CSCI5444: Intro to Theory of Computation, Fall 2019 HW 9 -Take home Final (due by the end of fall semester 2019)

This homework contains three problems. **Read the instructions for submitting homework on the course webpage**.

Collaboration Policy: For this homework, each student should work independently and write up their own solutions and submit them.

Read the course policies before starting the homework.

- Homework 9 servers as a take-home final and tests your familiarity with the material of the last two weeks of classes.
- Each student must submit individual solutions for these homework problems.
- Please carefully read the course policies on the course web site. If you have any questions, please ask in lecture, or by email. In particular:
  - Please submit all your homework questions in PDF form, preferably typed up in LaTeX.
  - Submit separately stapled solutions, one for each numbered problem, with your name and ID clearly printed on each page.
  - You may use any source at your disposal: paper, electronic, human, or other, but you must write your solutions in your own words, and you must cite every source that you use (except for official course materials). Please see the academic integrity policy for more details.
  - No late homework will be accepted for any reason.
  - Answering "I don't know" to any (non-extra-credit) problem or subproblem, on any homework or exam, is worth 25% partial credit.
  - Unless explicitly stated otherwise, every homework problem requires a proof.

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1 Required problems

1. (25 PTS.)
Let A be an oracle such that when input a boolean formula ϕ in 3CNF, $A(\phi)$ gives a 2-approximation to the number of satsifying assignments to ϕ. Given 10 3CNF formulas $\phi_1, \cdots, \phi_{10}$, describe a polynomial time algorithm that uses only a single query to A to decide which of $\phi_1, \cdots, \phi_{10}$ are satisfiable. (Note: You may assume that A can operate on any boolean formulas of any sort, so that you dont have to worry about coming up with a 3CNF formula to give to A. Getting to a 3CNF formula is fairly tricky.)

2. (50 PTS.)
(GHZ paradox.) Consider the following game: Alice, Bob, and Charlie are given input bits a, b, and c respectively. They are promised that $a \oplus b \oplus c = 0$. Their goal is to output bits x, y, and z respectively such that $x \oplus y \oplus z = a \vee b \vee c$. They can agree on a strategy in advance but cannot communicate after receiving their inputs.

(a) Show that in a classical universe, there is no strategy that enables them to win this game with certainty.

(b) Suppose Alice, Bob, and Charlie share the entangled state

$$1/2(|000\rangle - |011\rangle - |101\rangle - |110\rangle)$$

Show that now there exists a strategy by which they can win the game with certainty. [Hint: Have each player measure its qubit in one basis if its input bit is 0, or in a different basis if its input bit is 1.]

3. (25 PTS.)
(Conjugating CNOT.) Show that if you apply Hadamard gates to qubits A and B, followed by a CNOT gate from A to B, followed by Hadamard gates to A and B again, the end result is the same as if you had applied a CNOT gate from B to A. The above illustrates a principle of quantum mechanics you have heard about: that any physical interaction by which A influences B can also cause B to influence A (so for example, it is impossible to measure a particles state without affecting it).