1. Five bit repetition code. /50

The five bit (classical) repetition code has code words 00000 and 11111.

(a) What is the parity check matrix for this code? Hint: it is a 4 × 5 matrix.

(b) Draw an encoding circuit for this code.

(c) If each bit is flipped independently with probability \( p \), what decoding strategy should you use?

(d) What is the logical error rate after decoding (assuming the decoder and error rate \( p \) from part (c)).

2. The stabilizers of Bell states. /50

Recall the Bell state \(|\phi_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\).

(a) \(|\phi_{00}\rangle\) is a simultaneous +1 eigenstate of two commuting two-qubit Pauli matrices, \(P_1\) and \(P_2\). What are \(P_1\) and \(P_2\)?

(b) Show that the other Bell states, \(|\phi_{x_1x_2}\rangle = (I \otimes X^{x_1}Z^{x_2})|\phi_{00}\rangle\) are eigenstates of the same \(P_1\) and \(P_2\) and report their eigenvalues.