Deutsch's problem revisited!

PHYS/CSCI 3090

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Come see us!

- Alexandra Kolla/ Graeme Smith: Friday 3:00-4:00 pm, JILA X317.
- Ariel Shlosberg: Tu/Th 2:00-4:00pm, DUANG2B90 (physics help room)
- Steven Kordonowy: Th 11am-12pm, ECAE 124.
- Matteo Wilczak: Wednesday, 1-2pm, DUANG2B90 (physics help room)

Email

- Email is not a very good way to get your technical questions answered.
- We *try* to respond to emails, but can't guarantee a quick and thorough response. Most questions require interaction to clarify where your understanding is and how to best answer your question.
- The best way to get your questions answered is to come to one of the 7 hours of contact time we have scheduled each week.

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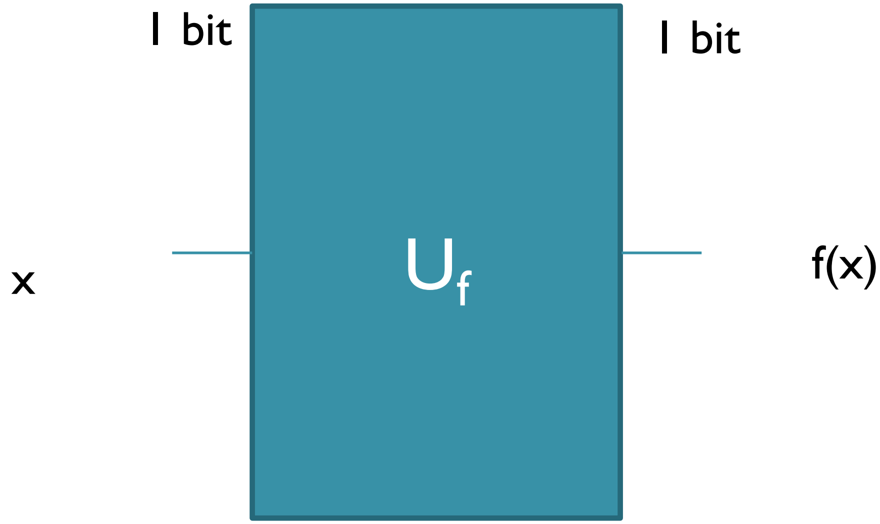
Last class:

Vectors, bras, kets, tensor products of vectors, tensor product of matrices

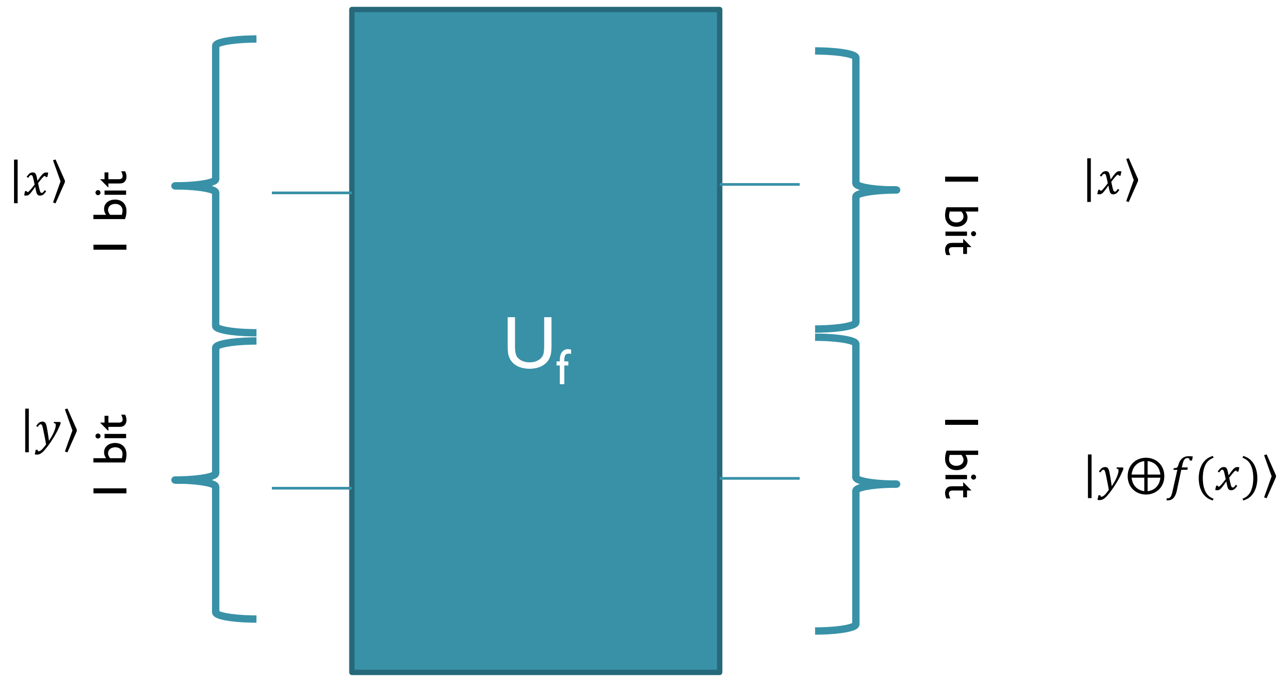
Today: Deutsch’s algorithm!

The setup: classical

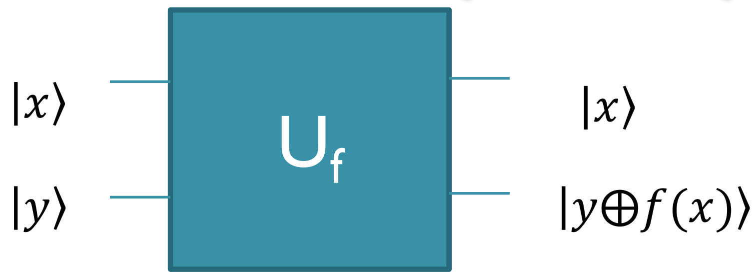


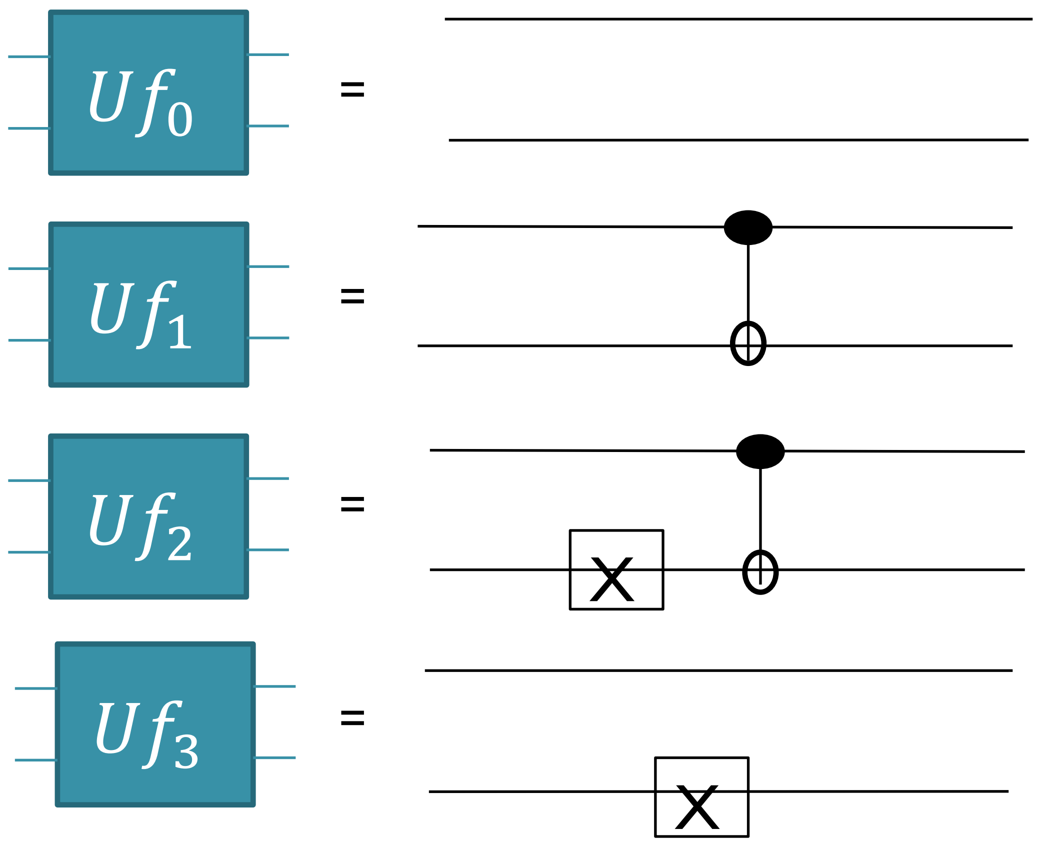
- Both input and output registers contain one bit.
- Functions f that take one bit to one bit

The setup: quantum



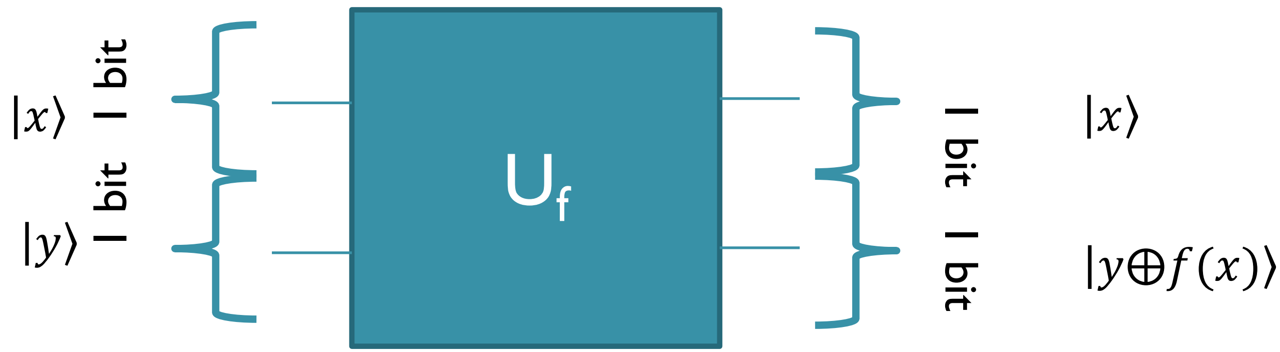
The setup, in quantum





$f(0)$	$f(1)$
0	0
0	1
1	0
1	1

The setup, in quantum

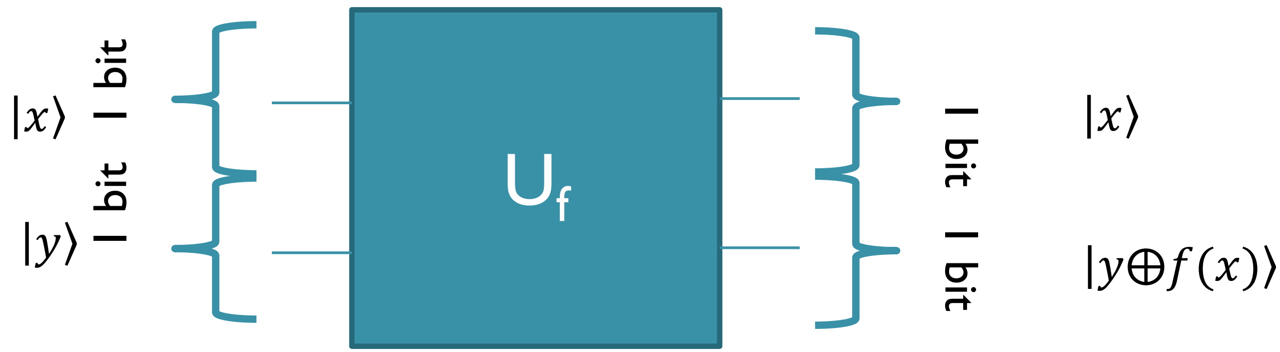


- U_f is one of 4 possibly functions (table).
- We are given a black box that calculates one of the 4 f's by performing

$$U_f|x\rangle \otimes |y\rangle = |x\rangle \otimes |y \oplus f(x)\rangle$$

- We are guaranteed that the black box performs one of the four computations, but we are not told which one.
- We are allowed to use the box only once, what can we learn about f?

The setup, in quantum



- Deutsch's problem

We want to learn if f is constant (f(0)=f(1), satisfied by f_0 and f_3) or not, with one application of black box.

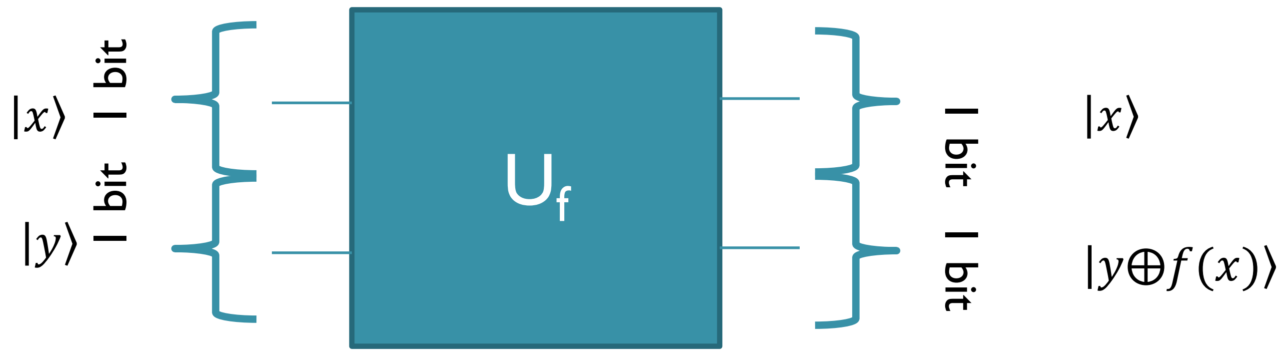
Classically

- We want to learn if f is constant ($f(0)=f(1)$, satisfied by f_0 and f_3) or not, with one application of black box.

- With a classical computer, we can either learn the value $f(0)$ or $f(1)$, so we can learn whether the function is one of (f_0, f_1) with $f(0)=0$ or (f_2, f_3) with $f(0)=1$.

- A classical computer needs two queries to U_f to determine if it is constant or not!

The setup, in quantum



- With a quantum computer we can do better! Without learning any information about the values of f(0) or f(1), we can determine with one application of the black box if f is constant or not.

Who cares?

- Well, maybe an alien ship arrived on earth and gave us one day to decide whether some complicated function is balanced or constant. They will vaporize us if we give the wrong answer!
- Even worse, it takes 23 hrs for us to compute the function! We can only evaluate it once before ZAP!
- Therefore, a quantum computer could save the world.

Just one query
Attempt 1: Superposition

- We could try preparing the input register in superposition of 0 and 1.

$$U_f(H \otimes I)(|0\rangle \otimes |0\rangle) = \frac{1}{\sqrt{2}}|0\rangle|f(0)\rangle + \frac{1}{\sqrt{2}}|1\rangle|f(1)\rangle$$

We can measure, and get either 0, f(0) or 1, f(1) but no improvement over classical

$$U_f(H \otimes I)(|0\rangle \otimes |0\rangle)$$

Just one query
Attempt 1: Superposition

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Attempt 1: Superposition

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We can measure, and get either 0, f(0) or 1, f(1) but no improvement over classical

Just one query
Attempt 2: Superposition |+>|+>

- We can pre and post process the state to yield what we want

$$U_f(\mathrm{H} \otimes H)(X \otimes X)(|0\rangle \otimes |0\rangle)$$

Superposition ++

- We can pre and post process the state to yield what we want

$$(\mathrm{H} \otimes H)(X \otimes X)(|0\rangle \otimes |0\rangle) = (\mathrm{H} \otimes H)(|1\rangle \otimes |1\rangle) =$$

$$(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle)\ (\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle)$$

$$= \frac{1}{2}(|0\rangle|0\rangle - |1\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|1\rangle)$$

Apply U_f

- We can pre and post process the state to yield what we want

$$U_f(H \otimes H)(X \otimes X)(|0\rangle \otimes |0\rangle)$$

$$= \frac{1}{2} U_f(|0\rangle|0\rangle - |1\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|1\rangle)$$

$$= \frac{1}{2}(U_f|0\rangle|0\rangle - U_f|1\rangle|0\rangle - U_f|0\rangle|1\rangle + U_f|1\rangle|1\rangle)$$

$$= \frac{1}{2}(|0\rangle|f(0)\rangle - |1\rangle|f(1)\rangle - |0\rangle|1 \oplus f(0)\rangle$$

$$+ |1\rangle|1 \oplus f(1)\rangle)$$

Apply U_f

$$|0\rangle|f(0)\rangle - |1\rangle|f(1)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|1\oplus f(1)\rangle$$

- Case 1: f(0)=f(1), output state is

$$|0\rangle|f(0)\rangle - |1\rangle|f(0)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|1\oplus f(0)\rangle$$
$$= (|0\rangle - |1\rangle)|f(0)\rangle - (|0\rangle - |1\rangle)\,|1\oplus f(0)\rangle$$
$$= (|0\rangle - |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$

Apply U_f

$|0\rangle|f(0)\rangle$ - $|1\rangle|f(1)\rangle$ - $|0\rangle|1\oplus f(0)\rangle$ +$|1\rangle|1\oplus f(1)\rangle$

- Case 2: f(0)≠f(1), $f(1) = |1\oplus f(0)\rangle$

output state is

$|0\rangle|f(0)\rangle$ - $|1\rangle|1\oplus f(0)\rangle$- $|0\rangle|1\oplus f(0)\rangle$ +$|1\rangle|f(0)\rangle$

= $(|0\rangle + |1\rangle)|f(0)\rangle - (|0\rangle + |1\rangle)\,|1\oplus f(0)\rangle$

= $(|0\rangle + |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$

Apply U_f

$$|0\rangle|f(0)\rangle - |1\rangle|f(1)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|1\oplus f(1)\rangle$$

- Case 1: f(0)=f(1), output state is

$$(|0\rangle - |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$

- Case 2: f(0) $\neq$ f(1), output state is

$$(|0\rangle + |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$

Apply U_f

$$|0\rangle|f(0)\rangle - |1\rangle|f(1)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|1\oplus f(1)\rangle$$

- Case 1: f(0)=f(1), output state is
  $$(|0\rangle - |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$
- Case 2: f(0) $\neq$ f(1), output state is
  $$(|0\rangle + |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$
- Apply H to the *input register*

Apply U_f

$$|0\rangle|f(0)\rangle - |1\rangle|f(1)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|1\oplus f(1)\rangle$$

- Case 1: f(0)=f(1), output state is

$$(|0\rangle - |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$

After applying H:

$$(H\otimes 1)(|0\rangle - |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$
$$=|1\rangle(|f(0)\rangle - |1\oplus f(0)\rangle)$$

- Apply H to the *input register*

Finally

$|0\rangle|f(0)\rangle$ - $|1\rangle|f(1)\rangle$ - $|0\rangle|1 \oplus f(0)\rangle$ +$|1\rangle|1 \oplus f(1)\rangle$

- Case 2: f(0) $\neq$ f(1), output state is

$$(|0\rangle + |1\rangle)(|f(0)\rangle - |1 \oplus f(0)\rangle)$$

After applying H:

$$(H \otimes 1)(|0\rangle + |1\rangle)(|f(0)\rangle - |1 \oplus f(0)\rangle)$$
$$=|0\rangle(|f(0)\rangle - |1 \oplus f(0)\rangle)$$

- Measure the *input register* t decide if f is constant (get $|1\rangle$), or not (get $|0\rangle$),

Uncertainty principle

$$|0\rangle|f(0)\rangle - |1\rangle|f(1)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|1\oplus f(1)\rangle$$

- Case 2: f(0) $\neq$ f(1), output state is

$$(|0\rangle + |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$

After applying H:

$$(H \otimes 1)(|0\rangle + |1\rangle)(|f(0)\rangle - |1\oplus f(0)\rangle)$$
$$= |0\rangle(|f(0)\rangle - |1\oplus f(0)\rangle)$$

- However, we learn nothing about output, since it is in uniform superposition of both values!

The whole operation

- $(\mathrm{H} \otimes 1)\, \mathrm{U_f}(\mathrm{H} \otimes H)(X \otimes X)(|0\rangle \otimes |0\rangle) =$
- Either $|1\rangle \frac{1}{\sqrt{2}}(|f(0)\rangle - |1 \oplus f(0)\rangle), \mathrm{f}(0) = \mathrm{f}(1)$
- Or $|0\rangle \frac{1}{\sqrt{2}}(|f(0)\rangle - |1 \oplus f(0)\rangle), \mathrm{f}(0) \neq \mathrm{f}(1)$
- Instead of querying the values of $f(0), f(1)$ algorithm queries the XOR $\mathrm{f}(0) \oplus f(1)$

Deutsch problem

- Fine, but who really cares? This is basically a quantum computing parlor trick. I don't care about saving myself one query!
- The idea----querying in superposition and putting the answer into the **input** register---is much broader and more powerful.
- There is a straight line from Deutsch's problem to Shor's factoring algorithm. We're going to follow it.

Monday: Bernstein Vazirani

- We'll see a different function (one bit output, but on more input bits) where you get an even bigger advantage by querying in superposition.
- Please read 2.2-2.4