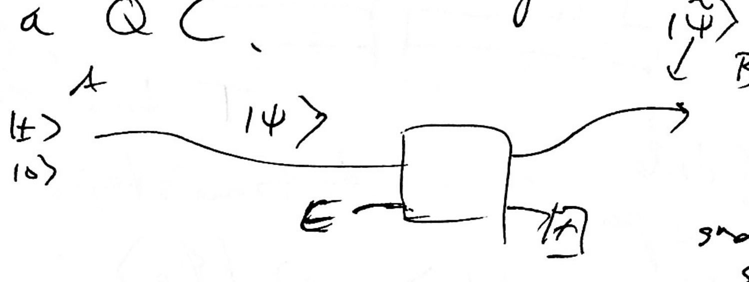
# Quantum Error Correction

HW 2 weeks

Why? quantum states are delicate

Any ~~...~~ interaction with it's environment will change the state of a QC.

Recall




small interaction, small change.

$\uparrow$ $|\uparrow\rangle$

O $|\downarrow\rangle$

2 level system in lab.

interacts with uncontrolled dofs changes the state.

Also: gates are imperfect.

CNOTs ~ ~~...~~ <99% SC

~99.9% ions.

contrast: ~$10^{-15}$ for classical computers.

99.9%: not good enough $10^{-15}$ is. why?

how good is good enough.

DISCUSS: ( pause to discuss.

answer: you should get the right answer to your computation

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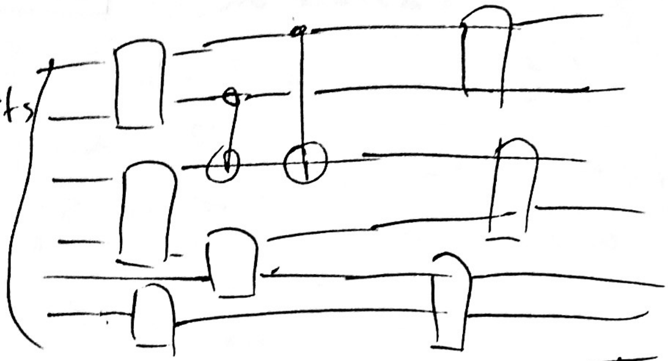
n bits



- T ~~time steps~~ gates !

separate into time steps:

initial state

ideal: $|0\rangle^{\otimes n} \longrightarrow |\phi_1\rangle \longrightarrow |\phi_2\rangle \cdots\cdots \longrightarrow |\phi_T\rangle$

real: $|\tilde{0}\rangle^{\otimes n} \longrightarrow |\tilde{\phi}_1\rangle \longrightarrow |\tilde{\phi}_2\rangle \cdots\cdots \longrightarrow |\tilde{\phi}_T\rangle$

good enough if $|\tilde{\phi}_T\rangle \approx |\phi_T\rangle$



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let ideal: $|\varphi_t\rangle = U_t|\varphi_{t-1}\rangle$ $1-\epsilon$ amplitude.

"error rate" $\epsilon$, but do $\tilde{U}_t$ instead. $\epsilon$ amplitude

$$\tilde{U}_t|\varphi_{t-1}\rangle = |\varphi_t\rangle + |E_t\rangle$$

$$|\tilde{\varphi}_1\rangle = |\varphi_1\rangle + |E_1\rangle$$

$$|\tilde{\varphi}_2\rangle = \tilde{U}_2(|\tilde{\varphi}_1\rangle) = \tilde{U}_2|\varphi_1\rangle + |E_1\rangle$$

$$= |\varphi_2\rangle + |E_2\rangle + \tilde{U}_2|E_1\rangle$$

$$\Rightarrow |\tilde{\varphi}_T\rangle = |\varphi_T\rangle + |E_T\rangle + \tilde{U}_T|E_{T-1}\rangle + \tilde{U}_T\tilde{U}_{T-1}|E_{T-2}\rangle + \cdots + \tilde{U}_T\tilde{U}_{T-1}\cdots\tilde{U}_2|E_1\rangle$$

Triangle ineq:

$$\| |\tilde{\varphi}_T\rangle - |\varphi_T\rangle \| \leq \| |E_T\rangle \| + \| |E_{T-1}\rangle \| + \dots + \| |E_1\rangle \|$$

$$\leq T\epsilon$$

okay as long as $T\epsilon \ll 1$.

$\Rightarrow \epsilon \ll \frac{1}{T}$

For QC, $\epsilon \approx .01$ $\Rightarrow T \ll 100$

For classical $\epsilon \approx 10^{-15}$

need $T \ll 10^{15}$

5 GHz = $5 \times 10^9$ gates per second,

$\Rightarrow 10^6$ seconds till an error (if you waited!)

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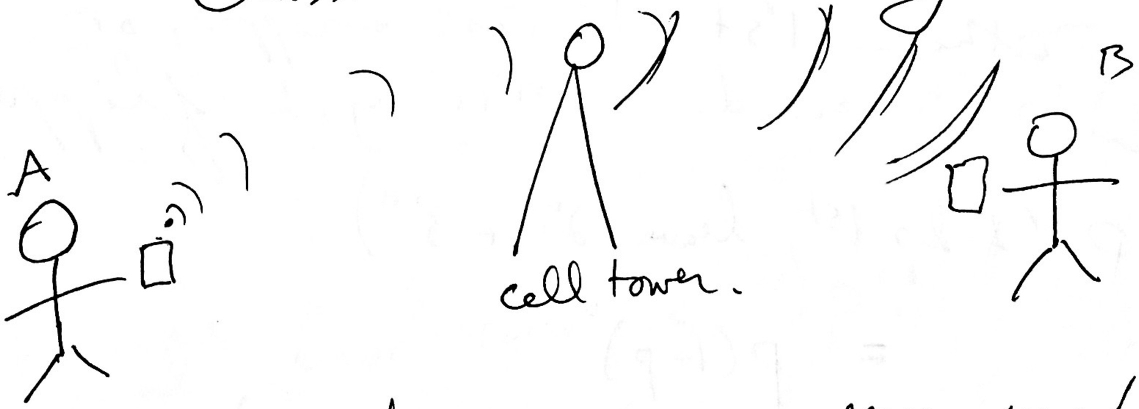
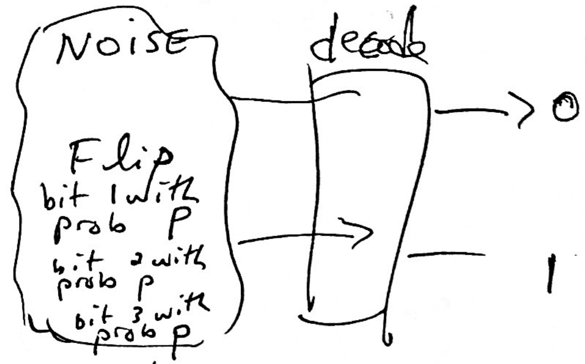
Classical Error Correcting Codes.




in communication scenarios, error correction is critical.

Simplest classical error correcting code: repetition.

| message | codeword |
| --- | --- |
| 0 | 000 |
| 1 | 111 |




$p \ll \frac{1}{2}$

Alice                Bob

How does this help?

__Question:__ you're Bob. You get message 011 what do you decode to? 0 or 1?



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011: could be either from 000 or 111.

2 ~~stories~~ your explanations: either 1'st bit got flipped, or second 2 bits got flipped.

more likely → Pr(flip 1st, leave 2nd + 3rd)

$$= p(1-p)^2$$

less likely → Pr( ~~flip~~ leave 1st, flip next 2) $= (1-p)p^2$.

So, adopt majority vote decoding:

| | | | |
| --- | --- | --- | --- |
| 000 | → 000 | 011 | → 111 |
| 001 | → 000 | 101 | → 111 |
| 010 | → 000 | 110 | → 111 |
| 100 | → 000 | 111 | → 111 |

Does it help.

1 bit: error with prob $p$.

3 bit rep code: ~~[illegible]~~ error in message if 2 or more bits are flipped.

message error prob

Pr(2 or more bits flipped)

= Pr(1+2 flipped, 3 not)
+ Pr(1+3 flipped, 2 not)
+ Pr(2+3 flipped, 1 not)
+ Pr(123 flipped)

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$$= p^2(1-p) + p^2(1-p) + p^2(1-p) + p^3 = 3p^2(1-p)+p^3$$

$$= 3p^2 - 2p^3 < 3p^2$$

"Logical" error rate $< 3p^2$.

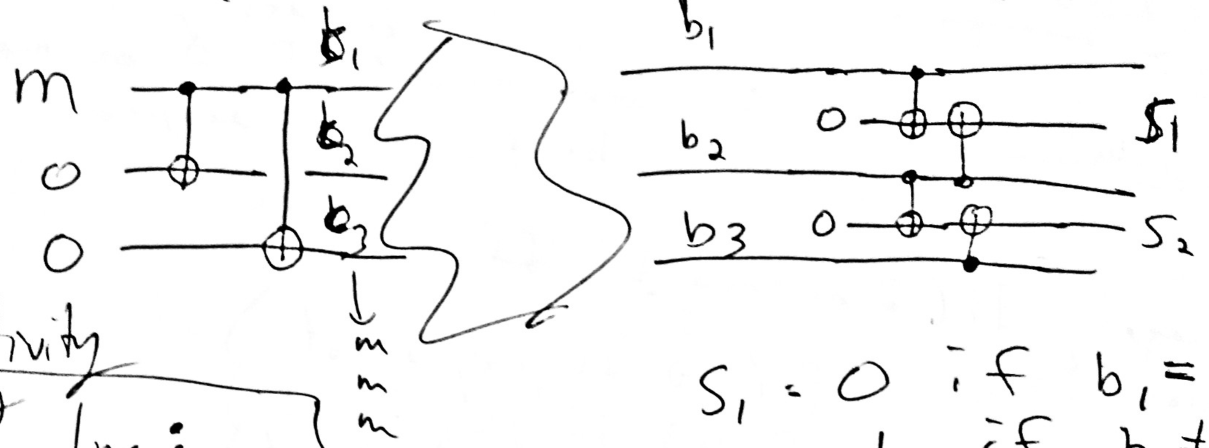
if $p$ small enough, $3p^2 \ll p$.

~~"3 bit co~~

"block length 3, 1 logical bit"

~~How to en~~ Encoding + decoding circuits.

Encode | Decode




~~Activity~~

Activity Decoding:

Discuss, rule to what is it terms of $s_1 + s_2$?

How to get back to original code word?

~~Q: is 1st bit right?~~

$S_1 = 0$ if $b_1 = b_2$

$\quad\;\; 1$ if $b_1 \neq b_2$

$S_2 = 0$ if $b_2 = b_3$

$\quad\;\; 1$ if $b_2 \neq b_3$

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answer

| | |
|---|---|
| $s_1 = 0$, $s_2 = 0$ => leave it | $s_1 = 1$, $s_2 = 0$ => flip 1 |
| $s_1 = 1$, $s_2 = 1$ => flip 2 | $s_1 = 0$, $s_2 = 1$ => flip 3 |

alternative description of repetition code.

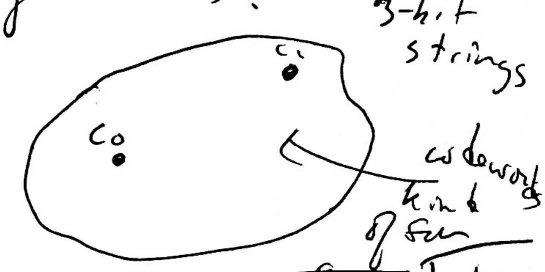
Parity Check Matrix: ~~$c \in C$ if~~ ~~$c \in \{0,1\}^3$ if~~

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$c \in \{0,1\}^3$ is in $R_3$ iff.

"syndromes."

$$Mc = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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## Next Time:

- can we do this for Q codes? seems hard, since $|\psi\rangle \mapsto |\psi\rangle|\psi\rangle|\psi\rangle$.
  no cloning precludes repetition
- how do we avoid collapsing logical qubit?
- HW due in 2 weeks