Quantum Communication: The Setting.

Charlie makes a state \( |\Psi\rangle \) and hands it to Alice. She wants to get it over to Bob, but she only has a classical channel. Charlie sends Alice \( |\Psi\rangle \) and in fact sends \( |\Psi\rangle = a |\uparrow\rangle + b |\downarrow\rangle \).

Alice sends this out to Bob via a classical channel. In fact, she can't do it perfectly.

In fact, \( \langle |\Psi| |\Psi\rangle \rangle^2 = \frac{2}{3} \).

Teleportation: if they share an EPR pair, measure.

Charlie sends Alice \( |\Psi\rangle \). In fact, which \( |\Psi\rangle \) did we get? \( b_1 = 3 \) or \( b_2 = 3 \)? Bob and Alice.
Let \( |\psi\rangle = a |0\rangle_C + b |1\rangle_C \)

\[
|\psi\rangle_C |\phi\rangle_{AB} = (a |0\rangle_C + b |1\rangle_C) \left( \frac{1}{\sqrt{2}} \left[ |00\rangle_{AB} + |11\rangle_{AB} \right] \right)
\]

\[
= \frac{1}{\sqrt{2}} \left[ a |00\rangle_{CA} + a |01\rangle_{CA} + b |10\rangle_{CA} + b |11\rangle_{CA} \right]
\]

\[
= \sum |\frac{1}{\sqrt{2}} (|\phi_0\rangle + |\phi_1\rangle) |0\rangle_B + \frac{1}{2} a \sum |\phi_1\rangle |0\rangle_B + \frac{1}{2} b \left( |\phi_0\rangle - |\phi_1\rangle \right) |1\rangle_B
\]

\[
\]

Write First:

\[
|\phi_{00}\rangle = \frac{1}{\sqrt{2}} \left[ |00\rangle + |11\rangle \right]
\]

\[
|\phi_{01}\rangle = \frac{1}{\sqrt{2}} \left[ |01\rangle + |10\rangle \right]
\]

\[
|\phi_{10}\rangle = \frac{1}{\sqrt{2}} \left[ |00\rangle - |11\rangle \right]
\]
\[
\frac{1}{2} \phi_{00} C_A \left( a_1 \phi_B + b_1 \phi_B \right) \\
+ \frac{1}{2} \phi_{01} C_A \left( a_1 \phi_B + b_1 \phi_B \right) \\
+ \frac{1}{2} \phi_{10} C_A \left( a_2 \phi_B^* - b_2 \phi_B^* \right) \\
+ \frac{1}{2} \phi_{11} C_A \left( a_1 \phi_B - b_1 \phi_B \right)
\]

When Alice measures \( C_A \), she collapses Bob to one of
\( 0, 1, X(\psi_B), XZ(\psi_B), Z(\psi_B), Z(\psi_B) \).

Tells Bob which one, and he can fix it up.
Next class:

There's a lot of randomness in QM. Measurement collapse, etc.

Would be nice if there was a more fundamental theory: maybe randomness in quantum mechanics is due to our ignorance of some underlying state that we don't know how to measure yet.

NOPE: LHUS inconsistent with QM (G-HZ - read 6.6)
Einstein Locality (aka Local Hidden Variables)

Einstein did not like entanglement.

\[ A \quad B \]

Alice can do a measurement that instantaneously changes the state of Bob. Can't send info but still seemed "iffy" to him. Einstein thought this meant QM was incomplete.

Wanted Local Realism:

A & B spacelike separated. Then, in a complete description of physical reality, an action on A cannot modify the description of B.

Entanglement violates this. Therefore QM is not complete.

What did Einstein want? Hidden variable theory (in today's language)
How about entangled states?

Einstein's idea: if Alice measures her half, gets more information about Bob's hidden variable (Y), which is why she updates her description of the state (but it's underlying value doesn't change).

Friday: no such theory can explain

E.g. given |0⟩, if we measure it in the (|θ+⟩ = \cos(θ)|0⟩ + \sin(θ)|1⟩)

|θ−⟩ = −\sin(θ)|0⟩ + \cos(θ)|1⟩

Pθ(|θ+⟩) = \cos^2(θ).

Suppose |0⟩ was more fundamentally