Convolutional Neural Networks (CNNs)

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https://dannagurari.colorado.edu/course/neural-networks-and-deep-learning-spring-2025/

Review

- Last class:
 - Model capacity: how it affects learning
 - Regularization: learning methods for improving model generalization
 - Hyperparameter selection: tuning to improve model performance
 - Programming tutorial
- Assignments (Canvas):
 - Problem set 1 grades are out
 - Review session will be held at 4pm
 - All regrade requests must be emailed to our TA, Nick Cooper (a comment in Canvas is not sufficient)
 - Problem set 2 due earlier today
 - Lab assignment 1 due a week from Thursday (in 9 days)
- Questions?

Today's Topics

- History of Convolutional Neural Networks (CNNs)
- CNNs Convolutional Layers
- CNNs Pooling Layers
- Pioneering CNN model: LeNet

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Inspiration: Neural Networks for Spatial Data

• Data where the order matters; e.g.,



Inspiration: Historical Context





https://braintour.harvard.edu/archives/portfolio-items/hubel-and-wiesel

Experiment Set-up:



https://www.esantus.com/blog/2019/1/31/convol utional-neural-networks-a-quick-guide-for-newbies Key Finding: neurons respond strongly only when light is shown in certain orientations



https://www.youtube.com/watch?v=OGxVfKJqX5E&ab_channel=RyanAbbott

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V1 physiology: direction selectivity



https://www.cns.nyu.edu/~david/courses/ perception/lecturenotes/V1/lgn-V1.html

Key Idea: cells are organized as a hierarchy of feature detectors, with higher level features responding to patterns of activation in lower level cells

featural hierarchy



https://bruceoutdoors.files.wordpress.com/2017/08/hubel.jpg



https://neuwritesd.files.wordpress.com/2015/10/visual_stream_small.png

Key Ingredients of CNNs





http://personalpage.flsi.or.j p/fukushima/index-e.html

"In this paper, we discuss how to synthesize a neural network model in order to endow it an ability of pattern recognition like a human being... the network acquires a similar structure to the hierarchy model of the visual nervous system proposed by Hubel and Wiesel."

- Fukushima, Neocognitron: A Self-organizing Neural Network Model for a Mechanism of Pattern Recognition Unaffected by Shift in Position. Biological Cybernetics, 1980.

Cascade of simple and complex cells identified by Hubel and Weisel:

Model with alternating convolutional layers and pooling layers:





Complex cells fire when any part of the local region is the desired pattern



Fukushima, 1980



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Motivation: Fully-Connected Layers Are Limited



Each node provides input to each node in the next layer

- Assume 3-layer model with 100 nodes, 100 nodes, and then 2 nodes
 - e.g., how many weights are in a 640x480 grayscale image?
 - 640x480x100 + 100x100 + 100x2 = 30,730,200
 - e.g., how many weights are in a 3.1 Megapixel grayscale image (2048X1536)?
 - 2048x1536x100 + 100x100 + 100x2 = 314,583,000

Motivation: Fully-Connected Layers Are Limited

Concern: many model parameters...

- increases chance of overfitting
- increases memory/storage requirements
- increases computational expense

Idea: Convolutional Layers



Fully-connected:

Convolutional:



each node receives input only from a small neighborhood in previous layer (and there is parameter sharing)

https://qph.fs.quoracdn.net/main-qimg-2e1f0071ca9878f7719ed0ea8aeb386d

Fully-Connected vs Convolutional Layers



Fully-connected:

Convolutional:

https://qph.fs.guoracdn.net/main-gimg-2e1f0071ca9878f7719ed0ea8aeb386d

Convolutional layers dramatically

reduce number of model parameters!

Key Ingredient 1: Convolutional Layers



Recall: Image Representation (8-bit Grayscale)



157	153	174	168	150	152	129	151	172	161	156	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	24	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	216	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
206	174	156	252	236	231	149	178	228	43	96	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	216
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

https://ai.stanford.edu/~syyeung/cvweb/tutorial1.html

Key Ingredient 1: Convolutional Layers





• A filter specifies the function for how to combine neighbors' values

https://www.jefkine.com/general/2016/09/05/backpropagation-in-convolutional-neural-networks/



Slides filter over the matrix and computes matrix multiplication



Slides filter over the matrix and computes matrix multiplication



Slides filter over the matrix and computes matrix multiplication



Slides filter over the matrix and computes matrix multiplication



Product = 1*1 + 1*0 + 1*1 + 0*0 + 1*1 + 1*0 + 0*1 + 0*1 + 0*0 + 0*0 + 1*1Product = 4



Filter

1	0	1
0	1	0
1	0	1

4	?	?
?	?	?
?	?	?





1	0	1
0	1	0
1	0	1

4	3	?
?	?	?
?	?	?



Filter

1	0	1
0	1	0
1	0	1

4	3	4
?	?	?
?	?	?





1	0	1
0	1	0
1	0	1

4	3	4
2	?	?
?	?	?





1	0	1
0	1	0
1	0	1

4	3	4
2	4	?
?	?	?




1	0	1
0	1	0
1	0	1

4	3	4
2	4	3
?	?	?



Filter

1	0	1
0	1	0
1	0	1

4	3	4
2	4	3
2	?.	?



Filter

1	0	1
0	1	0
1	0	1

4	3	4
2	4	3
2	3	?



Filter

1	0	1
0	1	0
1	0	1

4	3	4
2	4	3
2	3	4

Convolutional Layer

- Many neural network libraries use "convolution" interchangeably with "cross correlation"; these are technically different
- Examples in these slides show the "cross-correlation" function



Way to Interpret Neural Network





Convolutional Layer: Parameters to Learn

- For shown example, how many weights must be learned?
 - 4 (red, blue, yellow, and green values)
- If we instead used a fully connected layer, how many weights would need to be learned?
 - 36 (9 turquoise nodes x 4 magenta nodes)



Convolutional Layer: Parameters to Learn

Neocognitron hard-coded filter values... filter values are learned for CNNs





Filter





• e.g.,

Filter

Visualization of Filter

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0



Filter Overlaid on Image





• e.g.,



Filter

*

Weighted Sum = ?

Weighted Sum = (50x30) + (20x30) + (50x30) + (50x30) + (50x30) + (50x30)

Weighted Sum = 6600 (Large Number!!)

Filter Overlaid on Image



Image

• e.g.,

0	0	0	0	0	0	0
0	40	0	0	0	0	0
40	0	40	0	0	0	0
40	20	0	0	0	0	0
0	50	0	0	0	0	0
0	0	50	0	0	0	0
25	25	0	50	0	0	0

Filter

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

≭

Weighted Sum = ?

Weighted Sum = 0 (Small Number!!)

This Filter is a Curve Detector!

• e.g.,





Filter Overlaid on Image (Big Response!)



Filter Overlaid on Image (Small Response!)



	Filter	Feature Map
Identity	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	
Edge detection	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	Sold States

	Filter	Feature Map	
Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$		
Box blur (normalized)	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$		
Gaussian blur (approximation)	$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$		

https://ujjwalkarn.me/2016/08/11/intuitive-explanation-convnets/



Demo: http://beej.us/blog/data/convolution-image-processing/

Key Ingredient 1: Convolutional Layers



Can choose filters of any size to support feature learning!

Key Ingredient 1: Convolutional Layers



Filtered results are passed, with a bias term, through an activation function to create **activation/feature maps**

Key Ingredient 1: Convolutional Layers



Can have multiple filters (with a unique bias parameter per filter)

Key Ingredient 1: Convolutional Layer Summary



Neural networks learn values for all filters and biases in all layers

How Filters Are Applied to Multi-Channel Inputs



https://www.geeksforgeeks.org/matlab-rgb-image-representation/

How Filters Are Applied to Multi-Channel Inputs



 $n_H x n_W x n_C = 6 x 6 x 3$

Number of channels in a filter matches that of the input

https://indoml.com/2018/03/07/student-notes-convolutional-neural-networks-cnn-introduction/

Convolutional Layers Stacked

Can then stack a sequence of convolution layers; e.g.,



http://cs231n.stanford.edu/slides/2019/cs231n_2019_lecture05.pdf

Convolutional Layers Stacked

Can then stack a sequence of convolution layers, which leads to identifying patterns in increasingly larger regions of the input (e.g., pixel) space:



https://www.deeplearningbook.org/contents/convnets.html

Convolutional Layers Stacked

Can then stack a sequence of convolution layers, which leads to identifying patterns in increasingly larger regions of the input (e.g., pixel) space and mimicking vision system:

featural hierarchy

Higher level features are constructed by combining lower level features



https://bruceoutdoors.files.wordpress.com/2017/08/hubel.jpg

Problem #1: Input Shrinks

Why do the dimensions shrink with each convolutional layer?



Information is lost around boundary of the input!

http://cs231n.stanford.edu/slides/2019/cs231n_2019_lecture05.pdf

Solution: Control Output Size with Padding

• **Padding**: add values at the boundaries



https://software.intel.com/en-us/node/586159

Problem #2: Computation Expensive



Many computations to slide filter over every point in the matrix and compute multiplications

https://people.eecs.berkeley.edu/~jrs/189/lec/cnn.pdf

Idea: Reduce Computations with Stride

- Stride: how many steps taken spatially before applying a filter
 - e.g., 2x2











http://deeplearning.net/software/theano/tutorial/conv_arithmetic.html

Convolutional Layer Summary

- Hyperparameters:
 - Number of convolutional layers
 - For each layer, number of filters and their dimensions, padding type, & stride
- Model will learn values for:
 - Weights
 - Biases

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• Max-pooling: partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk

		- p	
1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

Single depth slice

max pool with 2x2 filters and stride 2



http://cs231n.github.io/convolutional-networks/#pool

• Max-pooling: partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk

		- p	
1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

Single depth slice

max pool with 2x2 filters and stride 2

6	8
3	4

http://cs231n.github.io/convolutional-networks/#pool

• Max-pooling: partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk



http://cs231n.github.io/convolutional-networks/#pool

- Max-pooling: partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk
- Average-pooling: partitions input into a set of non-overlapping rectangles and outputs the average value for each chunk

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

Single depth slice

Avg pool with 2x2 filters and stride 2



http://cs231n.github.io/convolutional-networks/#pool

- Max-pooling: partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk
- Average-pooling: partitions input into a set of non-overlapping rectangles and outputs the average value for each chunk

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

Single depth slice



3.25

2

5.25

2

Avg pool with 2x2 filters

and stride 2
Pooling Layer: Summarizes Neighborhood

- Max-pooling: partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk
- Average-pooling: partitions input into a set of non-overlapping rectangles and outputs the average value for each chunk
- And many more pooling options
 - e.g., listed here https://pytorch.org/docs/stable/nn.html#pooling-layers

Pooling for Multi-Channel Input



Pooling is applied to each input channel separately

https://indoml.com/2018/03/07/student-notes-convolutional-neural-networks-cnn-introduction/

Pooling Layer: Benefits

- Reduces memory requirements
- Reduces computational requirements

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Historical Context: Inspiration



By end of 1990s, LeNet read over 10% of checks in North America with millions every month

MNIST Dataset Challenge

- Goal: classify digit as 0, 1, ..., or 9
- Source: images collected by NIST from a total of 500 Census Bureau employees and high school students
- Dataset: 60,000 training and 10,000 test examples, pre-processed to be centered and same dimension; writers were different in the two sets
- Evaluation metric: accuracy (% correct)

0	0	0	0	0	0	0	0	D	٥	0	0	0	0	0	0
1	l	١	١	١	1	1	(/	1	١	1	1	١	1	1
2	ູ	2	2	ð	J	2	2	ደ	2	2	2	2	2	2	ス
3	3	3	3	3	3	3	3	3	3	3	З	3	3	3	З
4	4	٤	ч	4	4	Ч	ч	4	4	4	4	4	ų	¥	4
5	5	5	5	5	\$	5	5	5	5	5	5	5	5	5	5
6	G	6	6	6	6	6	6	Ь	6	Q	6	6	6	6	b
Ŧ	7	7	٦	7	7	ч	7	2	7	7	7	7	7	7	7
8	B	8	8	8	8	8	8	8	8	8	8	8	8	8	8
9	૧	9	9	9	ዋ	٩	9	٩	η	٩	9	9	9	9	9



tanh is used as the activation function

Multi-layer neural network



Multi-layer neural network



How many filters are between the input and hidden layer 1?



Training Procedure



Repeat until stopping criterion met:

- Forward pass: propagate training data through model to make predictions
- 2. Error quantification: measure error of the model's predictions on training data using a loss function
- 3. Backward pass: calculate gradients to determine how each model parameter contributed to model error
- 4. Account for weight sharing by using average of all connections for a parameter
- 5. Update each parameter using calculated gradients

Baydin et al. Automatic Differentiation in Machine Learning: a Survey. 2018

Training Procedure

Still descend an error surface, E, based on the chosen objective function (cross entropy loss)



Repeat until stopping criterion met:

- Forward pass: propagate training data through model to make predictions
- 2. Error quantification: measure error of the model's predictions on training data using a loss function
- **3. Backward pass**: calculate gradients to determine how each model parameter contributed to model error
- 4. Account for weight sharing by using average of all connections for a parameter
- 5. Update each parameter using calculated gradients

Training Procedure

Gradient computed for all values in all convolutional filters (i.e., model weights) as well as all bias terms

(covered in Section 6.3 of Kamath book and https://www.jefkine.com/general/2016/09/05/bac kpropagation-in-convolutional-neural-networks/) Repeat until stopping criterion met:

- Forward pass: propagate training data through model to make predictions
- 2. Error quantification: measure error of the model's predictions on training data using a loss function
- 3. Backward pass: calculate gradients to determine how each model parameter contributed to model error
- 4. Account for weight sharing by using average of all connections for a parameter
- 5. Update each parameter using calculated gradients

Yann Lecun. Generalization and network design strategies. Technical Report, 1989

Training Procedure (Key Novelty)



Repeat until stopping criterion met:

- 1. Forward pass: propagate training data through model to make predictions
- 2. Error quantification: measure error of the model's predictions on training data using a loss function
- 3. Backward pass: calculate gradients to determine how each model parameter contributed to model error
- 4. Account for weight sharing by using average of all connections for a parameter
- 5. Update each parameter using calculated gradients

https://www.jefkine.com/general/2016/09/05/backpropagation-in-convolutional-neural-networks/

LeNet Analysis

How many epochs are needed for training to converge?

Why might overfitting not arise with more training?

- Learning rate too large for the model to settle in a local minimum (instead oscillated randomly)



LeNet Analysis

All 82 mislabeled examples (correct answer on left, predicted answer on right):

Why might the model be making mistakes?

- Insufficient representation in the training data
- Ambiguity



Y. Lecun ; L. Bottou ; Y. Bengio ; P. Haffner; Gradient-based learning applied to document recognition; 1998

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