April 15 Quantum Error Correction

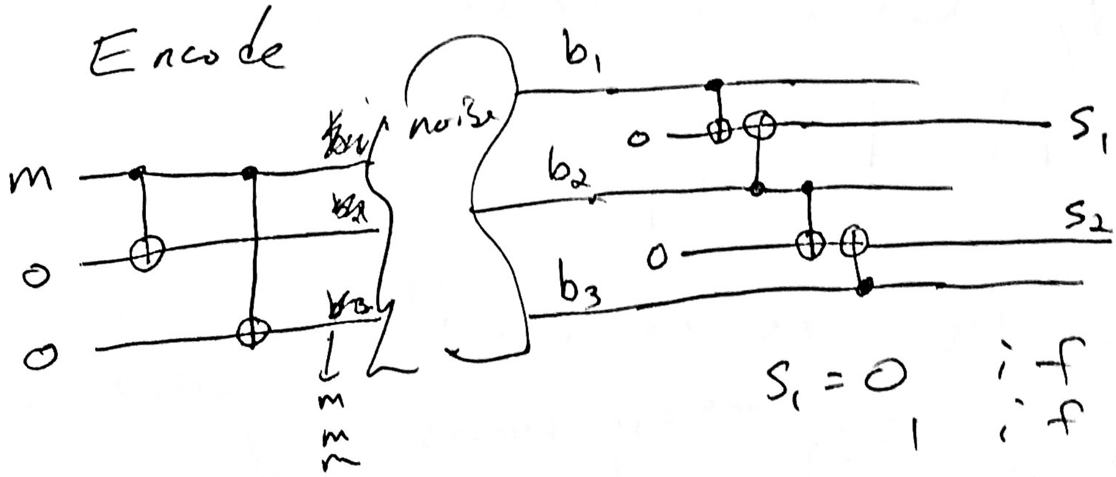
Last time: - need error correction for QC
- repetition code
$0 \to 000, \quad 1 \to 111$

Today: - more repetition code
- quantum noise models.

Announce: FCQ Next week.

Repetition code: encoding + decoding circuits.

Encode Decode



$s_1 = b_1 \oplus b_2$

$s_2 = b_2 \oplus b_3$

$s_1 = 0$ if $b_1 = b_2$
$\quad\;\; 1$ if $b_1 \neq b_2$

$s_2 = 0$ if $b_2 = b_3$
$\quad\;\; = 1$ if $b_2 \neq b_3$

Q: How to map back to original codeword?

A:

$s_1 = 0$, $s_2 = 0$ leave it	$s_1 = 1$, $s_2 = 0$ flip 1
$s_1 = 1$, $s_2 = 1$ flip 2	$s_1 = 0$, $s_2 = 1$ flip 3

3

Alternative description of repetition code

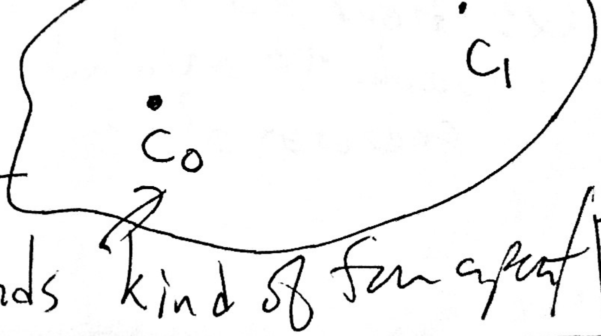
Parity Check Matrix:

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$c \in \{0,1\}^3$ is in R_3 iff

$$Mc = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ "syndromes"

$\{0,1\}^3$



Can't get from one to the other with 2 bit flips

Codewords kind of far apart

4

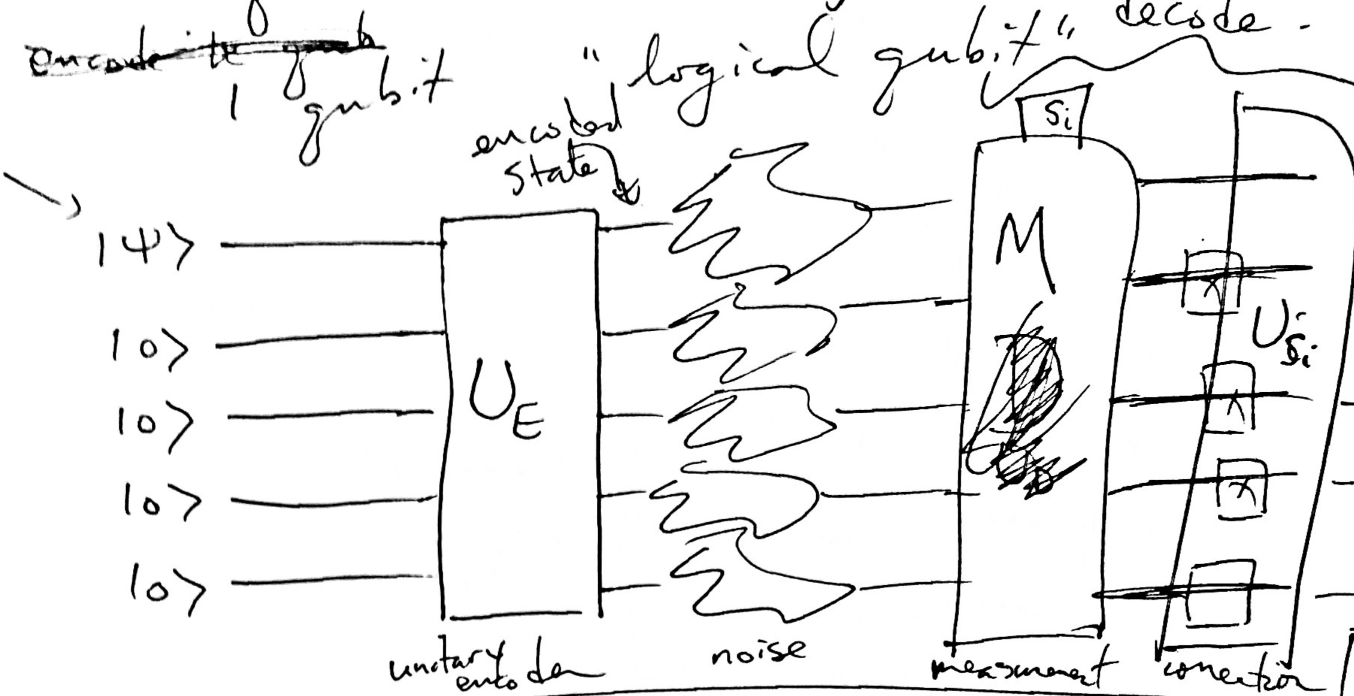
Quantum Error Correction

Basic picture:

n qubits "block length"

~~encode the qubit~~

1 qubit "logical qubit"



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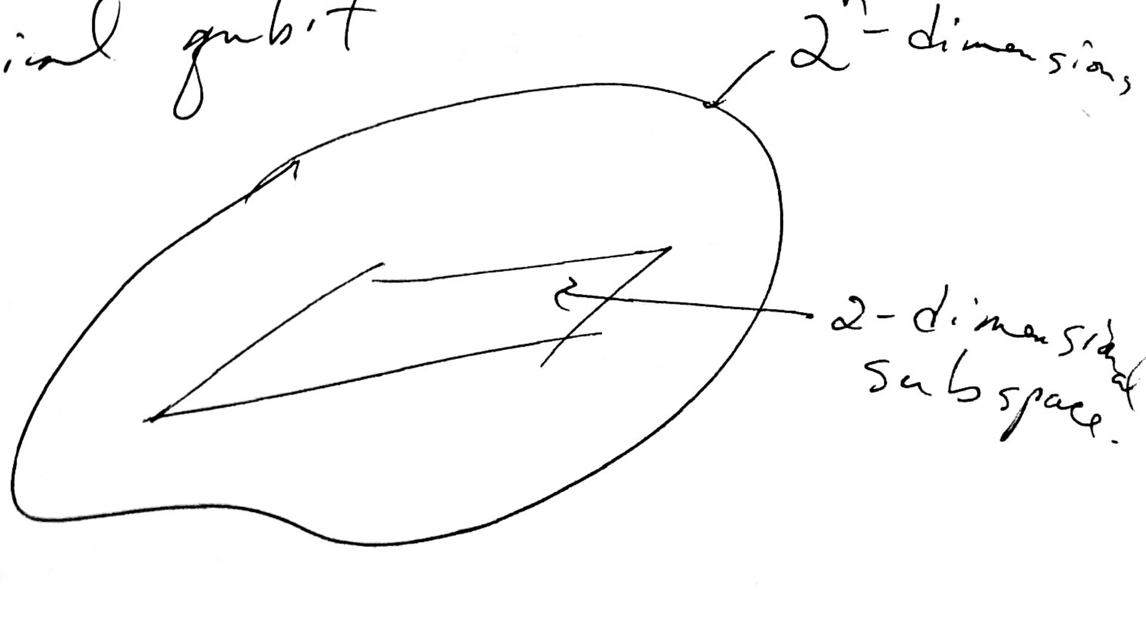
Quantum Error Correcting code

n qubits ~ 2^n-dim space

1 logical qubit



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What's the Noise model?

Good one:

on one qubit,

$$|\psi\rangle \xrightarrow{p} X|\psi\rangle$$
$$|\psi\rangle \xrightarrow{p} Y|\psi\rangle$$
$$|\psi\rangle \xrightarrow{p} Z|\psi\rangle$$
$$|\psi\rangle \xrightarrow{1-3p} |\psi\rangle$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = iXZ$$
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Why?

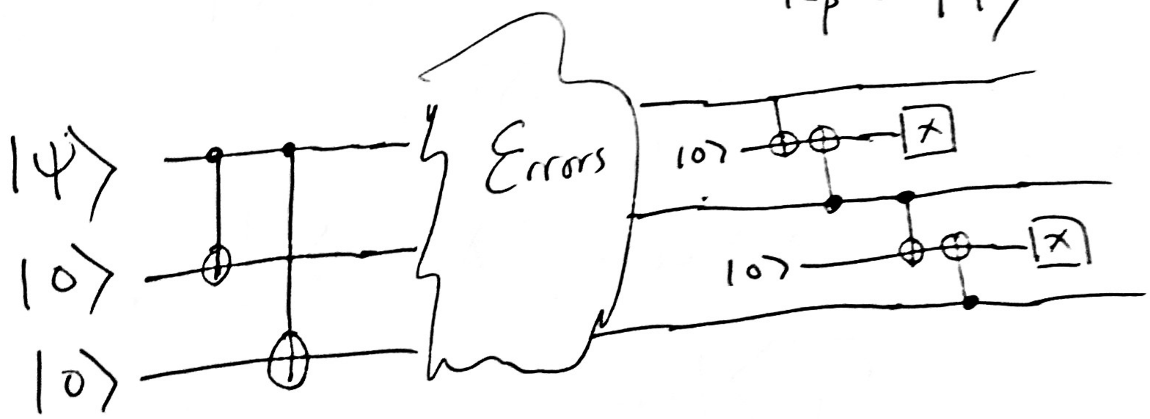
1) analogous to bit flips, but quantum.
2) $\mathbb{I}, X, Y, Z$ span space of 2×2 matrices, so good approximation to any nearly perfect op. e.g., covers $U_\epsilon = \cos\epsilon\,\mathbb{I} + i\sin\epsilon\,X$

7

First try: only correct X errors.

i.e.,

$$|\psi\rangle \xrightarrow{p} X|\psi\rangle$$
$$|\psi\rangle \xrightarrow{1-p} |\psi\rangle$$

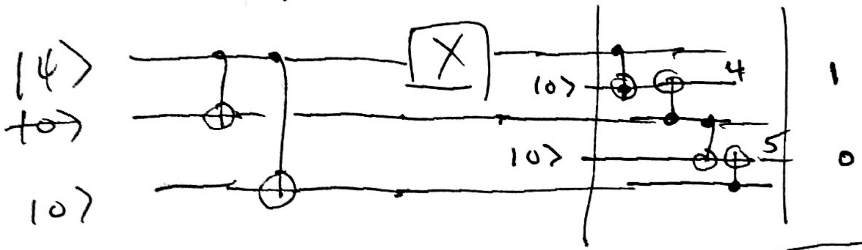


$$|\psi\rangle \rightarrow |\bar{\psi}\rangle = \alpha|000\rangle + \beta|111\rangle$$
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Cases: $\quad 0$ errors. get $0,0$, leave it alone.

case: $\quad 1$ error.

$$\alpha|000\rangle + \beta|111\rangle \to \alpha|100\rangle_{123} + \beta|011\rangle$$



$$\left(\alpha|100\rangle + \beta|011\rangle\right)|0\rangle_4|0\rangle_5 = \alpha|100\rangle|00\rangle + \beta|011\rangle|00\rangle$$

$$\to \alpha|100\rangle_{123}|10\rangle_{45} + \beta|011\rangle|10\rangle_{45} = \left(\alpha|100\rangle + \beta|011\rangle\right)|10\rangle_{45}$$

9

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measure 4 + 5. get $\quad s_1 = 1 \Rightarrow 1+2$ disagree

$s_2 = 0 \Rightarrow 2+3$ agree.

$\downarrow$

flip 1 with X

$$\cancel{\alpha|100\rangle|10\rangle} \quad \left(\alpha|100\rangle + \beta|011\rangle\right)|10\rangle_{45}$$

$$\to \left(\alpha\, X|1\rangle|00\rangle + \beta\, X|0\rangle|11\rangle\right)|10\rangle_{45}.$$

$$= |\Psi\rangle|10\rangle$$

2 or 3 errors: get $\alpha|111\rangle + \beta|000\rangle$ wrong

but prob $\approx 3p^2 \ll p$.

Note: - we measured the syndromes, but they didn't tell us about the state $|\psi\rangle$.

- both terms ~~$\alpha|\psi\rangle$ and~~ $(\alpha|011\rangle + \beta|100\rangle)|10\rangle$ same state

- this is critical for QEC.

- we want to learn what error happened, not what the state is.

if time: describe decoding as measuring $\begin{cases} Z_1 Z_2 1 \\ 1\ Z_2 Z_3 \end{cases}$.

which eigenvalues do these have..?

like parity check matrix

Next class:

- we covered X-type errors. what about Y and Z?
- we'll need to use even more qubits (9 qubit code)
- later: "stabilizer formalism" quantum parity checks.