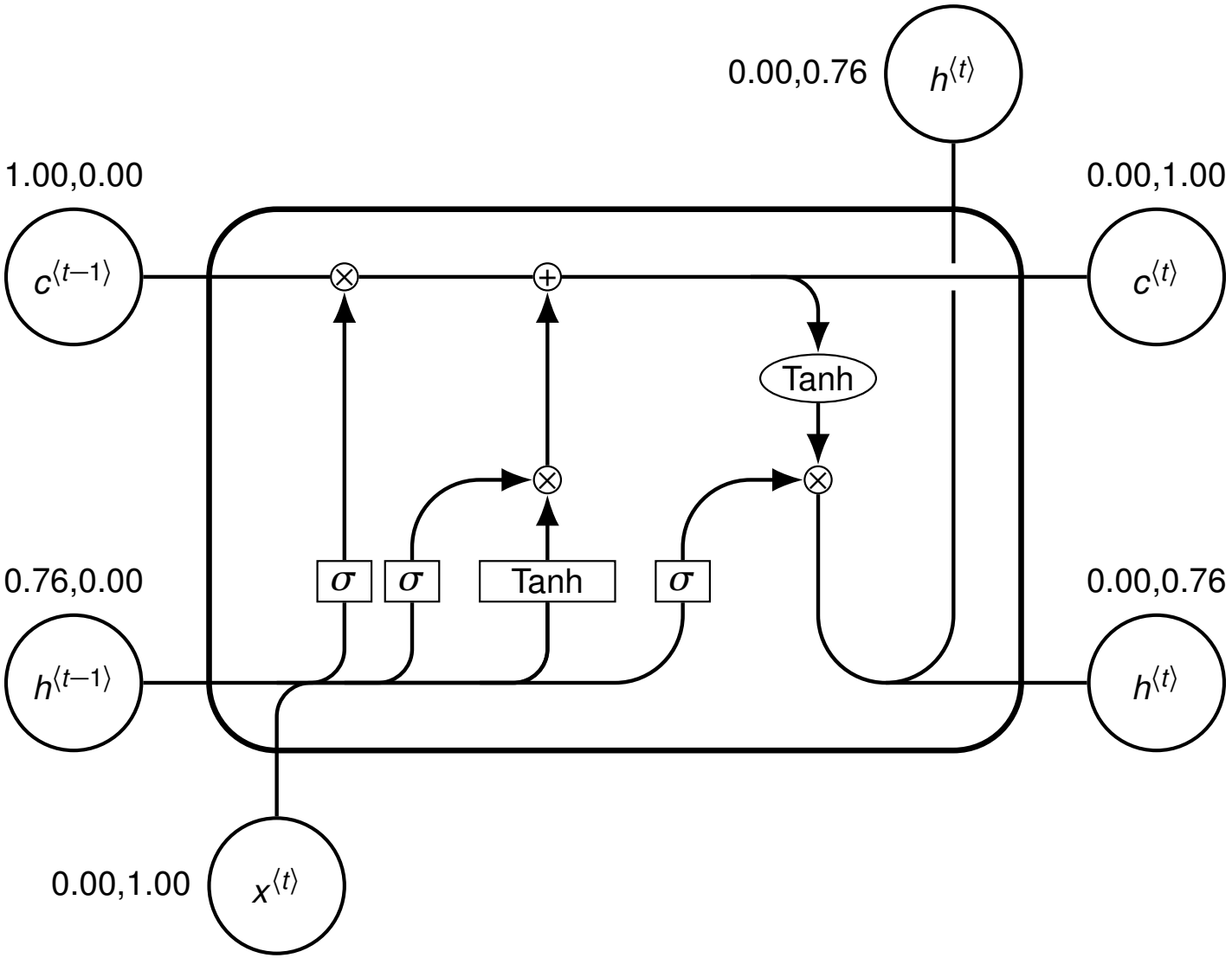
$$c_6 = [0.00, 1.00]^\top \tag{83}$$

- New hidden ($h_6$)

$$h_6 = [0.00, 0.76]^\top \tag{84}$$

- Prediction $y_6 = \text{softmax}(h_6) = 1$

## Summary at $t = 6$



## Input Gate at $t = 7$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top \qquad h^{(6)} = [0.00, 0.76]^\top$$

## Input Gate at $t = 7$: $i_1$

$$W_{ii} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ii} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hi} = \begin{bmatrix} 0.00 & 0.00 \\ 60.00 & 0.00 \end{bmatrix} \quad b_{hi} = \begin{bmatrix} 0.00 \\ -30.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top \qquad h^{(6)} = [0.00, 0.76]^\top$$

$$\begin{aligned} i^{(7)} &= \sigma(W_{ii}x^{(7)} + b_{ii} + W_{hi}h^{(6)} + b_{hi}) && (85) \\ &= \sigma([30.00, -30.00]^\top) && (86) \\ &= [1.00, 0.00]^\top && (87) \end{aligned}$$

## Forget Gate at $t = 7$: $f^{(7)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top \qquad h^{(6)} = [0.00, 0.76]^\top$$

## Forget Gate at $t = 7$: $f^{(7)}$

$$W_{if} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{if} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \quad W_{hf} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & -30.00 \end{bmatrix} b_{hf} = \begin{bmatrix} -30.00 \\ 0.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top \qquad h^{(6)} = [0.00, 0.76]^\top$$

$$f^{(7)} = \sigma(W_{if} x^{(7)} + b_{if} + W_{hf} h^{(6)} + b_{hf}) \tag{88}$$

$$= \sigma([-30.00, -22.85]^\top) \tag{89}$$

$$= [0.00, 0.00]^\top \tag{90}$$

## Output Gate at $t = 7$: $o^{(7)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top \qquad h^{(6)} = [0.00, 0.76]^\top$$

## Output Gate at $t = 7$: $o^{(7)}$

$$W_{io} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{io} = \begin{bmatrix} 30.00 \\ 30.00 \end{bmatrix} \quad W_{ho} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{ho} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top \qquad h^{(6)} = [0.00, 0.76]^\top$$

$$o^{(7)} = \sigma(W_{io}x^{(7)} + b_{io} + W_{ho}h^{(6)} + b_{ho}) \tag{91}$$

$$= \sigma([30.00, 30.00]^\top) \tag{92}$$

$$= [1.00, 1.00]^\top \tag{93}$$

## Memory Contribution at $t = 7$: $\tilde{c}^{(7)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top$$

$$h^{(6)} = [0.00, 0.76]^\top$$

## Memory Contribution at $t = 7$: $\tilde{c}^{(7)}$

$$W_{i\tilde{c}} = \begin{bmatrix} 30.00 & 0.00 \\ 0.00 & 30.00 \end{bmatrix} \quad b_{i\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} \qquad W_{h\tilde{c}} = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} \quad b_{h\tilde{c}} = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x^{(7)} = [1.00, 0.00]^\top \qquad h^{(6)} = [0.00, 0.76]^\top$$

$$\begin{aligned} \tilde{c}^{(7)} &= \tanh(W_{i\tilde{c}} x^{(7)} + b_{i\tilde{c}} + W_{h\tilde{c}} h^{(6)} + b_{h\tilde{c}}) && (94) \\ &= \tanh([30.00, 0.00]^\top) && (95) \\ &= [1.00, 0.00]^\top && (96) \end{aligned}$$

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

$$c_7 = f_7 \circ c_6 + i_7 \circ \tilde{c}_7 \tag{97}$$

(98)

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

$$c_7 = f_7 \circ c_6 + i_7 \circ \tilde{c}_7 \tag{97}$$

$$= [0.00, 0.00]^\top \circ [0.00, 1.00]^\top + [1.00, 0.00]^\top \circ [1.00, 0.00]^\top \tag{98}$$

$$\tag{99}$$

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

$$c_7 = f_7 \circ c_6 + i_7 \circ \tilde{c}_7 \tag{97}$$

$$= [0.00, 0.00]^\top \circ [0.00, 1.00]^\top + [1.00, 0.00]^\top \circ [1.00, 0.00]^\top \tag{98}$$

$$= [1.00, 0.00]^\top \tag{99}$$

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

$$c_7 = [1.00, 0.00]^\top \tag{97}$$

- New hidden ($h_7$)

$$h_7 \tag{98}$$

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

$$c_7 = [1.00, 0.00]^\top \tag{97}$$

- New hidden ($h_7$)

$$h_7 = o_7 \circ \tanh(c_7) \tag{98}$$

$$\tag{99}$$

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

$$c_7 = [1.00, 0.00]^\top \tag{97}$$

- New hidden ($h_7$)

$$h_7 = o_7 \circ \tanh(c_7) \tag{98}$$
$$= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) \tag{99}$$
$$\tag{100}$$

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

$$c_7 = [1.00, 0.00]^\top \tag{97}$$

- New hidden ($h_7$)

$$\begin{aligned} h_7 &= o_7 \circ \tanh(c_7) && (98)\\ &= [1.00, 1.00]^\top \circ \tanh([1.00, 0.00]^\top) && (99)\\ &= [0.76, 0.00]^\top && (100) \end{aligned}$$

## Forward message at time step 7

| $f_7$ | $c_6$ | $i_7$ | $\tilde{c}_7$ |
|---|---|---|---|
| $[0.00, 0.00]^\top$ | $[0.00, 1.00]^\top$ | $[1.00, 0.00]^\top$ | $[1.00, 0.00]^\top$ |

- Message forward ($c_7$)

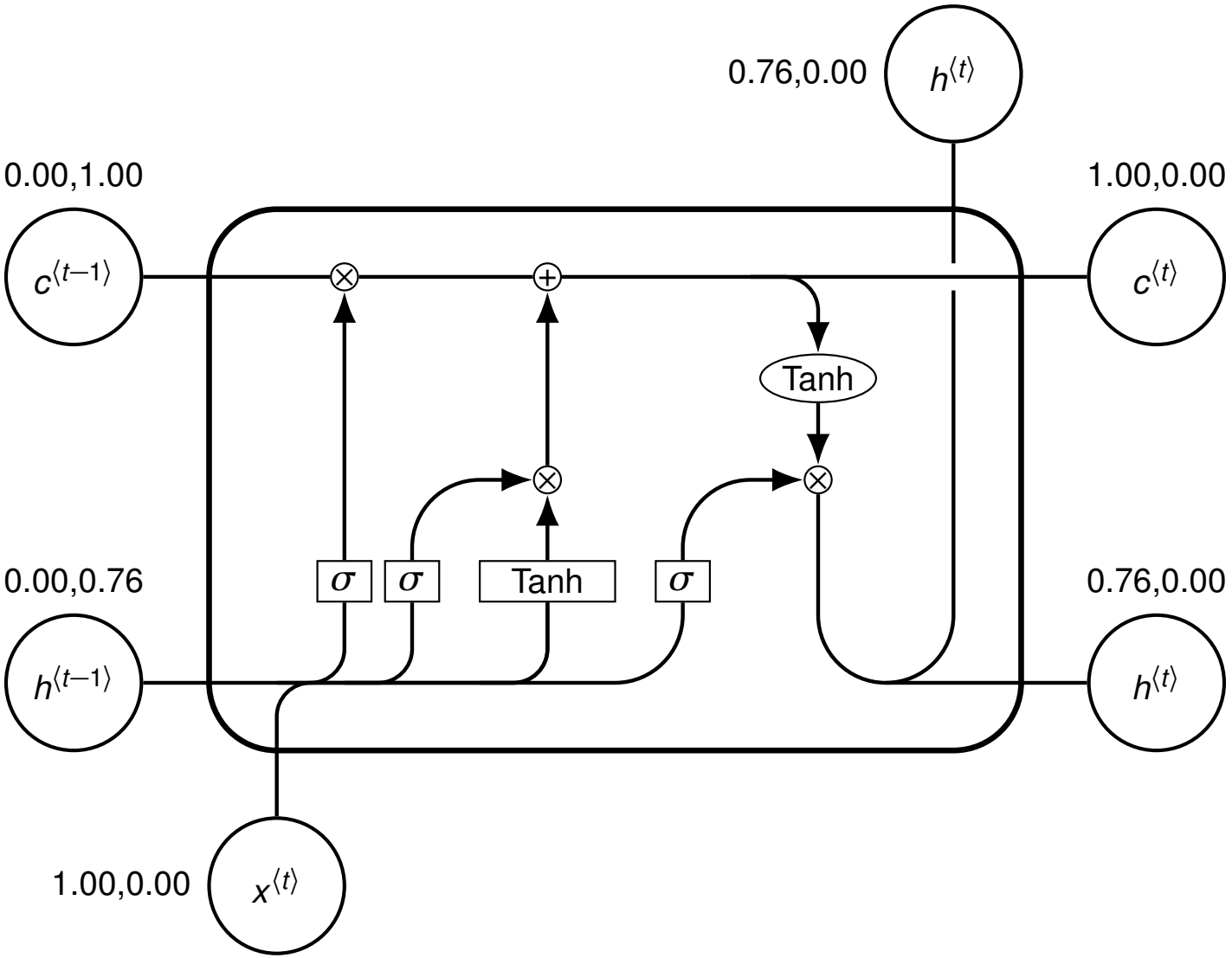
$$c_7 = [1.00, 0.00]^\top \tag{97}$$

- New hidden ($h_7$)

$$h_7 = [0.76, 0.00]^\top \tag{98}$$

- Prediction $y_7 = \text{softmax}(h_7) = 0$

## Summary at $t = 7$



## What's going on?

- What's the classification?
- What inputs are important?
- When can things be forgotten?
- How would other sequences be classified?