3090 Deep Breath Class (Bit of a review).

	Physics (kets)		Math (vectors
qubit	~~basis:~~ $\|0\rangle$	$\longleftrightarrow$	$\begin{pmatrix}1\\0\end{pmatrix}$
Basis for 2D space	$\|1\rangle$	$\longleftrightarrow$	$\begin{pmatrix}0\\1\end{pmatrix}$
State	$\alpha\|0\rangle+\beta\|1\rangle$	$\longleftrightarrow$	$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix}+\beta($

~~inner product;~~
row vs column vectors

$\langle\psi| = |\psi\rangle^{\dagger} \longleftrightarrow [\alpha^* \ \beta^*] = \begin{bmatrix}\alpha\\\beta\end{bmatrix}^{\dagger}$

$|\psi\rangle = \alpha|0\rangle+\beta|1\rangle$

$\langle\psi| = \alpha^*\langle 0|+\beta^*\langle 1|$

inner product

$(|\psi_1\rangle, |\psi_2\rangle)$ $\qquad (v_1, v_2)$

$= \langle\psi_1|\psi_2\rangle \longleftrightarrow = v_1^{\dagger} v_2, \ v_1 \cdot$

$|\psi_1\rangle = \alpha_1|0\rangle+\beta_1|1\rangle$

$|\psi_2\rangle = \alpha_2|0\rangle+\beta_2|1\rangle$

$\langle\psi_1|\psi_2\rangle \equiv (\alpha_1^*\langle 0|+\beta_1^*\langle 1|)(\alpha_2|0\rangle+\beta_2|1\rangle)$

$= \alpha_1^*\alpha_2+\beta_1^*\beta_2$

Physics.

Ma

$[\alpha_1^* \ \beta_1^*]\begin{bmatrix}\alpha_2\\\beta_2\end{bmatrix}$

$= \alpha_1^*\alpha_2+\beta_1^*\beta_2.$

Tensor Product

~~2 qubits~~

tensor product $\otimes$ of vectors $\otimes$

Physics

$|\psi_1\rangle \otimes |\psi_2\rangle$

$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$

$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$

Math

$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$V \otimes W = \begin{bmatrix} v_1 w_1 \\ v_1 w_2 \\ v_2 w_1 \\ v_2 w_2 \end{bmatrix} = \begin{bmatrix} v_1 \vec{w} \\ v_2 \vec{w} \end{bmatrix}$

$|\psi_1\rangle \otimes |\psi_2\rangle$

$= (\alpha_1|0\rangle + \beta_1|1\rangle) \otimes (\alpha_2|0\rangle + \beta_2|1\rangle)$

FOIL $= \alpha_1\alpha_2 |0\rangle\otimes|0\rangle + \alpha_1\beta_2|0\rangle\otimes|1\rangle + \beta_1\alpha_2|1\rangle\otimes|0\rangle + \beta_1\beta_2|1\rangle\otimes|1\rangle \longleftrightarrow \begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \end{bmatrix}$

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What is $|0\rangle\langle 1|$?

what kind of object

$U = \sum_{ij} u_{ij} |i\rangle\langle j|$

$V = \sum_{ij} v_{ij} |i\rangle\langle j|$

$U \otimes V = \left(\sum_{i_1 j_1} u_{i_1 j_1} |i_1\rangle\langle j_1|\right) \otimes \left(\sum_{i_2 j_2} v_{i_2 j_2} |i_2\rangle\langle j_2|\right)$

$= \sum_{i_1 j_1} \sum_{i_2 j_2} u_{i_1 j_1} v_{i_2 j_2} |i_1\rangle\langle j_1| \otimes |i_2\rangle\langle j_2|$

$= \sum_{i_1 j_1 i_2 j_2} u_{i_1 j_1} v_{i_2 j_2} (|i_1\rangle \otimes |i_2\rangle)(\langle j_1| \otimes \langle j_2|)$

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Tensor Product of Matrices

Math

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{pmatrix}$

Claim

5 mins

$$(U \otimes V)(|\psi\rangle \otimes |\varphi\rangle) = (U|\psi\rangle) \otimes (V|\varphi\rangle)$$

★ Task ~~easier~~ to write out LHS. let $|\psi\rangle = \sum_x \psi_x |x\rangle$, $|\varphi\rangle = \sum_x \varphi_x |x\rangle$ write out RHS.

LHS: $(U \otimes V)(|\psi\rangle \otimes |\varphi\rangle) = \left[\sum_{\substack{i_1 j_1 \\ i_2 j_2}} u_{i_1 j_1} v_{i_2 j_2} (|i_1\rangle \otimes |i_2\rangle)(\langle j_1| \otimes \langle j_2|)\right] \left[\sum_{x,y} \psi_x \varphi_y |x\rangle \otimes |y\rangle\right]$ yay

Do RHS first!

$$= \sum_{\substack{i_1 j_1 \\ i_2 j_2}} u_{i_1 j_1} \psi_{j_1} v_{i_2 j_2} \varphi_{j_2} |i_1\rangle \otimes |i_2\rangle$$

~~do RHS~~

$\boxed{\text{RHS}}$

$$U|\psi\rangle = \left(\sum_{\substack{i_1 j_1 \\ x}} u_{i_1 j_1} |i_1\rangle\langle j_1|\right)(\psi_x |x\rangle)$$

$$= \sum_{\substack{i_1 j_1 \\ x}} u_{i_1 j_1} \psi_x |i_1\rangle \underbrace{\langle j_1|x\rangle}_{\delta_{j_1 x}}$$

only keep ter where $j_1 = x$.

$$= \sum_{i_1 j_1} u_{i_1 j_1} \psi_{j_1} |i_1\rangle$$

$$V|\varphi\rangle = \sum_{i_2 j_2} v_{i_2 j_2} \varphi_{j_2} |i_2\rangle$$

$$U|\psi\rangle \otimes V|\varphi\rangle = \sum_{\substack{i_1 j_1 \\ i_2 j_2}} u_{i_1 j_1} \psi_{j_1} v_{i_2 j_2} \varphi_{j_2} |i_1\rangle \otimes |i_2\rangle$$