Introduction To Quantum Computing

PHYS/CSCI 3090

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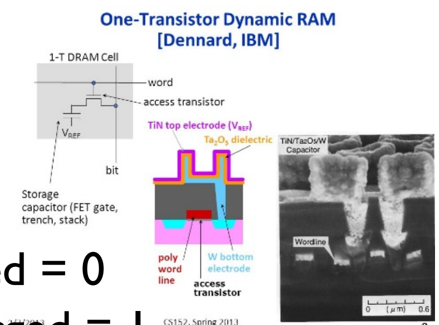
Come see us!

- Alexandra Kolla/ Graeme Smith: Friday 3:00-4:00 pm, JILA X317.
- Ariel Shlosberg: Tu/Th 2:00-4:00pm, DUANG2B90 (physics help room)
- Steven Kordonowy: Th 11am-12pm, ECAE 124.

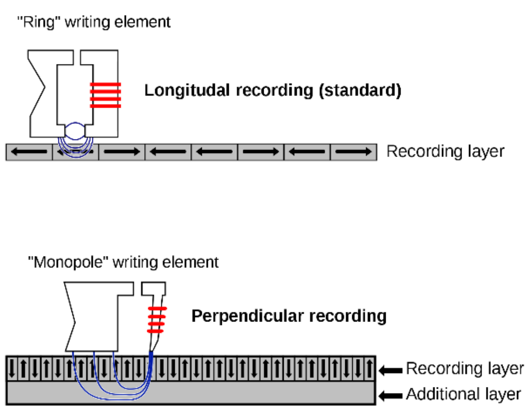
What is a bit?

Anything that can be in one of two states!

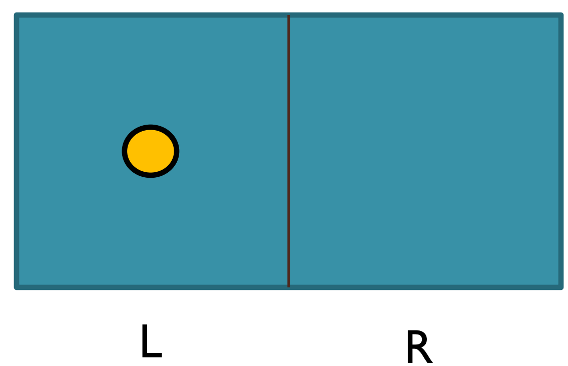
H = 0
T = 1



Charged = 0
Uncharged = 1



U = 0, D = 1



L = 0, R = 1

Representing bit-strings as vectors

- One bit: 0 or 1

  $|0\rangle$ or $|1\rangle$

  vectors: $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ or $|0\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

- Two Bits: 00, 01, 10, 11

$$|00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, |01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

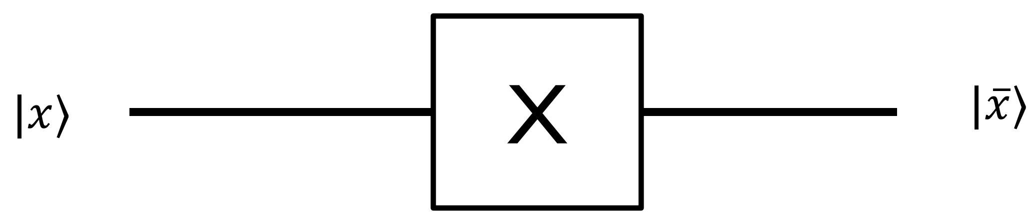
Manipulating one bit

- If someone hands you a bit, there are two ways you can process it to give another bit:

1) Leave it alone
2) Flip it

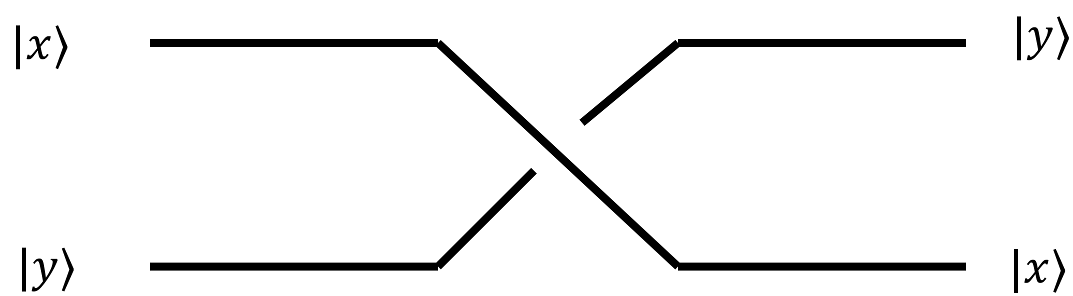
That's it. If we want more interesting computations, we'd better have more bits.

Flipping a bit: circuit diagram



Swapping Two bits

- Given two bits, what can we do? Swap them!



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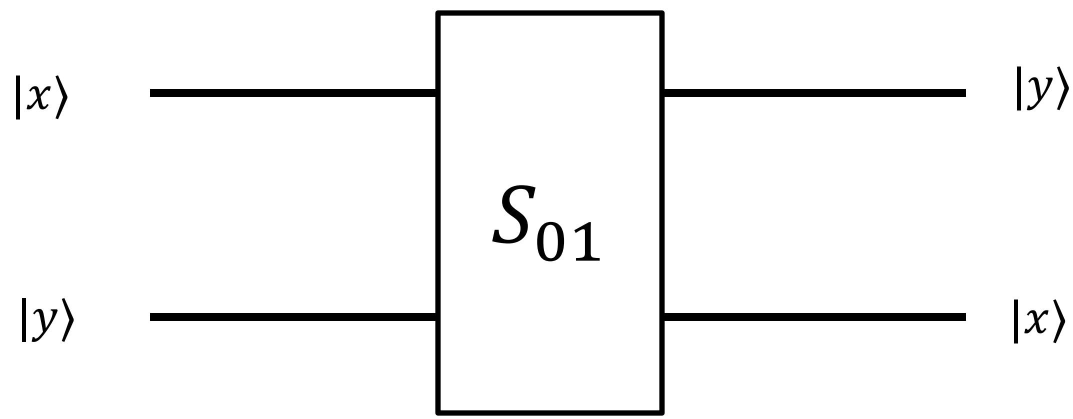


Swapping Two bits

- Given two bits, what can we do? Swap them!

$$S_{01}|x\rangle|y\rangle = |y\,\rangle|x\rangle$$

As a matrix

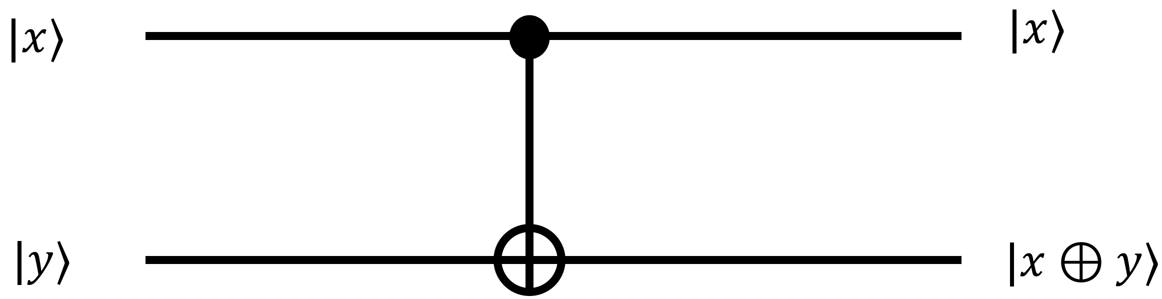
$$S_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Controlled NOT

CNOT: $|x\rangle|y\rangle \rightarrow |x\rangle|x \oplus y\rangle$

where $x \oplus y = (x + y) mod\ 2$

If x = 0, leaves y alone. If x = 1, flips y bit.



Concept Test

CNOT: $|x\rangle|y\rangle \rightarrow |x\rangle|x \oplus y\rangle$

What matrix represents CNOT?

A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Concept Test

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C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Concept Test: solution

- $\begin{bmatrix} & \\ & \\ & \\ & \end{bmatrix} \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix}$ CNOT swaps 10 and 11

00→ 00, 01→ 01, 10→ 11, 11→ 10

The matrix that does this is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Classical circuits: that's it.

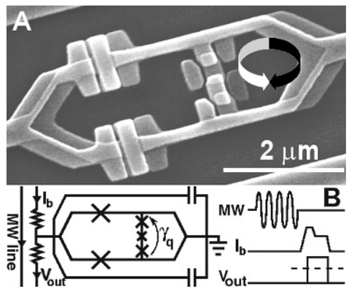
- Now you know about classical circuits, you can go off and study classical computation. Figure out how many gates you need to do stuff, what kind of gates you need, etc.
- We're not going to do that. We're doing to augment our model to include quantum effects. Qubits not bits.

What is a qubit?

- A **quantum** system whose state is a 2 dimensional complex unit vector
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1$$
- A classical bit is just a qubit, but which also satisfies $|\alpha|^2 = 0\ or\ 1$

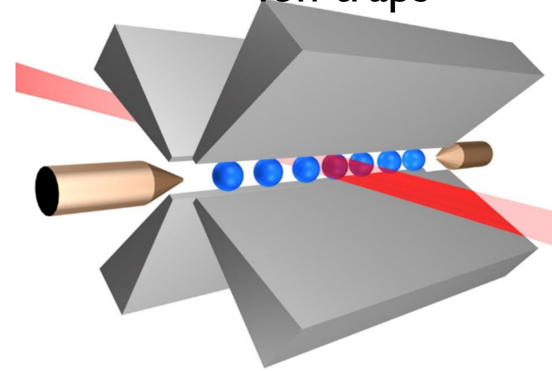
- Actually, all bits are really qubits.
But for some systems, natural physical processes drive an arbitrary state $|\psi\rangle$ to either $|0\rangle$ or $|1\rangle$ really fast. So it's hard to see the quantumness.

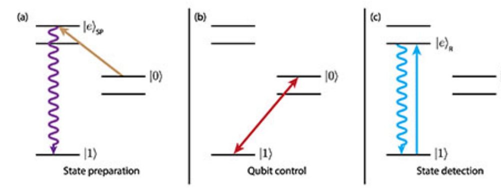
What is a qubit?

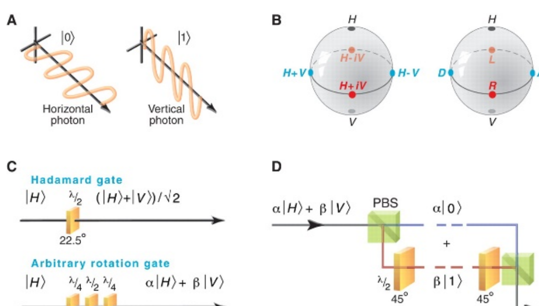


Superconducting circuits

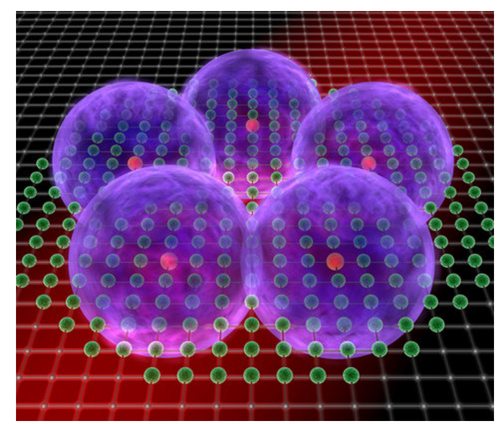
Ion traps







Polarization of a photon



Rydberg states: excited or not

What is a qubit?

- It would take a whole other course (with WAY more prereqs) to understand "which systems make good qubits?"
- For us, it's enough to know: people are getting pretty good at making systems with 10's of qubits right now. Plausible paths to 1,000's and 1,000,000's within 10's of years.
- Our goal is to understand the computational model that quantum theory implies, and better understand what quantum computers can do.

Levels of Abstraction

- I can drive a car. But I don't really know how it works inside (I couldn't fix it).
- I **do** know what happens if I turn the steering wheel or hit the breaks. Good enough for me! To me a car is a machine with 3-5 controls I need to worry about that gets me where I'm going.
- You all know how to use a computer. Many of you have written python code, used mathematica, C++, etc.
- Some of you know what a transistor is, how RAM works, etc. Others don't, but you can still use your computer effectively.

Our Level of Abstraction

- We are in the business of driving qubits, not building them or fixing them.
- For us, a quantum system is just something whose state is a complex unit vector.
- You might find it useful to think of a qubit as the polarization of a photon, two hyperfine states of an atom, etc.
- Actually, this is a very hip and modern (ahistorical) way to learn quantum mechanics---remove all the detailed physics to focus on the essential structure of quantum theory. ("spins first")

Axioms of Quantum Theory

Axiom 1 (states)

The state of a quantum system is a complex unit vector:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad |\alpha|^2 + |\beta|^2 = 1$$

2-dimensional vector: qubit

d-dimensional vector: qudit

Examples: states

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

sort of like "50% 0, 50% 1" but also different

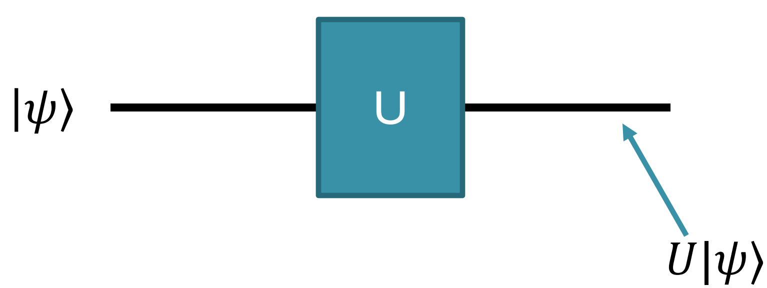
(we'll see more later in measurement axiom)

Axioms of Quantum Theory

Axiom 2 (dynamics)

The evolution of a closed system is described by a unitary matrix $U^t U = I$, where U^t is the conjugate transpose

$$|\psi\rangle \rightarrow U|\psi\rangle$$



Four fantastic unitaries

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathrm{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\mathrm{Y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \mathrm{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

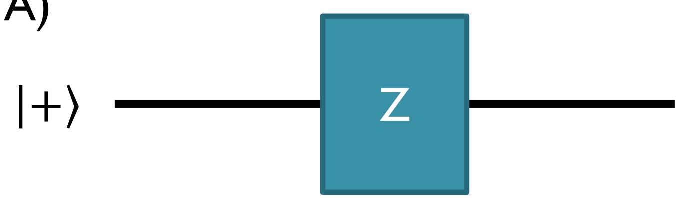
Check: are these unitaries?

$$Y^t Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

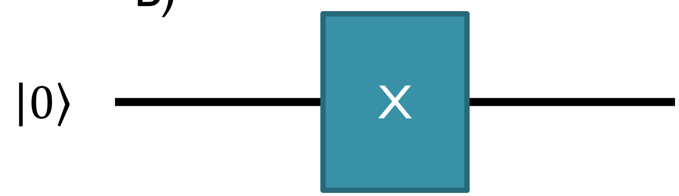
Concept test

Which prepares $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$?

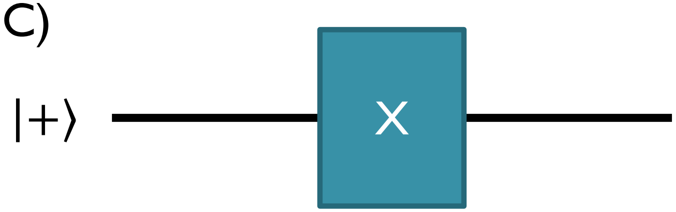
A)



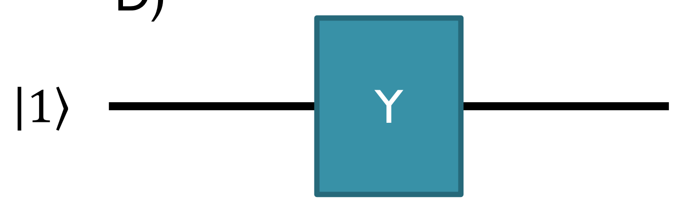
B)



C)



D)



Concept test

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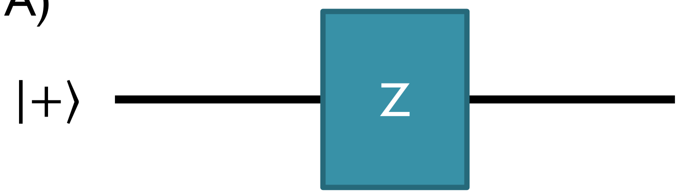
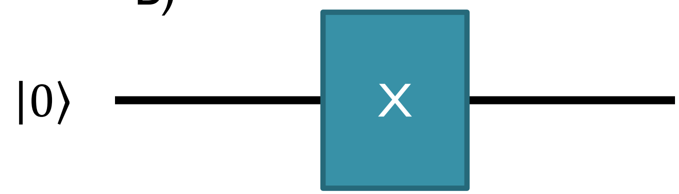
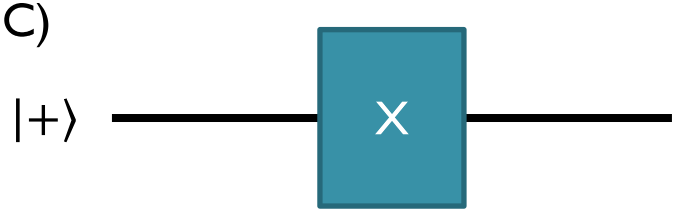


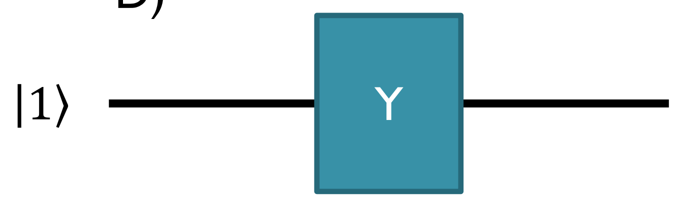
B)



C)



D)



$$X|+\rangle = X\left(\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \left(\frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle\right) = |+\rangle$$

Hadamard

- $H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

- $H|0\rangle = |+\rangle$ and $H|1\rangle = |-\rangle$

So far

Axiom 1: states are complex unit vectors

Axiom 2: evolution is multiplication by unitary

What's missing?

So far

Axiom 1: states are complex unit vectors

Axiom 2: evolution is multiplication by unitary

What's missing?

So far we have a theory of vectors that you can rotate. The state is a collection of complex numbers (so an infinite number of bits).

The next axiom tells us that the information we can extract about the state of a system is very limited.

Axioms of Quantum Theory

Axiom 3 (Measurements)

Can "measure" a system in any basis for its state space.

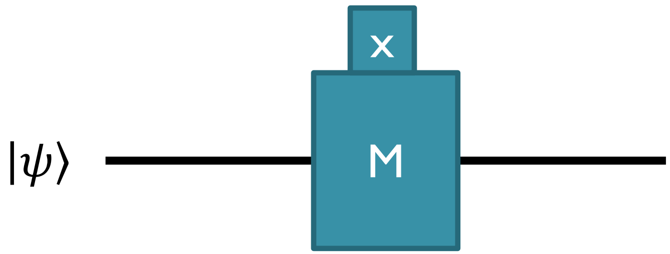
If you measure

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

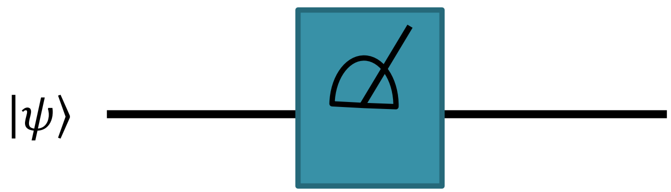
in the basis $\{|0\rangle, |1\rangle\}$, you get an outcome "x" with probability $|\alpha_x|^2$.

Furthermore, the state of the system "collapses" to $|x\rangle$.

Circuit diagram



Book's notation



More common notation

Axioms of Quantum Theory

Axiom 4 (Composite Systems)

If

A has a state in span($|a\rangle$), $a = 1 \ldots d_a$

B has a state in span($|b\rangle$), b$= 1 \ldots d_b$

AB has a state in span($|a\rangle \otimes |b\rangle$)

Tensor products

$$|\psi_A\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$
$$|\phi_B\rangle = \beta_0|0\rangle + \beta_1|1\rangle$$

$$|\psi_A\rangle \otimes |\phi_B\rangle =$$

$$\alpha_0\beta_0|0\rangle \otimes |0\rangle + \alpha_0\beta_1|0\rangle \otimes |1\rangle + \alpha_1\beta_0|1\rangle \otimes |0\rangle + \alpha_1\beta_1|1\rangle \otimes |1\rangle$$

$$= \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle$$

$$= \begin{bmatrix} \alpha_0\beta_0 \\ \alpha_0\beta_1 \\ \alpha_1\beta_0 \\ \alpha_1\beta_1 \end{bmatrix}$$

Today

- Two bit gates
- Quantum bits (qubits)
- a qubit is a system that obeys Axioms
    - State is a complex unit vector
    - Evolution is multiplication by unitary
    - Measurement is probabilistic, "collapses" state
    - Tensor product for combining systems

For Friday

Read the rest of Chapter 1.

On Friday, we will talk more about gates and measurements on multiple qubit systems.

We'll look at more examples of one and two qubit gates that will be useful for studying computation.

Colloquium Today 4pm, G1B20

- **"Quantum Clocks With Ultracold Molecules"**
- Presenter: Tanya Zelevinsky, Columbia University
- Superpositions ($|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$) of vibrational states of a molecule to make a clock.