CSCI 7000-005 Computational Complexity
Problem Set 2

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due September 27, 2018

Collaboration Policy: The homework can be worked on in groups of up to 3 students each (2 would be optimal, but 1 and 3 are both accepted).

One submission per team is sufficient. Please write the solution for each of the problems on a separate sheet of paper. Write your team's names and id on each submission and please **staple** all the sheets together.

Submissions should be written in LaTeX, unless your handwriting is indistinguishable from LaTeX.

Homework is due before the end of class, September 13. Only one late homework per person will be allowed. If you submit more than one homework late, you will get no grade for the excess late homeworks.

Problem 1 (25 pts.)

Prove that $\mathcal{ZPP} = \mathcal{RP} \cap \text{co-}\mathcal{RP}$.

Problem 2 (25 pts.)

Prove that if $\mathcal{NP} \subseteq \mathcal{BPP}$ then $\mathcal{NP} = \mathcal{RP}$. Hint: Show a $\mathcal{RP}$ algorithm for 3SAT using the assumption.

Problem 3 (25 pts.)

Prove the Chernoff bounds stated in Theorem 5 that appears in the Randomized Computation lecture notes (https://people.eecs.berkeley.edu/ luca/cs278-08/lecture03.pdf)

Problem 4 (25 pts.)

Recall that $EXP = DTIME(2^{n^{O(1)}})$.

1. Prove that if P=NP, then $\Sigma_k = P$ for all k.
2. (Harder, for 20 pts Extra Credit) Prove that if EXP $\subseteq$ P/Poly, then EXP is in Σ_2.
3. Prove that if P=NP, then EXP $\subsetneq$ P/Poly.

I will give full points to the a solution that proves (1) and then uses (1) and (2) to prove (3). I strongly recommend you at least attempt to prove (2), and if you do prove it, I will give you 20 extra floating points that can be used towards any other homework. I will also give points for partial attempts. The proof of (2) appears in the book, but I want you to attempt to prove it independently.