Problem 1 (25 pts.)

Consider the following decision problem, that we call $U$: we are given in input $(M, x, t, l)$ where $M$ is a Turing machine, $x \in \{0, 1\}^*$ is a possible input, and $t$ and $l$ are integers encoded in unary, and the problem is to determine whether there is a $y \in \{0, 1\}^*, \|y\| \leq l$, such that $M(x, y)$ accepts in at most $t$ steps. Show that $U$ is NP-complete.

Problem 2 (25 pts.)

Define a language $L$ which belongs to SIZE$(O(1))$ and is undecidable.
Problem 3  (25 pts.)

Recall that NEXP is defined by

\[ NEXP = \bigcup_{c} NTIME(2^{n^c}), c \geq 1 \]

Give a definition of NEXP that does not involve non-deterministic Turing machines, analogous to the verifier definition of NP seen in class, and prove that your definition is equivalent to the above definition using non-deterministic Turing machines.

Problem 4  (25 pts.)

Prove that if \( P = NP \), then \( EXP = NEXP \).