1. CNOT (/25).

1. Consider a CNOT gate whose control qubit (first input) is \( \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \) and target qubit (second input) is \( \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \). What are the states of the two output qubits?

2. Repeat when the control qubit is \( |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \).

3. Describe the action of the CNOT gate on the first qubit when the target qubit is \( \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \).

4. Describe the action of the CNOT gate on the first qubit when the target qubit is \( \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \).

5. This brings us to an important concept. We can consider what the action of a quantum gate looks like when we write the qubits in a different basis. Show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.
2. Series of CNOT (/25). Write the unitary matrix describing the quantum circuit shown below:

3. 3 Qubits (/25). Consider a system of 3 qubits. If we apply a CNOT gate to the first two qubits and a phase flip Z to the last qubit, write the unitary transformation that is applied to the composite system.