CSCI 6214— Randomized Algorithms— FALL 2021
Problem Set #2

Problem 1:

This problem shows that Markov's inequality is as tight as it could possibly be.

- Given a positive integer k, describe a random variable X the assumes only non-negative values such that $Pr[X \geq kE[X]] = \frac{1}{k}$
- Can you give an example that shows that Chebyshev's inequality is tight? If not, explain why not.

Problem 2:

Generalize the median-finding algorithm to find the k-th largest item in a set of n items for any given value of k. Prove that your resulting algorithm is correct, and bound its running time. Bonus: bound the probability of failure of this algorithm.

Problem 3:

- Suppose we have an algorithm that takes as input a string of n bits. We are told that the expected running time is $O(n^2)$ if the input bits are chosen independently and uniformly at random. What can Markov's inequality tell us about the worse-case running time of this algorithm on inputs of size n?
- Let X be the sum of Bernoulli random variables, $X = \sum_{i=1}^{n} X_i$. The X_i need not be independent. Show that

$$E[X^2] = \sum_{i=1}^{n} Pr[X_i = 1]E[X|X_i = 1]$$

- Use the equation above to provide another derivation of the variance of a binomial random variable with parameters n and p.