Last Class: Dense Coding

Sending classical inf.

1) without ent~~g~~ ~~tgnt~~ sending 1 qubit can: communicate 1 bit and no more

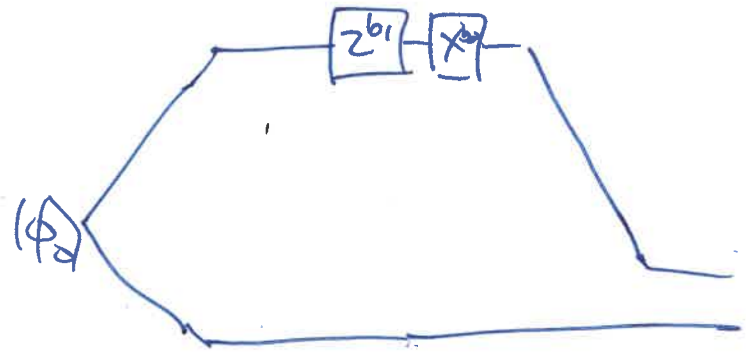
2) with ent ($|\phi_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$) sending

$|\phi_0\rangle_{AB} = |\phi_0\rangle$

$|\phi_1\rangle_{AB} = (X \otimes I)|\phi_0\rangle$

$|\phi_2\rangle_{AB} = (ZX \otimes I)|\phi_0\rangle$

$|\phi_3\rangle_{AB} = (Z \otimes I)|\phi_0\rangle$



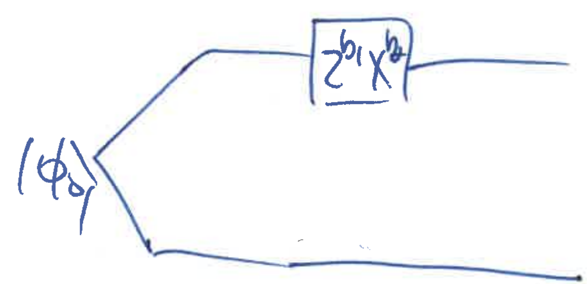
one of $|\phi_i\rangle$

how to distinguish $|\phi_i\rangle$'s?

~~1st how to make them~~

$\cancel{|\phi_{b_1 b_2}\rangle = (Z^{b_1} X^{b_2}) \otimes I |\phi_0\rangle}$

$|\phi_{b_1 b_2}\rangle =$



Properties of CNOTs:

Why? $\quad CNOT = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$

$$\Rightarrow (CNOT)(Z\otimes I) = (|0\rangle\langle 0| \otimes I)(Z\otimes I) + (|1\rangle\langle 1| \otimes X)(Z \otimes I)$$

$$= (|0\rangle\langle 0| \otimes I)\cancel{Z} - |1\rangle\langle 1| \otimes X$$

$$(Z\otimes I)(CNOT) = (Z \otimes I)($$

$\boxed{3}$

Activity

Activity

$$LHS = \cancel{(|0\rangle\langle 0| \otimes I) + |1\rangle\langle 1| \otimes}$$

$$(|0\rangle\langle 0| \otimes I)(X \otimes I) + (|1\rangle\langle 1| \otimes X)(X \otimes I)$$

$$= |0\rangle\langle 1| \otimes I + |1\rangle\langle 0| \otimes X$$

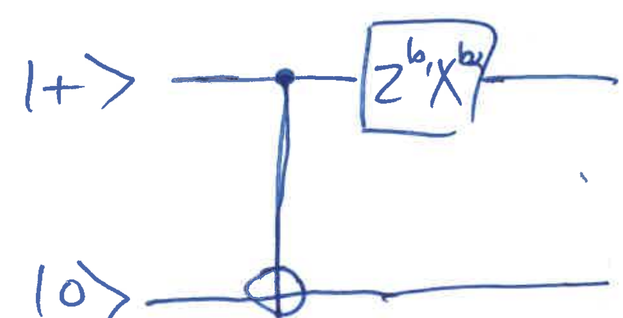
$$RHS = (X \otimes X)(|0\rangle\langle 0| \otimes I) + (X \otimes X)(|1\rangle\langle 1| \otimes X)$$

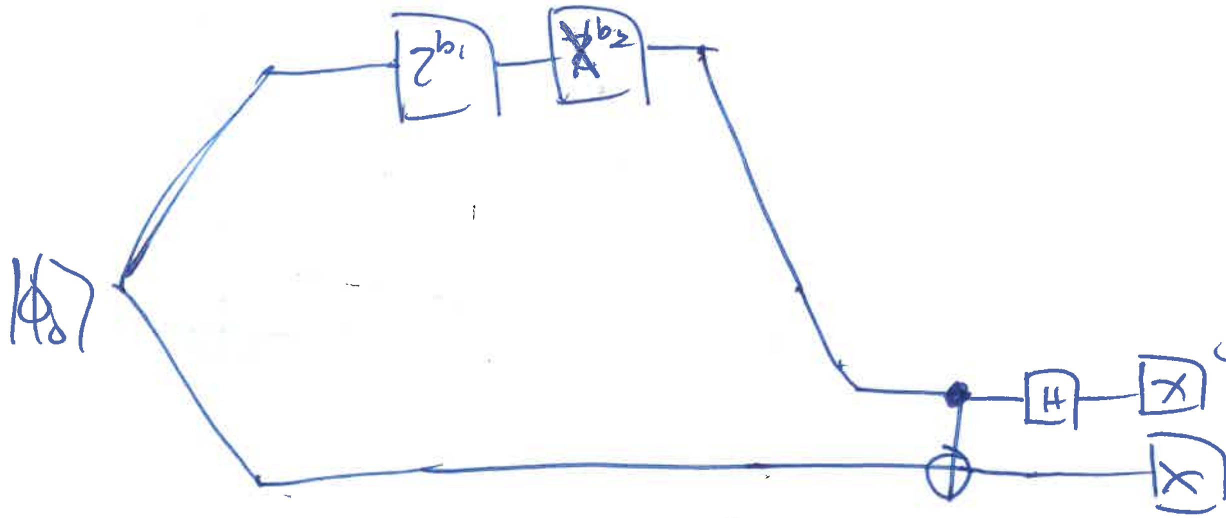
$$= |1\rangle\langle 0| \otimes X + |0\rangle\langle 1| \otimes I$$

4

Another way to make $|\phi_{b_1,b_2}\rangle$:

$\Rightarrow$ $|\phi_{b_1,b_2}\rangle =$

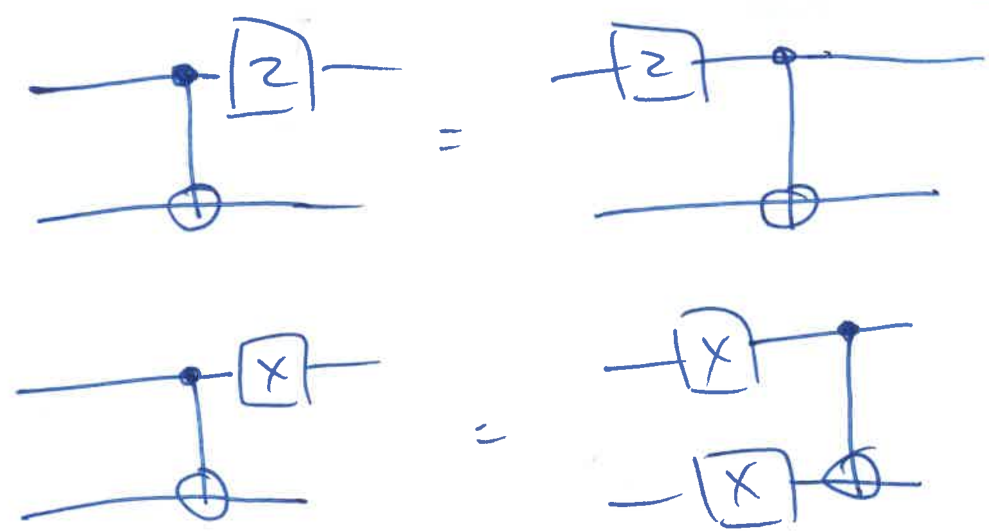




$\{|0\rangle, |1\rangle\}$ measurements.

8

Aside:



start with $|\psi_1\rangle \otimes |\psi_2\rangle$ unitary preserves

This entanglement is useful. Activity.

Can we recognize it?

Which is entangled?

$$\frac{1}{2}\left[|00\rangle + |10\rangle + |01\rangle + |11\rangle\right] = |+\rangle|+\rangle$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right) = \frac{1}{\sqrt{2}}\left[\left(|0\rangle+|1\rangle\right)|0\rangle + \left(|0\rangle - |1\rangle\right)\otimes|1\rangle\right]$$

$$= \frac{1}{2}\left[|00\rangle + |10\rangle + |01\rangle - |11\rangle\right]$$

can't make ent locally.

b/c, $(U|\psi\rangle)\otimes(V|\psi\rangle)$ still product. $[|0$

SVD

$$|\psi\rangle_{AB} = \sum_{ij} \alpha_{ij} |i\rangle|j\rangle$$

$$A = U \Sigma V^{+}$$

let $A_{ij} = \alpha_{ij}$

$$= \begin{pmatrix} \sqrt{\sigma_1} & & \\ & \ddots & \\ & & \sqrt{\sigma_d} \end{pmatrix} \geq 0$$

(V^{+}).

$$\sum_k \sqrt{\sigma_k} \left(\sum_i U_{ik} |i\rangle\right) \otimes \left(\sum_j (V^\dagger)_{kj} |j\rangle\right)$$

$$= \sum_k \sqrt{\sigma_k} \; (U|k\rangle) \otimes (V^*|k\rangle)$$

$$= \sum_k \sqrt{\sigma_k} \; |\tilde{k}\rangle \otimes |\tilde{k}\rangle$$

entangled iff more than one σ_k is $\neq 0$.

$\boxed{\text{GO TO EXAMPLE}}$

#2

~~Next class:~~

~~More apps of entanglement.~~

~~6.5 – Teleportation~~